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Stationarity test by auto-regression equation estimation: an industrial workshop communication example

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Abstract
Stationarity Test by Auto-Regression Equation Estimation: An Industrial Workshop Communication Example

The AR and ARMA estimations remain well-known tools for determining an underlying mathematical expression for a series at hand. Once a mathematical equation is obtained, it is possible to derive a transfer function between the input and output. If the poles of this transfer function reside on or outside the unit circle, the series generated would not be stationary. Such a series with a significant number of samples generated would cause integer overflows for a simulation activity. For the study, the MTBF observation purposed validation series is considered from an Industrial Embedded PC Communications Test-Bed. These are compared on time and frequency to AR and ARMA estimated ones. The AR estimation possesses a better match to the original, with a 57-degree polynomial maintaining the stationary property.

Keywords: *Auto-Regression (AR), Moving Average (MA), Auto Regression Moving Average (ARMA), Internet of Things (IoT), Homoscedasticity, Heteroscedasticity, stationarity, MTBF.*

Öz
Öz Bağlanım Denklem Tahmini ile Durağanlık Testi: Bir Endüstriyel İşletişim Örneği

AR ve ARMA tahminleri, eldeki bir seri için temel bir matematiksel ifadeyi belirlemek açısından iyi bilinen araçlar olmaya devam etmektedir. Matematiksel bir denklem elde edildiğinde, girdi ve çıktı arasında bir transfer fonksiyonu türetmek mümkündür. Bu transfer fonksiyonunun kutupları birim çemberin üzerinde veya dışında yer alıyorsa, üretilen seri durağan olmayacaktır. Önemli sayıda numunenin üretildiği böyle bir seri, bir benzetim etkinliği için tamsayı taşmalarına neden olacaktır. Çalışma için, Hatalar Arası Ortalama Zaman (HAOZ) gözlem amaçlı doğrulama serisi, bir Endüstriyel Gömülü PC İletişimi Sınama yatağından alınmıştır. Bunlar zaman ve sıklık açısından AR ve ARMA tarafından tahmin edilenlerle karşılaştırılır. AR tahmini, durağan özelliği koruyan 57 derecelik bir polinom durumunda, orijinal gözlemle daha iyi bir eşleşmeye sahiptir.

Anahtar sözcükler: *Öz Bağlanım (AR), Hareketli Ortalama (MA), Öz bağlanım hareketli ortalama (ARMA), Cisimlerin İnterneti (IoT), eş-varyans, varyans değişkenliği, durağanlık, HAOZ*

1. Introduction

Auto-regression Series are primarily employed for modeling the stock market exchange or currency values. They are mostly affiliated with the economics arena. An ancillary benefit of adhering to this technique is the possibility of generating new data by predicting underlying characteristics. These anticipated series have some consideration for error and irregular fluctuation either. With the mentioned qualities, adding more samples to a series or generating a whole new one at the desired length is possible. The generated data can later be employed for simulation and modeling purposes. A series developed for such an effort, at significantly high indexes, need a stable character or stationarity property. This quality would guarantee that the sequence generated would remain reasonably within certain limits. Suppose the originating one for the predictions possesses these qualities. In that case, it may be possible to talk of an Independent and Identical Distributed (IID) character. This quality would require at least the series need be stationary so that each sample could be coming from the same distribution without a trend or seasonality.

The generality of the available academic work is about stationary conditions and tests, which aim to identify if a unit root exists. This task is mainly done with available tests without identifying the auto-regression series that are auxiliary to this work. Therefore, whenever a particular mathematical tool is employed, its exceptions, assumptions, and technique limit the quality of a test outcome. The prediction, the precision of prediction, prediction errors, and following the original series too closely are sources of error. Even though they are generally accepted tests, predicting a random event to the uttermost precision is not evident in its benefits. Still, some interesting issues can be of importance. For example, the degree of the considered polynomial, AR, or ARMA process could be an interesting factor. Generally, tests are all deciding for these factors in built-in libraries. However, for modeling and simulation purposes, they have importance along with the polynomials.

