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Polar Codes Applications for 5G Systems

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LDPC codes,
5G systems.

Abstract: Improving data transmission over wireless channels has been the main obsession for many researchers for years. Since Shannon has discovered his theorem of information transmission, a lot of work has been done to attain his limit nevertheless no one has succeeded. Polar codes are the first provably codes that arrive to near Shannon's limits of capacity, therefore, they have most of the researchers' interest to be studied in 5G systems. As 5G systems require significant improvements in channel capacity, polar codes are the promising techniques that have the ability to offer these improvements. In this paper, we focus on studying polar codes in 5G systems. Moreover extracting polar codes in three common channels are studied. These channels are Binary Erasure Channel (BEC), Binary Symmetric Channel (BSC) and Additive White Gaussian Noise (AWGN) channel. Decoding information via polar codes is done by many methods, however, we give details about four common methods which are Successive Cancellation (SC) decoding, Successive Cancellation List (SCL) decoding, Cyclic Redundancy Check - Aided Successive Cancellation List (CRC-SCL) decoding and Adaptive Successive Cancellation List (Adaptive-SCL) decoding. We compare between them according to systems complexity and Block Error Rate (BLER). In this study, we provide details about the trial system of 5G encoder and decoder with polar codes and the general communication system with these codes.

5G Sistemler için Kutupsal Kodları Uygulamaları

Anahtar Kelimeler
Kutupsal kodlar,
Turbo kodları,
LDPC kodları,
5G sistemleri.

Özet: Kablosuz kanallar üzerinden veri aktarımının iyileştirilmesi yıllar boyunca birçok araştırmacı için problem olmuştur. Bu problemlerden biri kanal kapasitesidir. Shannon teoremi keşfedildiğinden beri Shannon kapasitesine erişmek için çok çalışma yapılmış ancak bu teorik sınıra ulaşılamamıştır. Kutupsal kodlar Shannon kapasite sınırına ulaşabilen ilk uygulanabilir kodlardır. 5G sistemlerinde veri artışı kutupsal kodların diğer kodlara göre kullanımını daha öne çıkarmıştır. Bu çalışmada Toplamsal Beyaz Gauss Gürültüsü (AWGN), İkili Simetrik Kanal (BSC) ve İkili Silen Kanal (BEC) üzerinden kutupsal kodların iletimi incelenmiştir. Kutupsal hata kodlama işlemi birçok yöntemle yapılır, çalışmamızda dört yaygın yöntem incelenmiştir. Bu yöntemler "Successive Cancellation (SC)" kodlama, "Successive Cancellation List (SCL)" kodlama, "CRC-Aided Successive Cancellation List (CRC-SCL)" kodlama ve "Adaptive Successive Cancellation List (Adaptive-SCL)" kodlamadır. Kutupsal kodlar kanal kapasitesinde çalışan kodlar olması dolayısıyla, çalışmamızda kutupsal hata kodların üretimi ve 5G sistemlerinde kutupsal kodların uygulaması yapılmıştır. Bu yöntemlerin Blok Hata Oranı (BLER) ve sistemin karmaşıklığına göre karşılaştırılması yapılmıştır. Bu çalışmada, 5G deneme sistemlerindeki kodlayıcı ve kod çözücüde kullanılan kutupsal kodlar ile genel iletişim sistemi hakkında ayrıntılı bilgi verilmiştir.

1. Introduction

Shannon in [1] wrote and derived the mathematical formulation of digital communications systems. In his main work, formulated the acceptable limit of reliable transmission of information. He proved that it is possible to receive the transmitted bits correctly. We add extra bits to the data in order to help us figure out the correct bits with a high rate (R) of probability. He first calculated the channel's capacity C(W) which is considered a high threshold. It means that if R < C(W), information sent at rate bits per channel with no errors or a low number of errors.

Figure 1. The basic communication model

From figure 1, the output from the source is a random process (X). This defined as source entropy H(X). If the system's source entropy H(X) is less than channel capacity C(W) i.e. H(X) < C(W), source input data will be sent through the channel reliably. In contrast, if the system's source entropy H(X) has a value larger than that of channel capacity C(W) i.e. H(X) > C(W), information cannot be sent reliably through the channel and the system loses the information through the channel. Referring to [2] Shannon showed that we can measure the distortion in a system by calculating the difference between two main parameters one is the X_n and the other is source output compressed representation.

Figure 2. Simple communication model

Figure 2 shows a simple communication model as Shannon drew. The source encoder compresses the data input into a suitable form with low quantity of distortion as low as possible then the output data χ_n goes through a block called channel encoder. In the channel encoder, the system puts extra bits into the data χ_n in order to improve the system's reliability against noise that comes from different sources. The output from the channel encoder X_n is sent through a channel. At the receiver, the channel decoder receives the data Y_n and remove the noise with the help of the extra bits that added in the transmission side. After that, the source decoder decodes the data γ_n into its original form. In order to reach a safe level of transmission in this system, the blocklength (N) should be large enough, therefore, for designing a practical system we should design a system that works with an acceptable complexity and low space [3].

In [4], [5], [6], the definition of polar codes is introduced and the method of encoding and decoding as well, however, in the first part of this paper, we summarize the definition of polar codes according to all of these references in order to make it easy understanding why these codes are being researched to be used in 5G systems. In [7], a general tutorial of 5G systems is presented and it focuses on the requirements of 5G systems in general nevertheless we study the requirements of 5G systems according to the chosen channel coding scheme. In [8], [9], [10], polar codes are studied in case of 5G systems and they suggest that polar codes are the codes that are suitable to 5G systems and ignoring that these codes have advantages and disadvantages. In this paper, we focus on studying advantages and disadvantages of polar codes in 5G systems and presents a comparison between the three suggested codes to be used in 5G systems which are LDPC, turbo, and polar codes. Moreover, we detail the trial encoding and decoding scheme of polar codes in 5G systems and in an overall communication system.

In this paper, we focus on studying the definition of polar codes and their encoding and decoding process. As polar codes are considered a promising technique in 5G systems, we compare between polar codes, LDPC, and turbo codes according to many important criteria such as BLER, error floor, system complexity, etc.

2. Material and Method

Before polar codes, turbo codes and LDPC codes have been used in practice and they have an excellent performance but they are not able to achieve capacity except in case of BEC [11]. In 2009, Arıkan has made tremendous steps when proved that we can use the channel polarization theory to extract codes with the property of achieving capacity with low encoding and decoding complexity [4]. To summarize the polar codes and their construction: (1) the general channels at the input which considered B-DMCs are combined together into a channel vector and then they are split up into channels which they are virtual channels and they are polarized into either zero or one. As the codes' blocklength get larger the capacity of these channels tends to be either near zero (useless channels) or to be near one (perfect channels). This theorem is called channel polarization and in this paper, we detail information about it. (2) transmitting polarized bits by the virtual channels gives the chance to get codes that achieve capacity and these codes are polar codes and they decoded under many methods. Let us suppose that the input alphabet X is a discrete memoryless channel with {0, 1}. Dealing with channel parameters directs us to define two important parameters. The first one is the symmetric capacity I(W) and the other parameter is Bhattacharyya parameter Z(W). Bhattacharyya parameter gives us the measure of channel reliability.

