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NON – LINEAR BEHAVIOUR OF SIDESWAY FRAMES AS AFFECTED BY COLUMN RIGIDITY

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ÖZET

Betonarme kolonlarının etkili bir şekilde burkulması üzerine bir çalışma sunulmaktadır. Bunlara etki eden faktörler ise eksenel yük, boyuna donatı yüzdesi, hacimsel fret donatı yüzdesi, yükseklik/genişlik oranı beton dayanımı ve en önemlisi ise creept sayılabilir . Yukarda bahsi geçen parametrelerin etkisini gösteren beton ile ilgili genel formülasyonundan da faydalanılmıştır. Gerçekçi bir şekilde basınç–moment eğrisi (N-M-K) arasındaki bağıntı da geliştirilmiştir. Gözlemler EI'nin kolonlar üzerine en etkili faktörlerin eksenel yük, boyuna çelik yüzdesi, ve creept olduğunu göstermiştir. EI'nin kolonlar için ACI'nın genel ifadesi malzeme non-lineer, çatlama ve creept için değiştirilmiştir. Bu değiştirilmiş EI ifadesi, moment büyütme faktörlerinin (ki bunlar birden fazla test edilmiş çerçeve faktörleridir) tahmininde kullanılmıştır.

SUMMARY

The effective flexural rigidity of reinforced concrete columns is studied as affected by the level of axial load, longitudinal steel percentage, volumetric percentage of stirrups, width/height ratio, concrete strength and most importantly creep. Use is made of a general expression for the σ-ε relationship of concrete which reflects the effect of the above stated parameters. Realistic thrust-moment-curvature (N-M-K) relationships are developed. Observations on the findings show that the level of axial load, longitudinal steel percentage and creep are most effective on column EI. The general form of the ACI expression for column EI has been modified for material non-linearity, cracking and creep. This modified EI expression has been used to predict the moment magnification factors of several tested frames. The results are in good agreement.

INTRODUCTION

In the design of earthquake resistant structures, conceptual correctness is much more important than mathematical precision. If the nature of damage of structures which have been subjected to earthquakes are carefully studied, the principles of conceptual correctness can be formulated.

Extensive research data have proven that each and every earthquake is "one of a kind" which leads to the anticipation that all future earthquakes will also be one of a kind. Therefore, it seems wiser that the design engineer make observations on the structural behavior and damage under earthquakes and design accordingly.

Observations and laboratory tests have shown that it is possible to design structures to minimize the damage on the non-structural elements as well as the structural system. Considering the fact that non-structural parts of a building compose a significant part of the cost of the structure, preventing damage of such elements gains significant importance. This necessitates a change of view on part of the design engineer. Instead of focusing attention on the character of the ground motion, emphasis must be placed on understanding the structural behavior under earthquakes.

Preventing damage on the structural system as well as the non-structural elements is only possible by controlling the sway. In doing so, controlling relative sway (defined as the sway at any point along the height of the structure divided by the height of the specied point) becomes a useful measure to monitor.

At this point, the design engineer must realistically answer the following question: What is the relationship between relative sway and degree of non-structural damage? Compiled data of laboratory experiments have been presented in Fig.1 [1]. In the figure, the level of damage is represented by an index where level 1 corresponds to damage beyond repair. When relative sway is small, there is a large scatter of damage depending on the type of the partition. However, when relative sway exceeds 1%, the degree of damage beyond repair definitely occurs. In technical literature, it has been suggested that relative sway be restricted to 1.5% to prevent damage to the structural as well as non-structural elements [2].

