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A GENERAL DESCRIPTIVE CHARACTERIZATION OF RETURN GENERATING PROCESS

GETİRİ SÜRECİNİN GENEL BETİMSEL KARAKTERİZASYONU

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ABSTRACT

The purpose of this theoretical paper is to obtain a general form of return generating process, in a way functional for real-life market participants and empirical researchers, by incorporating the findings of behavioral finance and imperfect information models. Conventional models of return generating process leave a large portion of time-series variation in realized returns unexplained, relying on a vague error term, very large compared to what the model can account for. I propose, building on Elton (1999), a general form of return generating process that specifies the components of error term based on findings in the noisy rational expectations and behavioral finance literature. My contribution is to extend the I_t term, added by Elton, into a general term, which I name *Market's Perception* of information, and to characterize it. The form suggested here provides a decisive methodology for testing behavioral theories of under- and overreaction collectively. The suggested methodology requires accurate recognition of information events externally (directly), but is promising to resolve debates on behavioral finance theories and market efficiency.

Keywords: characterizing return generating process, descriptive, behavioral, market's perception of information, *JEL classification:* G14

ÖZET

Bu teorik çalışmanın amacı, getiri sürecini, finansta davranış teorileri ve eksik enformasyon modellerinin bulgularını dahil ederek, piyasa katılımcıları ve ampirik araştırmacılar için işlevsel bir şekilde tanımlamaktır. Mevcut getiri süreci modelleri, gerçekleşen getirilerdeki zaman-değişimleri açıklamasız bırakarak, modelin açıklayabildiği varyasyona göre çok daha büyük ama boş bir hata terimine yüklemektedir. Bu çalışmada Elton (1999)'un önerisi geliştirilerek, getiri sürecinin, hata teriminin bileşenlerini gürültülü rasyonel beklentiler ve finansta davranış teorileri alanlarındaki bulgular ışığında betimleyen genel bir formu önerilmektedir. Çalışmanın literatüre katkısı, Elton tarafından önerilen I_t terimini geliştirerek genel hale getirmek, ve *Piyasanın Enformasyon Algılaması* adı verdiğim bu terimi karakterize etmektir. Burada önerilen form, eksik ve aşırı reaksiyon davranışsal teorilerinin nihai sonuç çıkarılabilecek şekilde test edilmesini mümkün kılacaktır. Önerilen metodoloji, enformasyon olaylarının doğrudan (dışsal) teşhisini gerektirmekle birlikte, davranış teorileri ve piyasa etkinliği konularındaki tartışmaları çözüme kavuşturma potansiyeline sahiptir.

Anahtar Sözcükler: *Getiri sürecinin karakterize edilmesi, betimsel, davranışsal, piyasanın enformasyon algılaması.*

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INTRODUCTION

An ultimate critique against the behavioral finance approach is that "random anomalies cannot replace the basic theory of finance" (see Fama, 1998: 283-306). In this paper, I attempt to overcome this critique by formulating a general form of return generating process that collectively incorporates the findings of behavioral under- and overreactions literature and imperfect information models onto the conventional basic form implied by random walk. An earlier formal attempt to incorporate behavioral properties into finance theory was by Shefrin and Statman (1994: 323-349). Here, however, I follow a more simple descriptive approach, functional for real-life market participants and empirical researchers. This approach will enable decisive empirical tests of whether the occurrences of under- and overreactions are unpredictable random events, or they develop predictably as outcome of conditions specified by behavioral finance theories; and whether they can be modeled, with empirical success, as a function of variables related to information structure and participant characteristics at a point in time.

The conventional form of return generating process is:

$$R_t = E_{t-1}(R_t) + e_t \tag{1}$$

where R_t is the realized return at period t, $E_{t-1}(R_t)$ is expected return at t conditional on information available at $t-1$ and e_t is unexpected return, implied to be i.i.d. (independently identically distributed) by the strict form of market efficiency. This form permits time variation in expected returns but requires error term to be random.