The series subject to this study comes from our previous studies. It is a validation purpose mean time between failure (MTBF) observation series from an Embedded PC Industrial Shop Communications physical testbed in Yucesan et al. [1][2]. It is a composite of a couple of experiments with slightly seasonal outcomes affected by ambient temperature. Nevertheless, they were statistically possessing stationary and exponential distribution characteristics. In our early studies, the series was subject to some existing known test techniques for stationarity and underlying existing classical random distributions involving Open Platforms Collaborations – Unified Access (OPC UA). The existing studies in the literature generally consider some known techniques applied to identify a unit root existence therefore stationarity yet studies which actually model a series with a known model (i.e. AR / ARMA) for any purpose are not too common.

The document organization is as follows: Section 1 Introduction gives acquaintance to the topic, followed by a brief literature survey Section 2. Theoretical Background. The Material and Methods in Section 3 describe the material at hand and the techniques employed. The Results and discussion in Section 4 present the results and support the initiation of a discussion on the topic. The text finalizes with Section 5. Conclusion and Recommendations.

2. Theoretical Background

There are studies on auto-regression series parameter estimations. Nevertheless, few studies comparatively identify these. Barman et al.[3] compare estimated parameters of AR, MA, and ARMA series based on three different information criteria. Besides them, the Regression Scores, Mean Absolute Error (MSE), Mean Squared Error (MSE), Median Absolute Error (MeAE), Root Mean Square Error (RMSE), and statistical stationarity tests are also considered.

There is time to time trends and seasonality among the observation or resourcing series. These properties generally can be visualized as a persistent increase in values as the trend name goes along. Seasonality means that a set of characters or values are happening for a period. Nevertheless, for a while in another duration, it could be repeated or not. The stationarity is also affected by the timescale of the series tested [4]. Longer times scales can include regular repetitions of a temporary trend or a seasonal effect. Stationary series would possess the property that the mean and variances are statistically persistent [5][6]. However, the series having this property does not guarantee stationarity. The student-t test is a valuable technique for a control for mean and variance levels [7]. Standard deviations, along with the variance, for the stationary character are essential as well. Therefore test on homoscedasticity is employed to control this factor [8][9][10].

Nevertheless, the observed series that do not have these qualities can be augmented to see the underlying characters. Pre-filtering [11], differencing and mean adjustment [12][13][14] type techniques change natural characteristics. However, they reveal a hidden and persistent character.

3. Material and Methods

Aim is still to test for stationarity in this study, however, extra step to actually generate a series matching the originating one been considered. This technique not only tell if the series is stationary or not by identification of a unit root existence, also yields a basis for a future simulation of the data collected in the physical testbed. A comparison of AR and ARMA expression based generation of the predicted series is more indicative of the characteristics of the observations at hand, getting us to know the system better.

In this section material at hand and the techniques employed for simulation is briefly outlined. The techniques AR/ARMA, which will serve as the basis for identification is presented. The reliability series observation method is outlined and the methodology employed for the study is described later on.

3.1. Auto-Regression (AR)

The auto-regression series in the simple form is as in equation (1).

It is expressed in the *z-domain*.

$$A[Z]Y[Z] = E[Z] \quad (1)$$

Assuming E[Z] is white noise, the term becomes constant in this domain. Where Y[Z] is the output of the transfer equation. Therefore the input term is not considered for an AR series. The terms of A[Z] indicate the leading terms of every output lag. This represents z^{-i} for increasing $i \in [1, n]$ and $n = 57$ in our final case outcome that is detailed in later on sections.

3.2. Auto-Regression Moving Average (ARMA)

The auto-regression series moving average is also based on an equation of various output lags with different constant leading terms. However, the white noise term also possesses leading constants. Therefore providing a weighted average for error. Again in the *z-domain*, the ARMA equation is expressed as in equation (2).

$$A[Z]Y[Z] = C[Z]E[Z] \quad (2)$$

As in the AR equation, the A[z] terms are the leading terms for the lag constants, and Y[z] is the output. E[z] represents the error modeled as white noise. Therefore C[z] represents the leading terms for the relevant lags of the error.