Equations 1 and 2 explain two important terms that define polar codes which are symmetric capacity I(W) and the Bhattacharyya parameter Z(W) respectively as,

$$\mathrm{I(W)} = \sum_{\mathcal{Y}\in \mathrm{y}} \sum_{\mathcal{X}\in \mathrm{x}} \frac{1}{2}\, \mathrm{W}(\mathcal{Y}|\mathcal{X}) \log \frac{\mathrm{W}(\mathcal{Y}|\mathcal{X})}{\frac{1}{2}\mathrm{W}(\mathcal{Y}|0) + \frac{1}{2}\mathrm{W}(\mathcal{Y}|1)} \quad (1)$$

$$\mathrm{Z(W)} = \sum_{\mathcal{Y}\in \mathrm{y}} \sqrt{\mathrm{W}(\mathcal{Y}|0)\mathrm{W}(\mathcal{Y}|1)} \quad (2)$$

The relationship between I(W) and Z(W) is,

$$\mathrm{I(W)} \geq \log\left(\frac{2}{1 + \mathrm{Z(W)}}\right) \quad (3)$$

$$\mathrm{I(W)} \leq \sqrt{1 - \mathrm{Z(W)}^2} \quad (4)$$

$$\mathrm{I(W)} \geq 1 - \mathrm{Z(W)} \quad (5)$$

which the equality occurs if the channel W is BEC. Equations 3, 4, and 5 suggest that there is reverse relationship between I(W) and Z(W) i.e. if the value of I(W) is close to one, the value of Z(W) is close to zero and in this case Binary Discrete Memoryless Channels (B-DMC) is in its good state and if the value of I(W) is close to zero, the value of Z(W) is close to one and in this case the B-DMC is in its bad state i.e. the reliability of the channel is very low. Figure 3 shows the relationship between I(W) and Z(W). From figure 3, we can conclude that the acceptable channels lie in the shaded area as equations 3, 4, and 5 suggested. Before we introduce the generator matrix, let us define the Kronecker product that is used to create the generator matrix of polar codes.

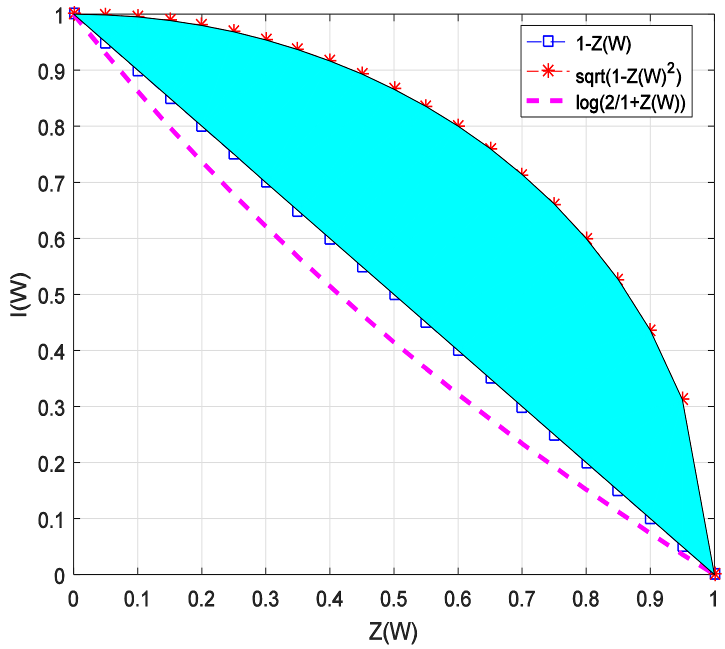

Figure 3. The relationship between I (W) and Z(W) as suggested by equations 3, 4, and 5

Definition 1. If we have a matrix A with (m) rows and (n) columns and a matrix B with (k) rows and (l) columns, the Kronecker product is defined by,

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \quad (6)$$

We denote $A^{\otimes n}$ to Kronecker product which means multiplying matrix A by itself (n) times i.e.

$$A^{\otimes n} = \underbrace{A \otimes A \cdots \otimes A}_{n} \quad (7)$$

2.1. Code Construction

Code construction is set to identify the values of the vectors of information bits ($\mathcal{A}$) and the vector of frozen bits ($\mathcal{A}^c$) based on the channel capacity of individual channels. Polar code construction is different from channel to channel according to the parameters of the channels such as variance (σ^2) in AWGN channel, (P) in BSC channel and (ε) in BEC channel. The following sections present extracting polar codes in the three common channels which are BEC, BSC, and AWGN.

2.1.1. Binary erasure channel (BEC)

Polar code construction of BEC of (N=8, K=4) is shown in figure 4 where the value of epsilon is ($\varepsilon = 0.4$). The Bhattacharyya parameter is set to be $Z = 0.4$ for the first W. Afterward the channel is distributed into two channels ($W_2^{(1)}, W_2^{(2)}$) and the measure of the reliability is calculated by $Z(W_2^{(1)}) = 2Z(W) - Z(W)^2$and $Z(W_2^{(2)}) = Z(W)^2$ [6]. The general formulas to calculate the recursive channel are,

$$Z(W_N^{(2i-1)}) = 2Z\left(W_{\frac{N}{2}}^{(i)}\right) - Z\left(W_{\frac{N}{2}}^{(i)}\right)^2 \quad (8)$$

$$Z(W_N^{(2i)}) = Z(W_{N/2}^{(i)})^2 \quad (9)$$

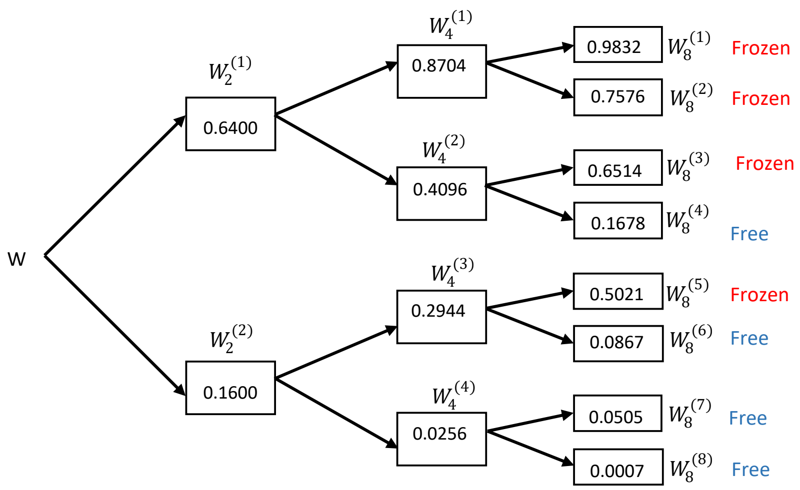

Figure 4. Code construction of BEC for N=8, K=4 and $\varepsilon = 0.4$

In the final step with $n = \log(N)$, all the possible reliability boundaries results appear, which in this example (n=3). All the eight possible results appear and according to this result, the bits are chosen depending on their amount of Bhattacharyya parameter. The bits whose amount of (Z_i) are low have a large quantity of capacity so these bits are chosen and the other bits are set to be frozen as shown in figure 4.

2.1.2. Binary symmetric channel (BSC)

The difference between BEC and BSC is that after applying channel transformation to BEC, BECs are received as well. In contrast for BSC after applying channel transformation, BSCs are received in case of $W_N^{(2i-1)}$ while in case of $W_N^{(2i)}$ BSCs are not received [12].

The channel input to BSC is $\mathcal{X} = \{0,1\}$ and the channel output is $\mathcal{Y} = \{0,1\}$ and crossover probability is known as (**P**). The channel's transition probabilities are $P(\mathcal{Y} = 0|\mathcal{X} = 1) = P(\mathcal{Y} = 1|\mathcal{X} = 0) = p$ and $P(\mathcal{Y} = 0|\mathcal{X} = 0) =$

$P(\mathcal{Y}=1|\mathcal{X}=1)=1-p$. The Bhattacharya parameter in BSC is calculated using $Z=\sqrt{p-(1-p)}$ and extracting polarized channels is done as shown in figure 5. Distribution of the Bhattacharyya parameter is done according to equations 8 and 9.