However, this form of return generating process is not consistent with many stylized facts on observed behavior of returns, such as simultaneous short-lag positive and long-lag negative autocorrelation in R_t. Moreover, most models of expected return leave a very large proportion of the variation in actual returns to an unexplained error term[1], offering little functionality for real-life practitioners, and providing little help to resolve the debate on market efficiency and dual hypothesis problem in empirical tests of asset pricing models. These models are especially silent in explaining time-variation in market returns, heavily relying on an error term, very large relative to what the model can account for.

Several researchers, including Haugen and Baker (1996: 401-439), note that abnormal returns are mostly the result of unexpected information

[1] Even the recent models that allow for time-variation in both volatility of returns and reward to volatiliy can explain merely around 7% of variation in realized returns (Li, 1998: 431-446; Priestley, 2001: 1271-1286).

arrivals. Elton (1999: 1199-1201) notes that the common use of average realized returns as a proxy for expected returns in the tests of asset pricing models leads to frequent rejection of these models, and argues that new information arrivals are responsible for these rejections. He provides clear examples of paradoxes where it is not plausible to explain abnormal returns over quite long periods by time-varying risk: It is not plausible to argue, for instance, that US stock market returns in 90's were higher because US markets were more risky in this period. He suggests, as a solution, the following return generating process:

$$R_t = E_{t-1}(R_t) + I_t + \epsilon_t \qquad (2)$$

where I_t is an information event[2]. Elton emphasizes that the variation which results from the I_t term can be far larger than that resulting from $E(R_t)$ and ϵ_t terms, and leads to sampling errors even in long sample periods unless $E(I_t)$ converges to zero.

The inadequacy of Brownian motion to characterize realized returns has been noticed by many researchers. While most of them follow a quantitative approach and develop non-standard versions of Brownian motion (see Corazza and Malliaris, 2002: 65-98 for a review of this literature), we follow a descriptive and qualitative approach to extend Elton's basic form with a synthesis of recent developments in behavioral finance and imperfect information models. Our purpose is to provide a more comprehensive account of what drives actual returns, particularly at high-frequency.

CHARACTERIZING RETURN GENERATING PROCESS: A DESCRIPTIVE APPROACH

Building on Elton (1999: 1200) and adding a behavioral element to allow for inevitable imperfections, I suggest:

$$R_t = E_{t-1}(R_t) + MP_t(I_t) + \epsilon_t \qquad (3)$$

where $E_{t-1}(R_t)$ is the *true* expected return, and $MP_t(I_t)$, a conversion function with time-varying form, stands for *Market's Perception* at time *t*, of the information publicly available at *t*. It converts the information set (i.e.; the incremental difference of the new information over extant information set), as perceived by the representative market participant, into price change implications. This model can be conceptualized as the general form with a special case of perfect information processing where $MP_t(I_t) = I_t$. We

[2] I_t can be operationalized as the log price change required by the new information arriving at time t.

deliberately leave I_t as an unmodeled external variable, which needs to be observed directly from the information flow into the market[3].

We obtain the ultimate form of our return generating process based on the following characterization of financial markets: Since public information is instantaneously reflected into market prices[4], price setting agents (speculators) have little benefit from studying publicly available information set as it will no more be driving prices. Rather, they have great incentives in getting clues of or forecasting the future information set (the set that will be available in the future). Thus, the main driver of market price is changes in beliefs about the future information set I_T ($T > t$) and arrivals of private information that shape these beliefs, rather than the public information. This is a consequence of market participants' adaptation to the semi-strong form of efficiency[5]. Our description is different from perfect information models which hold public information surprises the sole cause of price changes. The difference is most apparent in the tasks of price setting market participants and the resulting inevitable behavioral imperfections, as explained below.