3.3. The Reliability observations series

The observations were made in a peer-to-peer communication physical testbed, mimicking an industrial plant workshop. This plant supposedly makes polling of a past of data from a server software over an Embedded Personal Computer with the Windows Operating System. The experiment involved starting with a reset. Later this query for a history of the progressing counter information is polled as an array every 15 to 20 seconds. The number of successful repetitions of this poll activity is counted and recorded. The series has been obtained to predict an MTBF figure.

3.4. Methodology

First, AR and ARMA series were generated using MATLAB built-in AR and ARMAX generators accordingly to the above definitions. These tools provide some information criteria and MSE-type values as a basis for comparison. The *Fit Percent* has been considered as the main criteria in this study. Besides this one, tools provide a simultaneous Loss Function final parameter, MSE, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and a Final Prediction Error (FPE) figure. Those are yielding very similar outcomes. A better fit over the observations is aimed to be obtained using the outcomes. The maximum absolute value of the transfer function poles is monitored with increasing AR/ARMA degrees of prediction. Based on this maximum pole, the prediction degree of the polynomials, therefore, maximum lag, is increased. As the polynomial increases its degree, the maximum absolute value of the poles remains inside the unit circle condition to a point. After this point, they leave the unit circle, leaving the stationary zone. Therefore the degree of prediction polynomial with the maximum fit and the absolute value of the maximum pole remaining within the unit circle identifies the maximum degree of the prediction and decision for the AR or ARMA polynomial.

The obtained maximum degree of the polynomial and series type based on the conditions mentioned above are later compared on Fourier domain response and time-domain match for the predicted series and originating observation series.

4. Results and Discussions

4.1. Results

The progress of AR and ARMA fit predictions and their corresponding maximum absolute value poles with the increasing prediction polynomial degree is provided in figure 1.