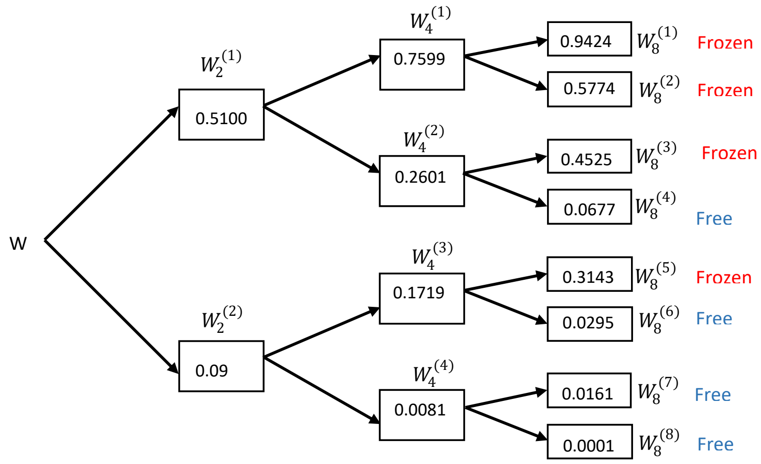

Figure 5. Code construction of BSC for N=8, K=4 and $p=0.1$

2.1.3. Additive white gaussian noise (AWGN) channel

The output of the AWGN is represented by $y=x+\omega, x\in\{+1,-1\}$ where ω is the noise with zero mean and variance of $[\![(\sigma]\!]^2)$ [13]. The channel transition probability is computed by,

$$P(y|x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{-([\![y-1)]\!]^2}{2\sigma^2}\right] \tag{10}$$

The Bhattacharya parameter in AWGN channel is calculated using $Z=\exp(-E_c/N_o)$ where E_c is the energy of transmitting a bit and N_o is the noise power. Extracting polarized channels is done as shown in figure 6. Distribution of the Bhattacharyya parameter is done according to equations 8 and 9.

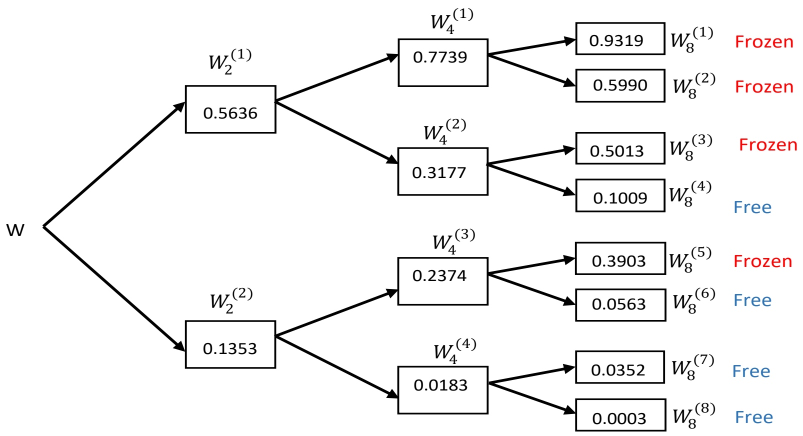

Figure 6. Code construction of AWGN for N=8, K=4 and $(\sigma^2=0.5)$

2.2. Recursive Channel Transformation

In channel combining and splitting, N times of channels (W) transform into $W_N^{(1)},\dots,W_N^{(N)}$. In this section, we care about breaking the produced channels recursively into single step channel transformation. In the first step, we get $(W_2^{(1)},W_2^{(2)})$ where $W_2^{(1)}:\chi\to\tilde{y}$ and $W_2^{(2)}:\chi\to\tilde{y}\times\chi$ which they are produced by the first step transformation of the two channels $W: x\to y$, $(W,W)\to(W_2^{(1)},W_2^{(2)})$ where,

$$W_2^{(1)}(\mathcal{Y}_1^2|u_1)\triangleq\sum_{u_2}W_2(\mathcal{Y}_1^2|u_1^2)=\sum_{u_2}\frac{1}{2}W(\mathcal{Y}_1|u_1\oplus u_2)W(\mathcal{Y}_2|u_2) \tag{11}$$

$$W_2^{(2)}(\mathcal{Y}_1^2, u_1|u_2) \triangleq \frac{1}{2}W_2(\mathcal{Y}_1^2|u_1^2) = \frac{1}{2}W(\mathcal{Y}_1|u_1 \oplus u_2)W(\mathcal{Y}_2|u_2) \tag{12}$$

which $u_1, u_2 \in \chi$ and $\mathcal{Y}_1, \mathcal{Y}_2 \in y$. Generally it is written $(W_N^{(i)}, W_N^{(i)}) \rightarrow (W_{2N}^{(2i-1)}, W_{2N}^{2i})$,

$$W_{2N}^{(2i-1)}\left(\mathcal{Y}_1^{2N}, u_1^{2i-2}\middle|u_{2i-1}\right) = \sum_{u_{2i}} \frac{1}{2} W_N^{(i)}(\mathcal{Y}_1^N, u_{1,o}^{2i-2} \oplus u_{1,e}^{2i-2}|u_{2i-1} \oplus u_{2i})W_N^{(i)}(\mathcal{Y}_{N+1}^{2N}, u_{1,e}^{2i-2}|u_{2i}) \tag{13}$$

$$W_{2N}^{(2i)}\left(\mathcal{Y}_1^{2N}, u_1^{2i-2}\middle|u_{2i}\right) = \frac{1}{2} W_N^{(i)}(\mathcal{Y}_1^N, u_{1,o}^{2i-2} \oplus u_{1,e}^{2i-2}|u_{2i-1} \oplus u_{2i})W_N^{(i)}(\mathcal{Y}_{N+1}^{2N}, u_{1,e}^{2i-2}|u_{2i}) \tag{14}$$

In equations 13 and 14 $u_{1,o}^{2i-1} = (u_1, u_3, \dots, u_{2i-1})$ and $u_{1,e}^{2i-2} = (u_2, u_4, \dots, u_{2i-2})$ and these equations are the same of equations 11 and 12 if $u_{2i-1} \rightarrow u_1$, $\left(\mathcal{Y}_1^N, u_{1,o}^{2i-2} \oplus u_{1,e}^{2i-2}\right) \rightarrow \mathcal{Y}_1$, and $u_{2i} \rightarrow u_2$, $\left(\mathcal{Y}_{N+1}^{2N}, u_{1,e}^{2i-2}\right) \rightarrow \mathcal{Y}_2$.

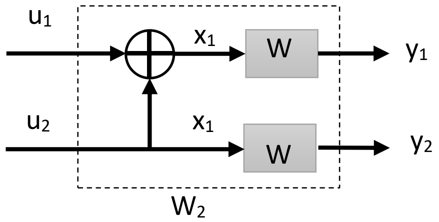

Figure 7. The Recursive construction of channel vector when (W=2)

Figure 7 shows an example of creating polar codes in case of two channels. The transformation when N=8 is shown in figure 8. The graph is read from right to left. This shape is also called the Butterfly Patterns. From figure 8 in the most right side point there are two channels in which they are independent copies of W and immediately next to them there are two channels $(W_2^{(1)}, W_2^{(2)})$ which they transformed into four copies of the independent channels and so forth until they arrive into N copies of independent channels [14].

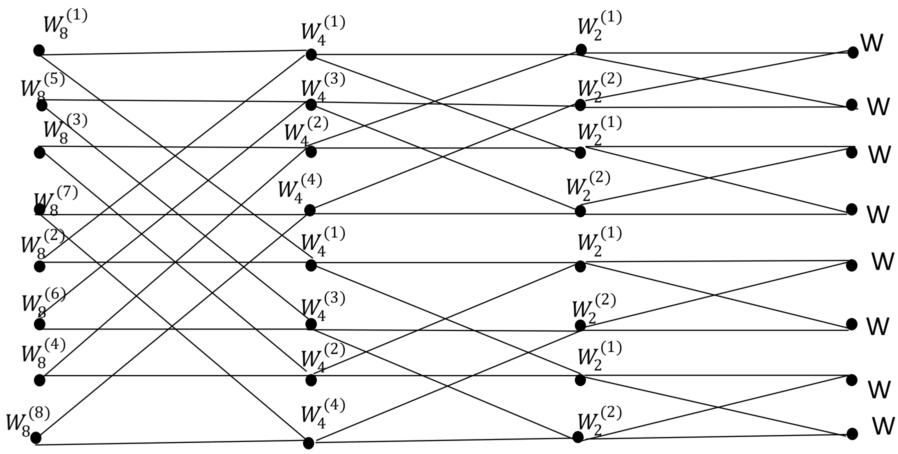

Figure 8. The channel transformation when (N=16)