Under conditions where public information is instantaneously reflected in price, the speculators who dominate in the price setting process are evolved, rather than accepting the martingale assumption and passively holding a well-diversified portfolio, to (i) actively search for clues of the future information set, investing in private information sources (ii) form beliefs about the future information set, investing in analysis and forecasting techniques or in experience. As a consequence of their activities, the process of incorporation of new information into prices is spread over time, starting before its arrival, even in some cases before the occurrence of the information event, and continuing for a while through the information assimilation phase, sometimes extending beyond public announcement. These tasks, however, create heterojenity in the information structure, and

[3] I_t is probably a random variable, but there is evidence that it is not i.i.d., rather, it exhibits short-term positive autocorrelation and long-term negative autocorrelation (e.g.; Smirlock and Starks (1988: 31-41); Fama and French, (2000: 161-183)). We believe that its distribution characteristics will dramatically vary accross samples depending on the market and data frequency in the analysis, and that sufficiently long samples such that $E(I_t)=0$ are difficult to observe in practice.

[4] See for supporting evidence in futures markets Ederington and Lee (1993: 1161-1191), in the FX market Andersen and Bollerslev (1998: 219-265), and in the US T-Bond market Fleming and Remolana (1999: 1901-1915).

[5] Our description is consistent with findings that the association between volatility and public informaton arrivals is positive but has low explanatory power (Berry and Howe (1994: 1331-1346), Mitchell and Mulherin (1994: 923-950)), and further, that there is no clear association between returns and public information arrivals (Andersen and Bollerslev (1998: 219-265)).

the latter task involves subjective judgment vulnerable to psychological mechanisms. Therefore, imperfect information structure is an inevitable property that follows from semi-strong form of efficiency, rather than a characteristic associated with inefficiency. It is the result of the continual search for undiscounted future information.

Adding a forward-looking property, to reflect the characterization above, to Equation 3, we obtain the ultimate form of our return generating process:

$$R_t = E_{t-1}(R_t) + MP_t(I_T) + \epsilon_t \qquad (4)$$

where I_T is the future information set that will be publicly available at time $T > t$. The use of T rather than $t+1$ to notate future time reflects event time scale rather than calender time scale.

We shall elaborate on this point: Let's assume that an information event I_T occurs at time t_0 and is publicly announced at T and fully reflected in the market price instantaneously after the public announcement. The task of speculation, we define, is to obtain or anticipate this information before T and trade on it, making the market price move towards its full information value earlier than T. If the market is perfect (i.e.; $T - t_0 \approx 0$), then the only available task for the price setting speculators is to form beliefs about a noisy estimate of I_T before $t_0 \approx T$ (i.e.; before the information event even occurs). In this case, only beliefs about the future information set will drive the market price, and the shape of the $MP_t(I_T)$ function will reflect the properties of belief formation and revision. If $T - t_0 > 0$ (i.e.; the market is imperfect), then some market participants with informational advantages may obtain private information, and some others will try to use the information that the former will reveal through their trades; in addition to those who act on their beliefs. In this case, market price will be driven by the private information in addition to beliefs, and the shape of the $MP_t(I_T)$ function will reflect diffusion properties of private information, as well as belief formation and revision patterns. The former case will be subject to behavioral imperfections studied by behavioral finance, and the latter to informational imperfections studied by noisy information models, in addition.

This description helps us withstand arguments that rationality will ensure $MP(I_t) \approx I_t$. As I_T is never known for sure at time t, but is to be predicted subjectively, rationality arguments do not follow. Rather, social cognitive factors, such as Keynes' beauty contest anology, happen to be at work; hence conjectures of the noise trader approach apply. This is because common private signals such as previous period's return, past and current information flows are positively but imperfectly correlated with I_T.

Specifying the $MP_t(I_T)$ function:

Now, we provide a suggestion for the form of the $MP_t(I_T)$ function, based on a synthesis of the literature on imperfect information models and behavioral finance. For simplicity, we assume a linear form. The form of the $MP_t(I_T)$ function, which is time-varying, depends on the information structure in the market (i.e.; whether private information exists and who has it) and properties of the information set (i.e.; significance, salience, arrival sequence, autocorrelation, etc.), at a particular point in time.