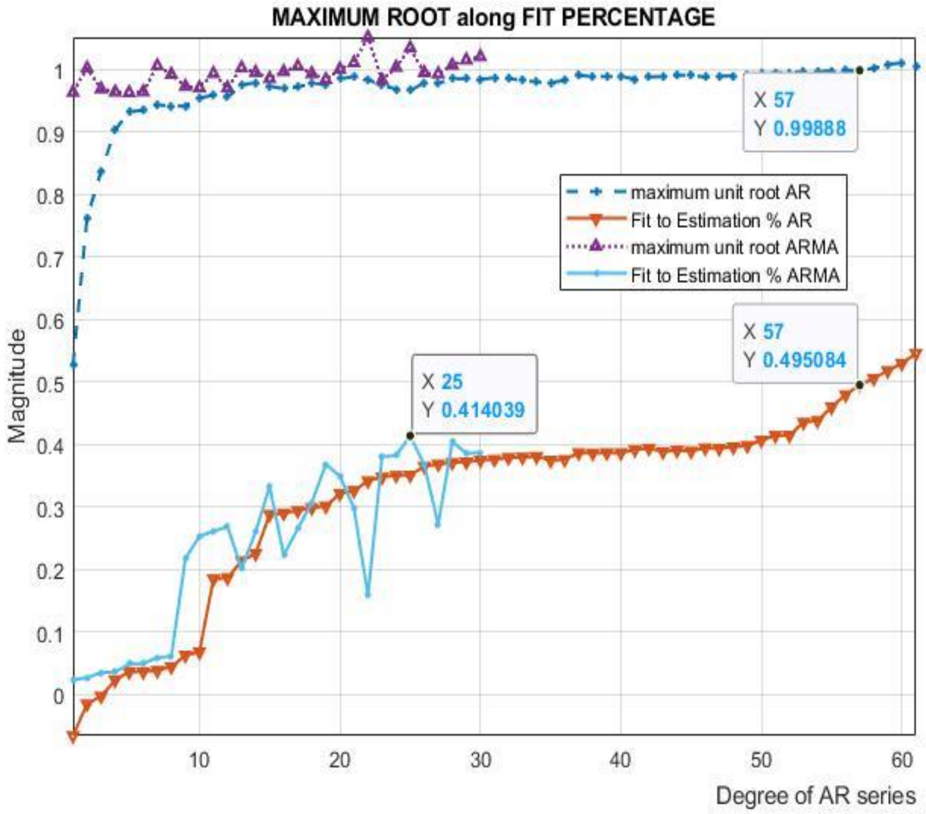

Figure 1. Progress of maximum poles for AR and ARMA along with fit percentages (lower lines)

In this figure, the inverse triangles represent AR fit percentage. The light-colored small '+' signed series represent ARMA fit percentage. The maximum ARMA fit percentage is around degree for the A[z] and C[z] series 25th sample, at roughly 41% fit percent. The above residing lines with straight triangles represent the maximum absolute value of the pole of the ARMA series. It has a darker shade. Interrupted lines with small '+' markers represent the progress of the maximum absolute value of the poles of the AR process. This maximum absolute value of the pole of the AR process surpasses one, namely, the unit circle boundary into the unstable region with the 58th A[z] degree.

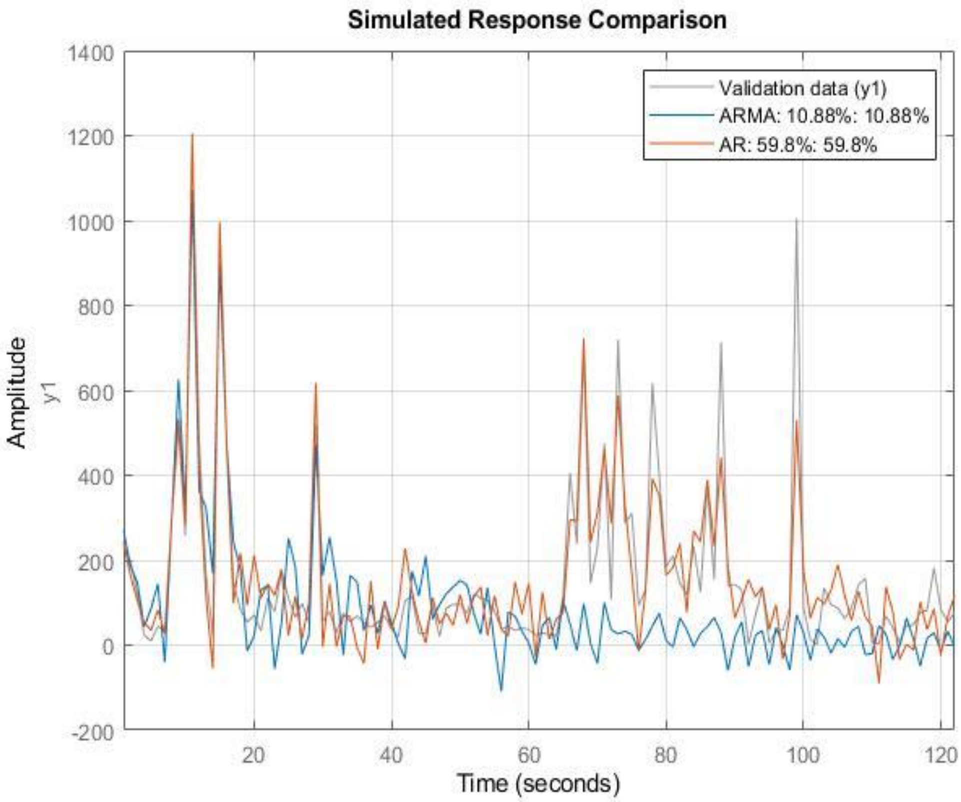

Figure 2. Progress of AR and ARMA predicted series along with originating series.