2.3. Channel Polarization

Channel polarization is the method in which proves that polar codes are able to achieve the symmetric capacity of B-DMC. By using chain rule to deal with symmetric capacity we have [14],

equation 16 proves that the process of channel transformation keeps the symmetric capacity of the uniform channels multiplying N times.

$$I(\mathcal{Y}_1^N; x_1^N, u_1^N) = I(\mathcal{Y}_1^N; x_1^N) + I(\mathcal{Y}_1^N; u_1^N|x_1^N) = I(\mathcal{Y}_1^N, u_1^N) + I(\mathcal{Y}_1^N; x_1^N|u_1^N) \tag{15}$$

as $I(\mathcal{Y}_1^N; u_1^N|x_1^N) = I(\mathcal{Y}_1^N; x_1^N|u_1^N) = 0$, hence $I(\mathcal{Y}_1^N; x_1^N) = I(\mathcal{Y}_1^N, u_1^N)$. Therefore,

$$I(W_N) = I(\mathcal{Y}_1^N, u_1^N) = I(\mathcal{Y}_1^N; x_1^N) = NI(W) \tag{16}$$

The following theorem proves that as $N \to \infty$, I(W) of every channel W is polarized to zero or one (useless or perfect). In figure 9, it is concluded that the distribution of the points starts to be zero and keeps on going for a while then converts to be one in case of the channel index is large.

Theorem (1) [4], for any B-DMC W, the bit-channel $W_N^{(i)}$ polarizes in the sense that, for any fixed $\delta \in \{0,1\}$, as N goes to infinity through power of two, the fraction of indices $i \in \{1, \ldots, N\}$ for which $I(W_N^{(i)}) \in (1-\delta, 1]$ tends towards I(W) and the fraction for which $I(W_N^{(i)}) \in [0, \delta)$ tends toward 1-I(W), that is,

$$\lim_{N\to\infty} \frac{\left|i \in [N]: I(W_N^{(i)}) \in (1-\delta, 1]\right|}{N} = I(W) \tag{17}$$

$$\lim_{N\to\infty} \frac{\left|i \in [N]: I(W_N^{(i)}) \in [0, 1-\delta)\right|}{N} = 1 - I(W) \tag{18}$$

Equations 17 and 18 are called channel polarization because the property of achieving capacity of polar codes depends on this phenomenon.

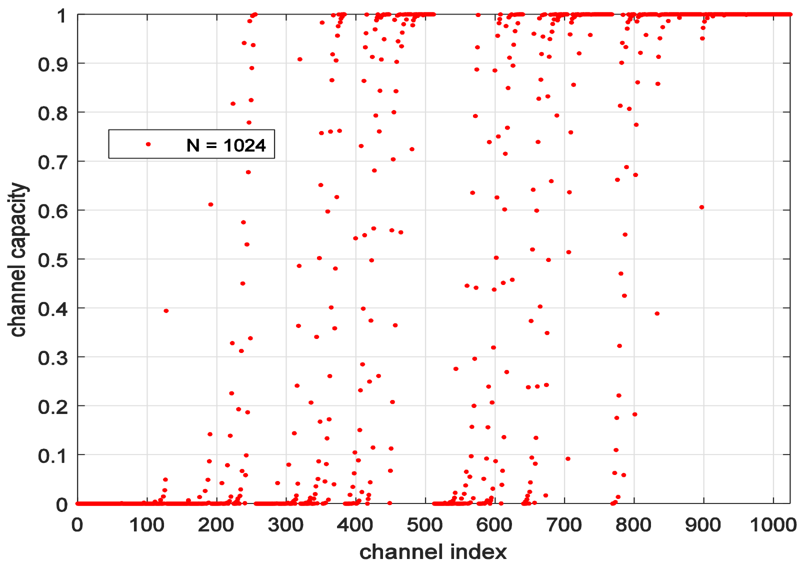

Figure 9. The plot of mutual symmetric channel vs. channel index in case of BEC ($\varepsilon = 0.5$)

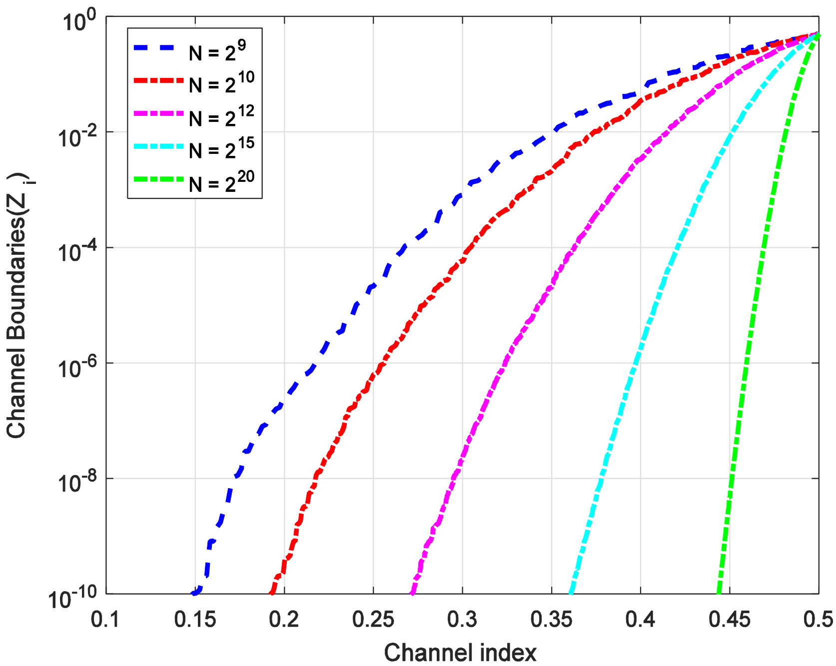

Figure 10. Channel index vs. channel reliability boundaries for n = [9, 10, 12, 16, 20] when BEC $\varepsilon = 0.5$

2.4. Polar Codes Coding

By using the polarization theorem polar codes are constructed in which they have the properties of achieving the channel capacity I(W). In channel polarization, two channels are created one is good and has a high quality of capacity (perfect) and the other is bad and has a low quality of capacity (useless). We choose the perfect channels and upload data on them in which the Bhattacharyya parameter is as low as possible i.e. $Z(W_N^{(i)})$ is close to zero.

2.4.1. Polar codes encoding

In a B-DMC W, we are able to encode the polar codes with the help of a simple organizing method. The space complexity of the encoder is equal to O(N) and the time complexity is O(Nlog(N)) [4]. The generator matrix G can be defined by $G_N = B_N F^{\otimes n}$ where $n = \log(N)$, B_N is called the bit-reversal matrix and $F^{\otimes n}$ is the Kronecker product where $F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ as it is mentioned previously in definition 1. Figure 11 shows a simple example of an 8-bit polar encoding. If we have polar code with blocklength N, it consists of an input vector u_1^N i.e. $u_1^8 = [0\ 0\ 0\ 1\ 0\ 1\ 0\ 1]$ and the vector at the output is x_1^N i.e. $x_1^8 = [1\ 1\ 0\ 0\ 0\ 0\ 1\ 1]$. Transferring between the input vector and the output vector $u_1^N \rightarrow x_1^N$ is linear over Galois Field (GF(2)) such that,