The following time notation is used below: s: a short span of time, ℓ: a long span of time. Let's assume we are at time t, an information event happens at time t_0 and will be publicly announced at time T. If $t_0 > t$ (i.e.; the information event will happen in the future), market participants can only form beliefs about I_T; if $t_0 < t$ (i.e.; the information event has happened in the recent past), some of the market participants (informed traders) can obtain private information as well forming beliefs about I_T[6]. In addition, two other information events happened in $t\text{-}s$ (near past) and $t\text{-}\ell$ (far past), respectively. Note that information set builds cumulatively such that the information set at T also includes information available at all previous periods as well as the R_t series.

The first term of the $MP_t(I_T)$ function, in Equation 5 below, reflects gradual diffusion of private information (Hong and Stein, 1999: 2143-2171) and/or gradual incorporation of private information into price as a result of strategic behavior of informed traders (Kyle, 1985: 1315-1336) and/or gradual recognition of I_T by forecasting or beliefs (the process which I call ***gradual diffusion of beliefs***)[7]. The second term reflects the effect of underreaction (Barberis, Shleifer and Vishny, 1998: 307-343) or continuing overreaction (Daniel, Hirshleifer, Subrahmanyam, 1998: 1839-1870) to an information event in the near past, I_{t-s}. The third term reflects correction from overreaction (DeBondt and Thaler, 1985: 793-805) to an information event in the far past, $I_{t-\ell}$. The fourth term reflects the impact of positive

[6] This can only be the case if the market is imperfect, i.e.; $T - t_0 > 0$.

[7] Gradual diffusion of beliefs follows from conservatism bias and diversity of market participants in terms of their belief formation process.

feed-back trading (in other words, herding on previous period's return[8]) (e.g.; momentum traders in Hong and Stein, 1999: 2143-2171). The fifth term is pure public information surprises at time *t*, those impossible to anticipate or leak. The final term is temporary errors with no information content. The sum of the last two terms, which are both i.i.d., $I_t + c_t$, replaces, now justifiably, e_t in Equation 1.

$$MP_t(I_T) = \beta_{1,t} I_T + \beta_{2,t} I_{t-s} - \beta_{3,t} I_{t-r} + D_{4,t}\beta_4 R_{t-1} + I_t + \epsilon_t \qquad (5)$$

More details about the terms of Equation 5 are given below: Over an information assimilation phase, $d\beta_{1,t}/dt > 0$; in other words β_1 will grow from 0 towards 1 (or slightly above 1, to allow for overreaction to private information) as $t \rightarrow T$. A positive value of $\beta_{2,t}$ and $\beta_{3,t}$ will signify underreaction (or continuing overreaction[9]) and overreaction, respectively. $D_{4,t}$ is a dummy that takes the value 1 if $|R_{t-1}| > k$, and 0 if $|R_{t-1}| \leq k$, where k is a threshhold value. This formulation implies that past returns exceeding a certain threshhold value will trigger positive feed-back trading, a form of herding with the market. $\beta_{4,t}$ is a coefficient that reflects the dominance of noise traders in the market. Note that we capture the longer lag negative autocorrelation induced by positive feed-back trading under the third term, together with the correction from overreaction[10].

Under an alternative ex ante design, the values of $\beta_{2,t}$ and $\beta_{3,t}$ are to be identified externally by analyzing the information flow and applying arguments of behavioral finance theories; for instance via an expert system that relates ratings on factors leading to under- or overreaction to specific values of these coefficients. The value of $\beta_{2,t}$ depends on the properties of information arriving at time *t-s*: $\beta_{2,t} > 0$ if at least one of the factors suggested by behavioral finance theories to cause underreaction (such as statistical information, information contradicting extant beliefs, etc.) or continuing overreaction (salient, anecdotal information, etc.) has been

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[8] Froot, Scharfstein and Stein (1992: 1461-1482) show theoretically that when market participants have short horizons they may herd on a (potentially fundamentally irrelevant) subset of the information set, letting the market price deviate from the fundamental value. Nofsinger and Sias (1999: 2263-2295) find that institutional trades have a significant impact on the contemporaneous price, and that institutional traders do positive-feed trade. Bange (2000: 239-255) find that small individual investors positive feed-back trade, but their trades are negatively correlated with subsequent periods' returns. Säfvenblad (2000: 1275-1287) observes positive feed-back trading in the Swedish stock market following market moves that exceed a certain magnitude in absolute value.