Second, the time-domain representation of the prediction is provided in figure 2. The series move together in the first oscillatory group. However, we see that the ARMA with the maximum possible degree of prediction of 30 is residing low, lacking the ability to follow the high values in this season. Nevertheless, the AR is sufficient to follow the originating series with a 61-degree polynomial. The lightest shades belong to the initial validation or MTBF observation data series. The issue is reconsidered in the discussions section.

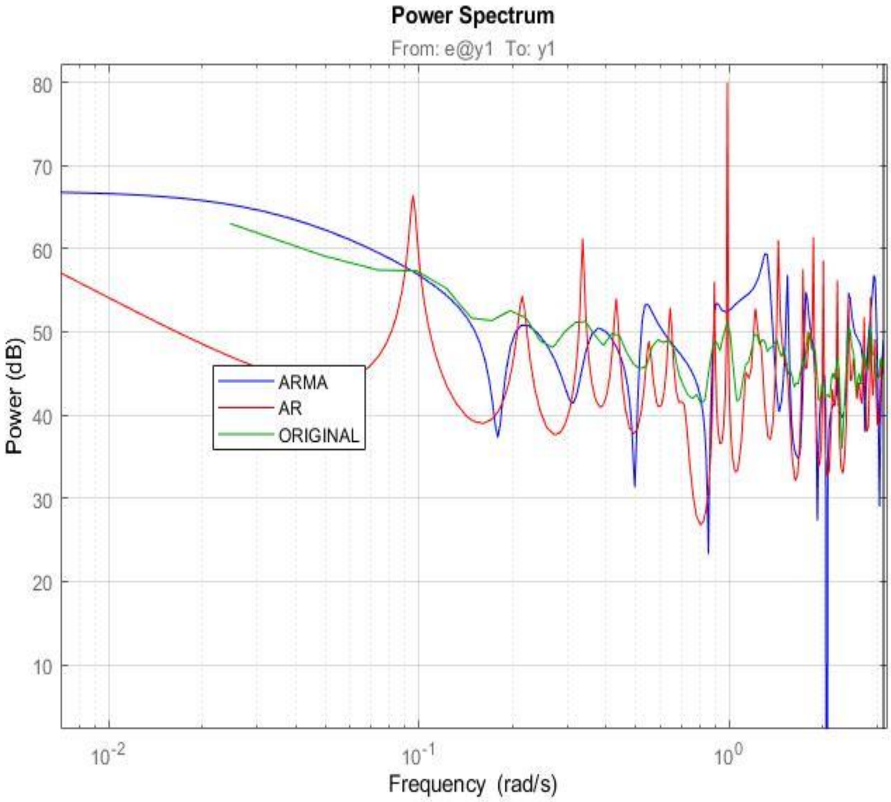

Figure 3. The spectrum of the AR, ARMA, and originating validation series

Third, the Fourier domain power spectrum of the estimations along with the originating series is available in figure 3. Here the original series has mediocre oscillations in the spectrum. AR series represented with red have the first peak and follow smooth oscillations of the original one with harsher but consistent peaks (zero). The Blue one having the first ditch (pole) is the one that is missing some of the characteristics of the original, especially in middle-range frequencies, which can account for the unfollowed season of the original.