$$x_1^N = u_1^N G_N \tag{19}$$

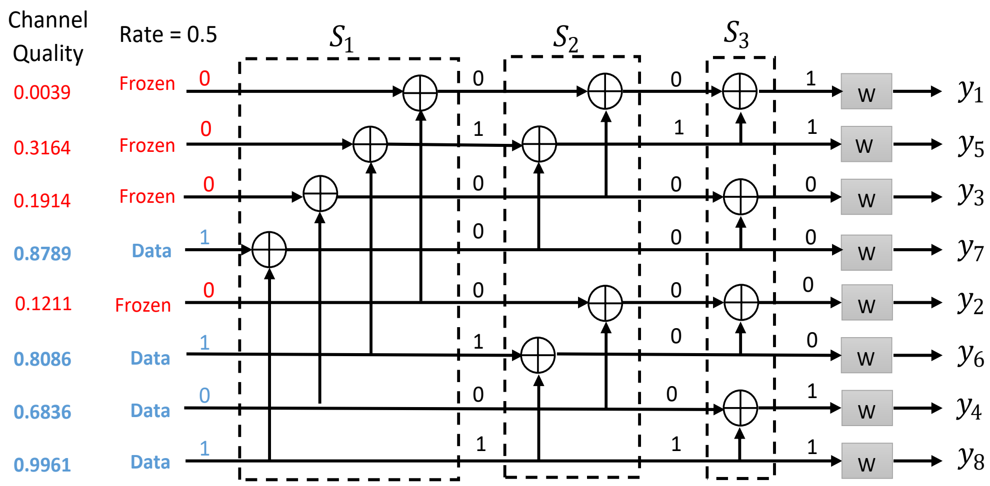

Figure 11. Polar encoding block diagram

2.4.2. Polar codes decoding

The polar codes under this assumption have a length of ($N = 2^n$) which contains the information length (K) and the frozen bits are ($N - K$). We say that $u = u_1^N$ is the information bits (information bits + frozen bits) and $x = x_1^N$ is the codeword vector produced from the encoding process and sent through the channel $W : \chi \rightarrow y$ where $\chi = \{0,1\}$. In the receiver, we receive $y = \mathcal{Y}_1^N$. The decoding process is applied to the vector y which gives $\hat{x}$ that has corresponding information to vector $\hat{u}$. Suppose $W_N^{(i)}$ is the virtual channel vector that observed by u_i in the process of recursive SC-decoding. The space complexity of the decoder is O(N) and the time complexity is O(Nlog(N)). In order to choose a decoding method, we take in mind the trade-off between two important parameters. One is the reliability and the other is the computation complexity. Under this section, we review the most common decoding methods and compare between them and which one of them achieve a better resistance to error than the main SC decoder.

2.4.2.1. Successive cancellation (SC) decoding of polar codes

SC decoder figures out the value of the transmitted data u_1^N as $\hat{u}_1^N$ by the help of the received vector $y \in Y$ in the channel $W : \chi \rightarrow y$. The information that comes to the receiver by the received codewords $\mathcal{Y}_1^N$ is solved by using Log Likelihood Ratio (LLR). SC decoder uses soft-decision decoding in calculating LLR values of the received bits by the help of channel LLR values. First, we calculate a sequence of LLR then SC decoder uses the hard-decision to figure out the value of received bit $\hat{u}_1^N$ successively from $\hat{u}_1$ to $\hat{u}_N$ i.e. the value of $\hat{u}_1^i$ is calculated by the help of the previous bit's value $\hat{u}_1^{i-1}$[4].

$$\hat{u}_i \triangleq \begin{cases} u_i, & if\ i \in A^c \\ h_i(y_1^N, \hat{u}_1^{i-1}), & if\ i \in A \end{cases} \tag{20}$$

where h_i is defined as $y^N \times x^{i-1} \rightarrow x$, and the decision is based on,

$$h_i(y_1^N, \hat{u}_1^{i-1}) \triangleq \begin{cases} 0, & if\ \dfrac{W_N^{(i)}(y_1^N, \tilde{u}_1^{i-1}|0)}{W_N^{(i)}(y_1^N, \tilde{u}_1^{i-1}|1)} \geq 1 \\ 1, & otherwise \end{cases} \tag{21}$$

2.4.2.2. Successive cancellation list (SCL) decoding of polar codes

Although the SC decoding method tries to attain to Shannon capacity level, it works well only with a large number of blocklengths and does not give a perfect performance with a small number or a moderate number of blocklengths [5].

Tal and Vardy in [5] introduced a new method to fix this problem. SC decoder takes just only one path from the decoding paths, but SCL decoder has the ability to activate L number of best decoding paths. In the case of L is large, Maximum Likelihood (ML) decoding is used. Because the complexity algorithm $O(L\ NlogN)$ depends on the list size (L), there is a trade-off relation between two important parameters the complexity and the performance of the SCL algorithm.

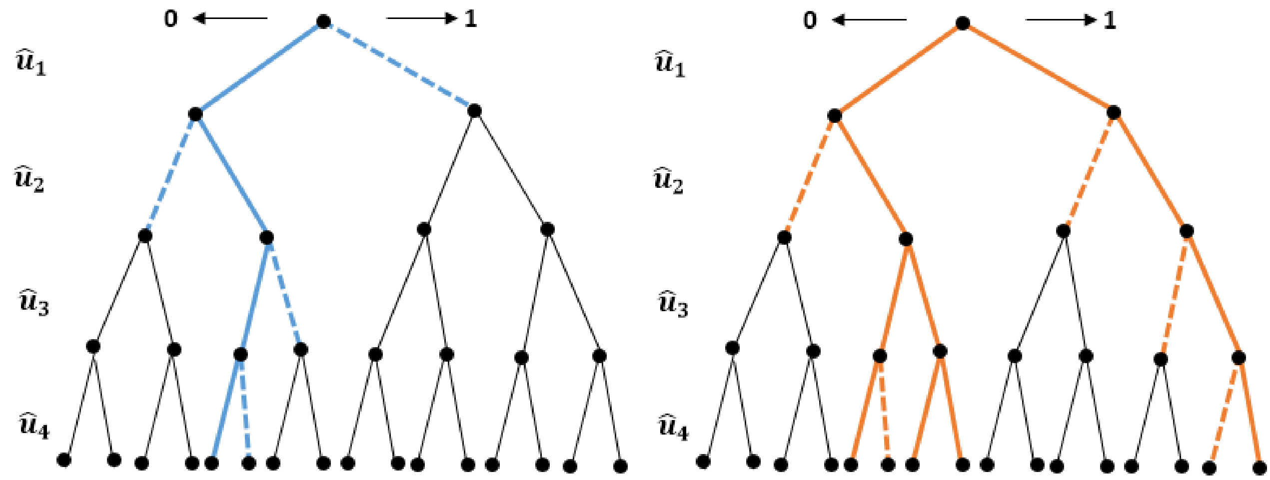

Figure 12. a) SC decoding. b) Successive cancellation list (SCL) decoding (list size L = 4). Bold face lines are Inspected paths. Dashed lines are inspected but discarded paths. Thin lines are never inspected paths.

Even though SC decoder takes one path to get the bits of $\hat{u}_{\mathcal{A}}$, SCL decoder takes L paths to reach into $\hat{u}_{\mathcal{A}}$. In every time, SCL distributes the path into two options ($\hat{u}_{\mathcal{A}} = 0, \hat{u}_{\mathcal{A}} = 1$) as shown in figure 12. As the number of paths reaches into the predefined list size L, SCL chooses the best paths and discards the rest. As each distributed path doubled, the number of tracks should be studied and we should prune each track accordingly. During this distribution process, the highest number of allowed tracks is equivalent to list size L [5][15].

2.4.2.3. CRC-aided successive cancellation list decoder

SCL decoders improve polar codes in many aspects in which they reduce their probability of error but it suffers from some drawbacks such as its performance cannot exceed the performance that restricted by ML. Moreover, in some cases the correct codeword does not belong to the most likely paths however it is in the list then we miss a codeword [16]. Tal and Vardy in [5] introduced a new method of using error detecting codes such as CRC codes. The new method aims to specify the correct codeword from the list in SCL decoder.

Figure 13 shows the block diagram of the polar codes together with the CRC encoder which connected with its list-CRC decoder. Transmitting and receiving data using this scheme are done as follows. Data bits of length k encoded by using CRC encoder to extract $K = k + r$ bits where (r) is parity check bits and it is added to help to identify the correct data at the decoder and it is added by CRC encoder. Then the polar encoder encodes the bits that come from CRC encoder and extracts the binary codeword that has a length of N.