[9] We are unable at *t* to distinguish between underreaction and continuing overreaction, we will however be able in later periods.

[10] This is because positive feed-back trading can reflect both information and noise. Only noise part is corrected subsequently.

observed to be at work in period *t-s*; and $\beta_{2,t} = 0$ otherwise. The value of $\beta_{3,t}$ depends on the properties of information arriving at time *t-ℓ*: $\beta_{3,t} > 0$ if at least one of the factors suggested by behavioral finance theories to cause overreaction (such as extraordinary event, absence of similar experience in the past, anecdotal information, presence of momentum trading, etc.) was observed to be at work in period *t-ℓ*, and $\beta_{3,t} = 0$ otherwise. To the extent that the values of $\beta_{2,t}$ and $\beta_{3,t}$ can be identified ex ante, our characterization of return generating process serves as a tool to forecast market's reaction to information, and ultimately realized returns. This will provide a decisive methodology to empirically test behavioral theories and help find a resolution to the debate raised by Fama (1998: 283-306) (i.e.; whether under- and overreactions are unpredictable random occurrances). Similarly, $\beta_{4,t}$ can be set externally as a function of public participation in the market (e.g.; measured by proxies such as the small trades or new entrants), which will provide a test of noise trader models. Alternatively, under the ex post design, the values of β_2 and β_3 ($\beta_{4,t}$) can be estimated from the data: Significantly positive values would be evidence of under- and overreaction (noise trading). This version will constitute a more direct test of market efficiency.

The form implied by perfect market models is: $MP_t(I_T) = I_t + \epsilon_t$ where ϵ_t is i.i.d. This is a special case of our general model, where all of the time-varying coefficients of Equation 5 are jointly equal to 0 (i.e.; there is no private information, no change in beliefs about the future information set, there has been no underreaction and no overreaction to past information, there is no positive feed-back trading). Extant literature, however, suggests that there are theoretical and/or empirical grounds to add these terms. Behavioral imperfections apply because market participants are not able to precisely identify terms of Equation 5, and to distinguish first term from other terms.[11]

The form suggested above is open to modification, if future research justifies to do so. For example, $\beta_{1,t}$ can be set to be a function of the correlation between informed traders' signals, to reflect the findings of Foster and Viswanathan (1996: 1437-1471).

[11] The empirical implementation of the study proposed here is underway in another study where an I_t series for about 3 years has been accumulated on a real-time basis at daily frequency. Preliminary results so far suggest that it is the first term of Equation 5, $\beta_{1,t}I_T$, that explains the largest portion of variation in realized returns.

DISCUSSION

The characterization of the return generating process proposed in this letter offers a more comprehensive description of the process driving realized returns, and, subject to empirical test, a more plausible account of variation in returns accross time. It allows for partial return predictability over information assimilation phases, whereas conventional models have difficulty to be consistent with return predictability findings in literature. Most importantly, it opens up a ground for more conclusive tests towards the resolution of the debate on behavioral finance theories and market efficiency; and to provide some help for the dual hypothesis problem. The empirical application of the ideas proposed here requires a careful external (direct) record of information events with their price change implications (our approach argues that statistical approaches to identify information events from unexpected returns may be misleading). It will require $MP_t(I_T)$ to be identified free of bias, preferably on a real-time basis. Though demanding, such research will be likely to resolve many issues resulting from ignorance of $MP_t(I_T)$ in extant finance literature. This letter merely formalizes this methodology, and leaves its justification via the long-taking task of empirical implementation, with its own problems and issues in recognizing $MP_t(I_T)$, to another ongoing study.

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