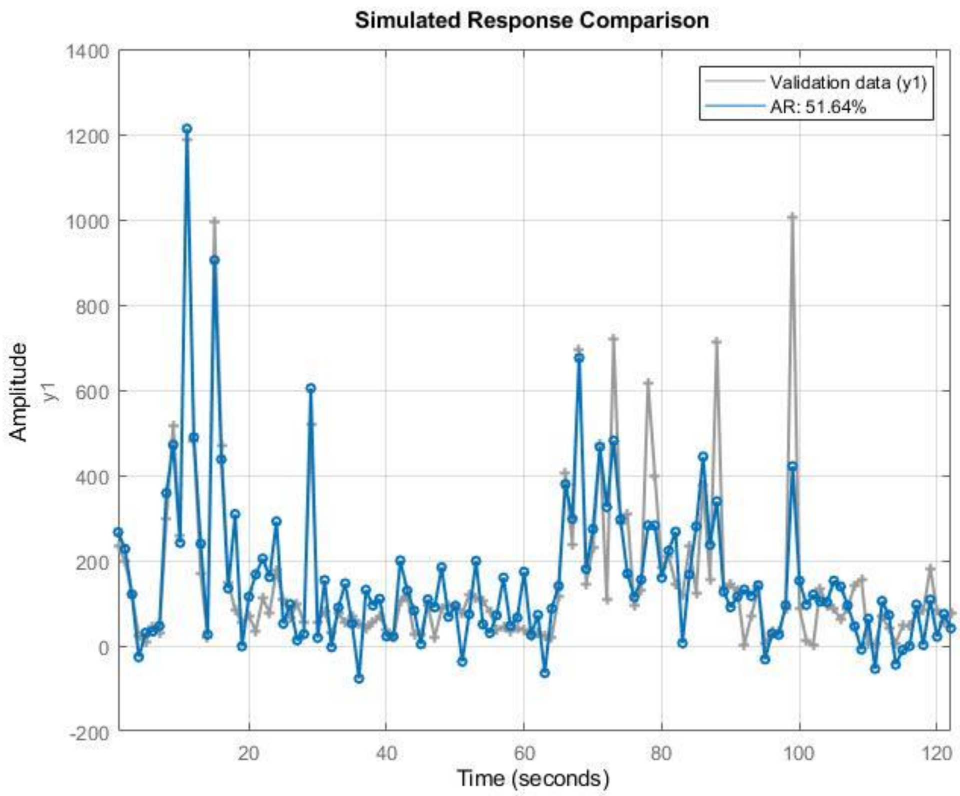

Figure 4. Time domain comparison of AR 57-degree with originating series

Fourth, from figure 1, we observe that the max AR pole is within the unit circle at the 57th degree of prediction polynomial. Finally, figure 4 includes the time domain procession of the original and AR series predicted with a 57-degree polynomial. Here the AR series are marked with a small 'o', and the validation (original) series are marked with '+'. AR series still follow the validation one.