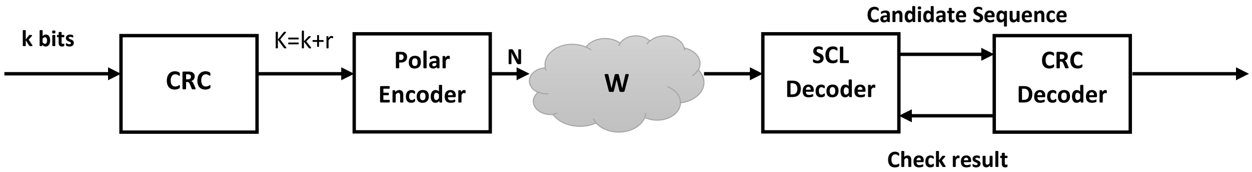

Figure 13. CRC-Aided Successive Cancellation List Decoder block diagram

At the receiver side, the receiver evaluates the likelihood and passes the resulted signal to the next block which is list-CRC decoder. In this step, (L) survived paths (candidate codeword) are specified from the available list decoder which tested by CRC detector and initializing from the most likelihood paths. As soon as the candidate bits are detected by CRC detector, the data bits (k) are determined [15][16][17].

2.4.2.4. Adaptive successive cancellation list decoding of polar codes

From [15][18], we can conclude that SCL decoder with a small number of (L) is decoded correctly however some frames require a large number of L to be decoded correctly. In order to minimize the complexity of the system and enhance the throughput of the decoder adaptive SCL decoder for polar codes with CRC was introduced. The idea of this method is to use a small number of L initially and increase it iteratively (when we do not have survival track passing CRC) till L arrives at a predefined amount of L_{max}.

Figure 14 shows the comparison of different decoders' schemes. The comparison was held in case of BLER in which the number of transmitted frames is equal to 100000 frames and the maximum number of error is 100. In this figure, there is a comparison between different decoders. The first one is SC in which L = 1. It is shown that it achieves the lowest amount of BLER compared to the other decoders, however, it performs the perfect amount of complexity which it is considered the best decoder in complexity achievement compared to the other decoders because as list size increases the complexity increases accordingly [5]. Systematic polar codes achieve better than SC but it is still not competitive to be used in 5G systems compared to other codes such as turbo codes and LDPC codes. The third decoder is SCL which L = 4. Because of the drawbacks found in SCL and explained in section 2.4.2.3, CRC was added and it achieves a better result than SCL alone. Moreover, SCL with CRC is considered the first step to the 5G wireless communication systems [19]. In our simulation, we used CRC = 16 and L = 4.

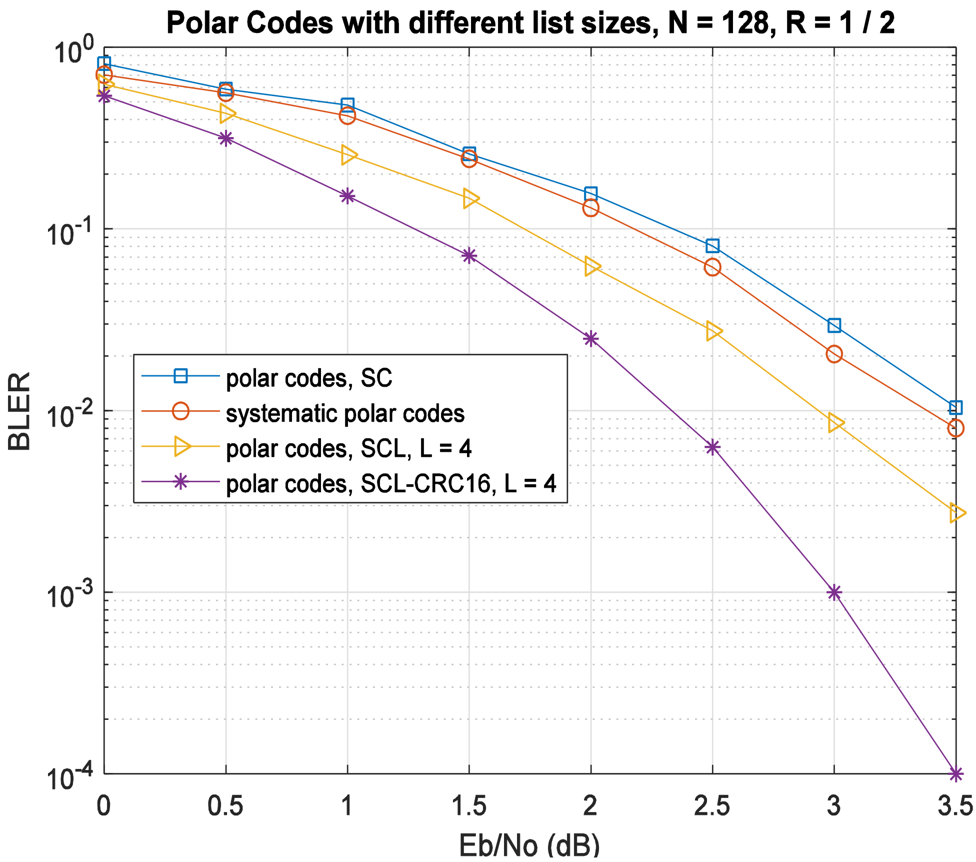

Figure 14. Comparison between different polar codes' decoders

2.5. 5G systems Requirements

5G wireless networks ask for more improvements in networks' structures regarding the quality of offered services, transmission's reliability and system's security. Three important parameters should be improved indeed in order to get high capacity performance in a km^2. The first one is that it requires a hundred times better data rates than previous generations. The second one is that it requires less amount of latency in which it should be 0.5 ms in the radio link. The third parameter is that it asks for a hundred times more connections (links).

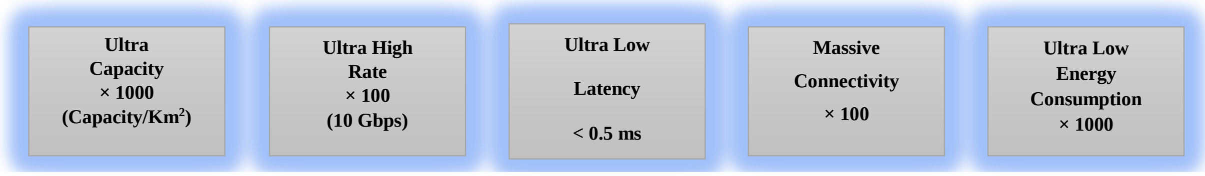

Figure 15. 5G performance targets

These parameters lead to decrease the amount of energy consumption which is better than that used by the worked generations nowadays. The required system performances of the 5G system are drawn in figure 15. We conclude that two vital challenges face the 5G wireless networks. One is that it should exploit the whole available spectrum including low bands to high bands and licensed to unlicensed bands. The second is that it should be flexible to offer suitable support for the different services, a large amount of connectivity and the large quantity of capacity, therefore the 5G system will be able to enhance the spectral efficiency, get high connectivity and decrease the amount of latency [20].

Channel coding is used to correct the error in received data that caused by noise, interference and the weak received signal, therefore, channel coding is very important in cellular communication systems. In 3G and 4G cellular communication systems, turbo codes are used as the channel code however in 3GPP (*3rd Generation Partnership Project*) standardization systems it is debating of either using LDPC codes or Polar Codes. The decision will be made according to the requirements including up to 20 Gbps of throughput in the downlink (DL) comparing to 1Gbps in 4G systems, very low amount of latencies, and more flexibility in order to offer more supports regarding broadband data, Internet of Thing (IoT), vehicular communication and cloud computing [21].

2.5.1. Latency

5G new radio (5G-NR) should have latency less 0.5 ms compared to 10 ms performed by 4G systems [22]. In fact, improving the latency will help to develop and achieve many new applications to 5G systems. These applications are targeted by 5G such as 3D video rendering, UHD screen work and play in the cloud, augmented reality, etc. Unlike humans, machines are very sensitive to delays, therefore low latencies will enable many applications related to machine communications such Massive machine type communications (mMTC) which deal with improving industry automation, self-driving cars, etc. Regarding the latency, it is proven in [23] that polar and LDPC codes outperform turbo codes in case of small packets meanwhile in case of large packets, turbo codes outperform polar and LDPC codes. Polar codes have been studied under 5G systems with different scenarios such as air interface, frame structure, and different modulation waveforms such as Orthogonal Frequency Division Multiplexing (OFDM) and Filtered OFDM (F-OFDM). The result of the test proved that polar codes outperform turbo codes in all cases which make them a strong channel coding candidate to be chosen in 5G systems [24]. The following sections explain the major requirements of 5G systems.

2.5.2. Error correction capability

5G-NR is supposed to achieve just only 1 error block in each 100000 transmitted blocks compared to 1 error block in each 10000 transmitted blocks in 4G communication systems. This improvement will make 5G more powerful in correcting transmitted error blocks and need not depend completely on Hybrid Automatic Repeat Request (HARQ) for retransmitting the error data bits. Although 5G is not depending completely on HARQ, it is still the main parameter to support more error-free transmission [8].

2.5.3. Flexibility

Flexibility is an important factor to design 5G-NR system. Improving the systems' flexibility will help to make 5G more distinctive than previous wireless generations. Flexibility will play the main role in improving applications such as enhanced mobile broadband (eMBB) that deal with IoT, a vehicle to everything (V2X) communications. To improve this factor, coding scheme should have the ability to support a wide range of rates (R) to be used efficiently in the communication systems. For instance, short data bits should be used in typical IoT applications while long data bits should be used in broadband data applications. In the rural areas, we need to use low coding rates because of the sparse distribution of base stations while in urban areas, we need to use high coding rates because of the ultra-dense population. It is required that channel code should support wide range of data block length (K) and data code rate (R) in order to avoid using wasteful data bits and avoid using code rate that causes noise, interference, and degradation in signal strength. Both wasted data bits and undesirable code rate will cause transmission of unwanted data bits and then wasting more spectrum which has a bandwidth, time duration, and energy. This will cause degradation in many important parameters such as throughput, latency, and capability of error correction. Therefore flexibility is an important factor and the chosen code scheme for 5G should have enough flexibility [25].

2.5.4. System complexity

The complexity of 5G-NR should be as low as possible. The system complexity plays the main role in the hardware requirements and the energy consumptions. The three suggested codes for 5G-NR are turbo codes, LDPC codes and polar codes which they have comparable complexity of the decoder, nevertheless, polar codes have the lowest complexity among the three codes because of the absence of iteration at the decoder side [19]. Performing a comparison between the three suggested codes in 5G systems is applied by calculating Maximum, Minimum and Addition operations (MaxMinAdd) [26],[27]. The 4G-LTE turbo decoder achieves 155 MaxMinAdd operations in one iteration [28], and it achieves around 4340 MaxMinAdd operations per data bit in case of applying 6 iterations and 28 iterations as a low degree of parallelism and full degree parallelism of turbo decoder respectively. LDPC codes, on the other hand, achieve 840 MaxMinAdd operations per data bit in case of 28 iterations and low coding rate as R = 1/2, and achieve 560 MaxMinAdd operations per data bit in the same iterations but high coding rate of 5/6 which is lower than that achieved by turbo decoders [29]. SC polar codes calculate the computation complexity by $log_2(K/R)/R$ MaxMinAdd operations. SC with R = 1/2 and K = 4096 has 26 MaxMinAdd per data bit which is the lowest complexity of the three codes.

Although polar codes have the lowest complexity among the three suggested codes, SC has a big problem with error correction capability which is very low and to fix this problem SCL was introduced and in case of L = 32 the complexity of the decoder is comparable to that produced by LDPC codes. Moreover, polar codes perform the lowest BLER compared to LDPC and turbo as shown in the result section in which the parameters that were used to get this result are summarized in table 1. We used the encoder that explained in section 2.4.1 and the decoder that explained in section 2.4.2.3 to extract the polar code's figure. We used the encoder and the decoder that explained in [30] to extract the turbo code's figure. We used the encoder and the decoder explained in [31] to extract the LDPC code's figure.

Table 1. parameters used to extract figure 19.

	Polar Codes	LDPC Codes	Turbo Codes
Blocklength (N)	256	256	255
Rate (R)	1/3	1/3	1/3
Iterations	None	20	2
Modulation	BPSK	BPSK	BPSK
List size-CRC	4-16	None	None

2.6. Applying Polar Codes to 5G Systems

Polar codes have many advantages that make them breakthrough codes in coding theory in which they have the ability to achieve Shannon capacity limit in case of having large enough code length size. Figure 16 shows the trial block diagram of transmitter and receiver with polar codes [32]. Polar codes have many advantages over Turbo codes that used in 4G include: (1) higher gain in which at the same equivalent level polar codes have lower Signal to Noise Ratio (SNR) than that produced by turbo codes [33]. This means higher efficiency and hence higher spectral efficiency will be improved as well. (2) polar codes do not have error floors, therefore, better reliability than turbo codes in which turbo codes have error floors because the sub-optimal algorithm that is used in turbo codes. (3) decoding in polar codes are handled by SC scheme that enormously reduces the complexity of encoding and decoding which is about O(NlogN) hence power consumption is lower than that is done by turbo codes. Turbo codes, LDPC codes and polar codes are three suggested codes for 5G-NR systems. Each one of them has advantages and disadvantages to be used in next-generation systems. Polar codes indeed are considered the promising technique because of the capacity it produce and the absence of error floor nevertheless they have drawbacks such as the supported blocklength has just only powers of two which later solved by using rate matching method [34]. The rate matching suggests a code (M) that is less than mother code (N) and the data bits between the mother code and the suggested code either punctured or shortened. Although the rate matching method solved the problem of coding rate, it causes degradation in the system's reliability. 3GPP standardization for 5G-NR has agreed to deploy polar codes for uplink and downlink control channel and LDPC codes for uplink and downlink data channel [22].

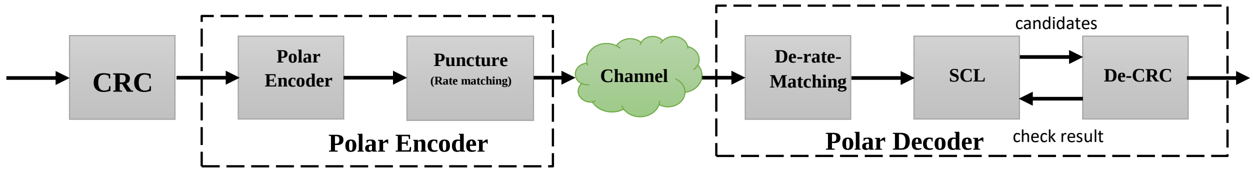

Figure 16. The framework of polar encoder and decoder in 5G trial system

Figure 17 shows the communication model of 5G systems with polar codes. This system is designed to use OFDM or F-OFDM with M subcarriers. Suppose a sequence of (K) bits gathered in (d) is encoded by using polar codes and produce (c) binary sequence with a blocklength of N and rate of $R = K/N$. The codeword (c) passes through the bit interleaver section in order to enhance the polarization [35].