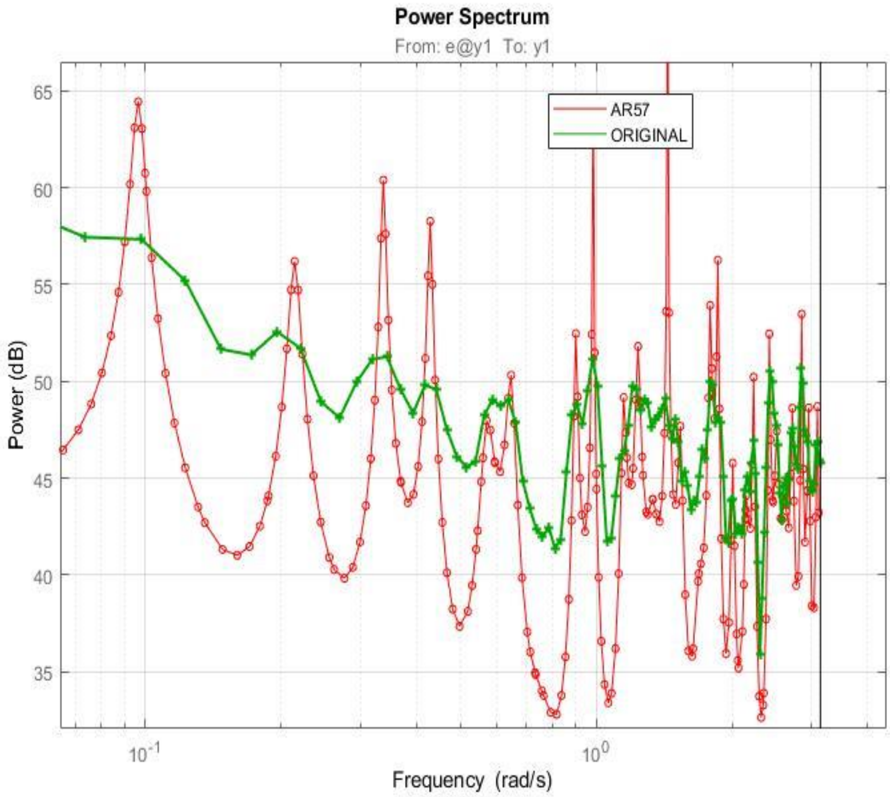

Figure 5. Time domain comparison of AR 57-degree with originating series

Fifth, figure 5 includes the Fourier domain power spectrum of the originating validation series, and the AR predicted series with a 57-degree polynomial. The series with minor 'o' marks are AR with a 57-degree polynomial, whereas the one with '+' marks is the original validation series. Here AR mainly responds to an increase in validation series with an increase in the shape resembling to that of a pole.

4.2. Discussions

Figure 1 presents an interesting result. The maximum pole of the AR series is pretty much constant. However, as the parameters of the series become more specifically modeling the original validation series, rather than the general statistical nature of the general behavior of the signal, the exact values of each sample take precedence. Such an over-modeling can cause prediction errors and mismatches by being at the limits of the prediction technique and the details of this instance of the experiments.

As visible from Figures 3 and 5, the ARMA series on mid-range frequencies, where spectral power is still significant, fail to follow the originating validation series closely. The mean is related to low-frequency components; however, this mid-range with moderate power would impact the variance of the predictions and simulations derived. The missing spectral components meant the ARMA series missed the second seasonal high variance high peak region, as seen in figure 2.

On the front of the AR model, the polynomials at 58, 59, 60, and 61^{st} degrees have maximum pole absolute value (radius of a complex pole) outside the unit circle. This observation should be due to over-modeling while observing the persistent plateau across the AR polynomial degrees or less and equal to 57. This maximum radius or absolute value of the maximum pole is around 0.9989 for the 57^{th}-degree AR polynomial. This value is around 0.9889 for 38^{th} degree AR polynomial. Therefore, assuming all the poles are within the unit circle and accepting stationarity is reasonable.

From figure 1 one can also observe that the ARMA model has significantly oscillatory performance as the max pole radius is once within and once out of the unit circle in a periodic manner. Observing the Fit Percentage of AR and the frequency mismatches of ARMA lead us to concentrate on AR. Therefore, with 57-degree AR having a good fit yet still stationary, one can model and simulate the scenario in the future. However, AR is not all smooth either in the Frequency domain front. The spectrum of AR with a 57-degree polynomial has a better match, as can be seen in figure 5. However, similar spectral components seem more powerful than the original series. Besides this, the times series seem to match with reasonable error as in figure 4.

All of these tell the series possesses AR property. The error signal inherent in the AR process does not span across time. Dirac delta is the time function version of the white noise. This outcome should also be related to the independent and identical character of the series since the error model is a simple and only variant of the current time. This simplicity can be considered, in a way, a Markovian Property. Another definition, the output's causality condition, being dependent only on its own and previous values, is observed in the AR series. The multiple lags considered for output could violate the Markovian property. However, error according to a probability distribution can change output value with nothing but the past state of the system. For the ARMA series, dependency on the other instances of the error challenges the Markovian Property by being dependent on the previous state of the system and different instances of the input. Therefore, this is in contrast to the situation of the IID property of being independent of other samples. The ARMA models do not yield consistent and closely fitting results. This misfit should not be considered along with the over-modeling caused error. Since that is a marginal condition, the misfit is a general situation. Possibly general acceptance of stationary condition implies IID property is still valid here.

5. Conclusions and Recommendations

The validation observations can be modeled with an AR series with reasonable mistakes. By neglecting the marginal conditions of the modeling boundaries, it has all poles within the Unit circle, which indicates stationary character. Being AR modeled and dependent on the single instant of the error function strengthens belief in the IID character. Employment of such a model for a simulation activity could be decent future work based on the findings.

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Kaynaklar

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