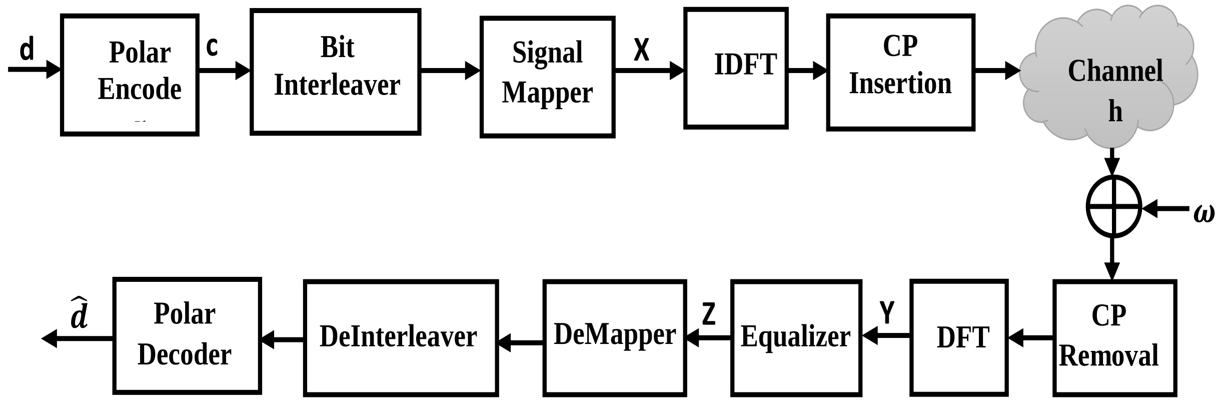

Figure 17. The Communication System Model of 5G with Polar Codes.

Let us suppose we have a signal with two-dimensional constellation χ which has a size of $|\chi| = 2^m$. The output bits from the interleaver section go to the mapper section with a sequence of $N_s = M = N/m$ in which they are in frequency-domain and $X = \{X_i \in \chi, 1 \leq i \leq N_s\}$. The transmitted energy per symbol is represented by the average of $E_s = E[|X_i|^2]$. Hence, the average SNR is equal to $SNR = E_s/N_o = \rho * E_b/N_o$, where ρ represents the spectral efficiency and $\rho = mR$, and E_b represents the average energy in every information bit. The X signal enters into Inverse Discerete Fourier Transform (IDFT) in order to obtain the symbols in time-domain. Then, Cyclic Prefix (CP) is used to delete the noise produced from the multipath propagation such as Inter-Symbol Interference (ISI). At the receiver side, the CP is removed and the signal is passed to Discrete Fourier Transform (DFT) to produce the symbols Y in which they are in frequency-domain which $Y = \{Y_i \in \Upsilon, 1 \leq i \leq N_s\}$, i.e.

$$Y^T = FhF^HX^T + F\omega = HX^T + W^T \tag{22}$$

The parameters in equation 22 are defined as, F is the unitary of DFT matrix, F^H is the IDFT matrix of size M, h *and* H represent the channel matrix of size $N_s \times N_s$ in case of time-domain and frequency-domain respectively, ω represents the independent and identically distributed (i.i.d) complex noise vector in which it has zero mean and $N_o/2$ variance, and $W = \{W_i\}$ represents the noise in frequency-domain where W has the same properties as noise in time-domain (ω) which $W_i \sim cN(0, \sigma_n^2)$. The produced signal Y from the DFT section passes through the equalizer in which Linear Minimum Mean Square Error (LMMSE) mostly use to eliminate the Inter-Carrier Interference (ICI).

Let us suppose that we use perfect Channel State Information (CSI) at the receiver and CSI is not known at the transmitter side. The output from the equalizer on the i-th subcarrier is produced by,

$$Z_i = U_iH_iX_i + U_iW_i \tag{23}$$

Which U_i represents the coefficient of equalizer and it is equal to $U_i = \frac{1}{H_i^H H_i + \sigma_n^2}$, and H_i denotes to the i-th row of frequency-domain of matrix (H). The symbols produced from the equalization section is equivalent to $\frac{\sigma^2}{||H_i||^2 + \sigma_n^2}$ and pass them to de-interleaver section then polar decoder section to decode these symbols and produce the equivalent transmitted data $\hat{d}$ [36].

3. Results

Polar codes are considered a promising technique to be used in 5G systems because of the advantages they provide. In this paper, we showed that polar codes are the first codes that have the ability to achieve the system capacity of Shannon limit. Furthermore, they have the lowest system complexity among the competitive codes that might be used in 5G systems which are LDPC and turbo codes. Polar codes are free of error floor compared to LDPC and turbo codes which make them preferred in 5G systems. Although they have many advantages, they suffer from drawbacks such as the equipment for designing decoders are complicated which make them expensive.

Figure 18 shows the BLER calculations of LDPC, turbo, and polar codes. In this figure, LDPC codes used 20 iterations, turbo codes used 2 iterations, and polar codes used L = 4 and CRC = 16. From the figure, it is clear that polar codes achieve better performance than turbo and LDPC codes. Turbo codes achieve better performance than LDPC, however, it is not preferred to be used in 5G-NR because of the high complexity it is performed. Polar codes achieve a better result in case of control channels and LDPC codes achieve a better result in case of data channels, therefore, 3GPP standardization decided to use polar codes in control channels and LDPC in data channels [22].

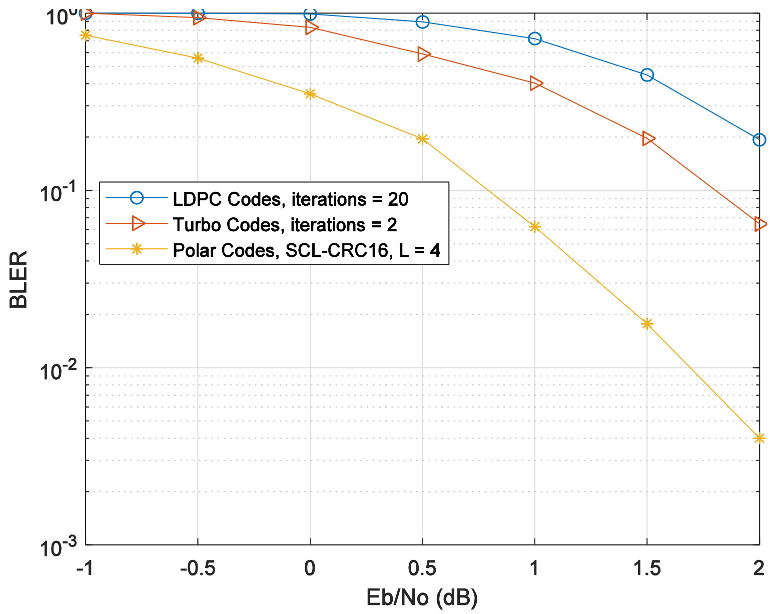

Figure 18. The BLER calculations of different code schemes.

4. Discussion and Conclusion

During this study, we discussed the new channel coding scheme, polar codes. Polar codes are the promising technique to be used in the next generations wireless communications systems. Polar codes are proven to be the codes that have the lowest complexity among other codes such as turbo and LDPC codes in case of SC decoder. There are intensive studies on polar codes to use them in 5G systems. We also presented a comparison between turbo, LDPC and polar codes in many aspects such as BLER, error floor, systems complexity. Polar codes achieved the lowest BLER in which we used SCL-CRC16 and turbo codes with 2 iterations and 20 iterations of LDPC codes. Although polar codes perform better than turbo and LDPC codes in many aspects such as the complexity, BLER calculation and the absence of error floor. The drawbacks of polar codes include the code length must be a power of two which cause coding rate problem. The complicated design of polar decoders make them expensive, therefore many studies still searching how to use these codes efficiently.

In this study, we explained clearly the channel polarization that produces polar codes. These codes are considered a breakthrough in coding systems because they are the first provably codes that attain Shannon limit. Polar codes are promising techniques that will be used in 5G systems because they have advantages which outperform turbo codes that used in previous generations of cellular communications systems. These advantages include higher gain than other codes and there are no error floors like turbo and LDPC codes. 5G systems seek more improvements because of the new applications it will offer, therefore the channel coding scheme should have the ability to correct the errors in the received data. Moreover, polar codes have the ability to work better in control channels, hence it is dedicated to control channels in 5G-NR systems. Regarding the complexity of the system, polar codes achieve the lowest complexity over turbo and LDPC codes. SCL decoder achieves better in systems performance however it increases the complexity and latency of the system. Accordingly, there are intensive studies on polar codes to find a decoder that performs better in both complexity and BLER calculations.

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