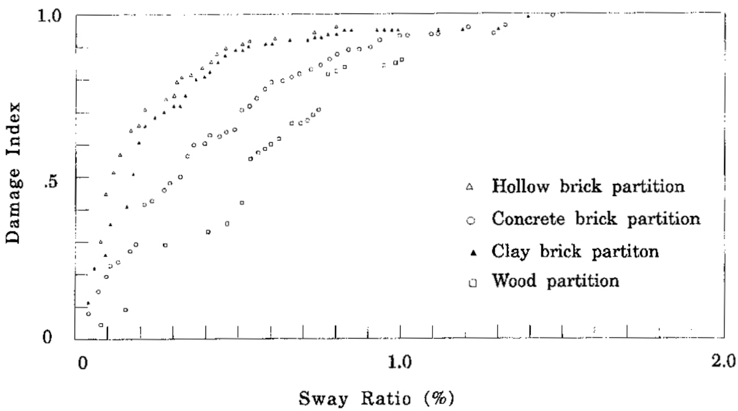


Fig.1. Relationship between partition damage and sway ratio.

Obviously, the above discussion calls for a realistic determination of the flexural rigidity of frames subject to sidesway. This flexural rigidity must consider all important parameters that affect the non-linear behavior of reinforced concrete, and the reinforced concrete column, in particular.

EFFECTIVE COLUMN FLEXURAL RIGIDITY

Reinforced concrete is a composite material which reflects many non-linearities. The behavior is further complicated by the existence of cracking, axial load and the elusive property called creep. All important parameters must be considered if a realistic expression to yield the effective flexural rigidity is desired.

Stress-Strain Relationship of Steel

Steel is assumed to be perfectly elasto-plastic. No strain hardening is considered.

Stress-Strain Relationship of Concrete

The primary σ-ε curve for concrete (specimens failed in 2-3 minutes) is quite well studied and established, a classical example being the "Hognestad stress block". However, the question of time-dependent variations of the σ-ε relationship of concrete is still not well answered. The main reason for this vagueness is that time-dependent effects are influenced by a great number of variables, some of which are quite difficult to control and formulate such as ambient humidity, temperature, aggregate type, cement type, etc. It now seems that time-dependent effects of concrete spring from the very basic texture of concrete, and there is developing research on the subject.

In this study, the general form of the σ-ε relationship of concrete in compression is given by [3],

$$\sigma = \frac{A\left(\varepsilon/\varepsilon_o\right) + \left(D-1\right)\left(\varepsilon/\varepsilon_o\right)^2}{1 + \left(A-2\right)\left(\varepsilon/\varepsilon_o\right) + D\left(\varepsilon/\varepsilon_o\right)^2} \tag{1}$$

ε = the compressive strain, ε_o = compressive strain corresponding to maximum stress and

E_c = modulus of elasticity. The parameters A and D are explained below.

$$A = \frac{E_c\,\varepsilon_o}{k_3\,f_{ck}} \tag{2}$$

where k_3 is the ratio of the maximum stress to the characteristic cylinder strength. The expression of k_3 can be expressed as below considering all possible factors,

$$k_3 = C\left(k_{3d}\right)\left\{k_{3t}\,\left(1+k_{3v}\right)\frac{1-C}{C} + 10^{-2}\left[0.7\left(\frac{5}{k_d}\right)^{0.5} + 1.5\left(1-0.25\right)\frac{s}{b_c}\right]\frac{g_c\ddot{f}_{yk}}{\left(f_{ck}\right)^{0.5}}\left(S_f\right)\right\} \tag{3}$$

$$C = \frac{b_e\,h_e}{b_w\,h} \quad \text{and} \quad S_f = \frac{2b_e\,h_e}{b_e^2 + h_e^2} \tag{4}$$

where s= spacing of lateral reinforcement, $\ddot{\rho}_c$ = volumetric ratio of lateral reinforcement calculated over enclosed core area, k_{3d} = size effect parameter, k_{3t} = time effect parameter, k_{3u} = cover effectiveness factor, k_d = depth of neutral axis, C = enclosed core ratio,

S_f = shape factor, $\ddot{f}_{yk}$ = characteristic yield stress of lateral reinforcement,

b_e, h_e = width and depth of inside to inside of lateral reinforcement.

Modulus of elasticity E_c is mainly affected by the concrete strength, creep and strain gradient. The expression for E_c is given as follows:

$$E_{ct} = \frac{256262\left(f_{ck}+1120\right)^{1/3}}{1+\Phi_c} \tag{5}$$

Creep effect is introduced into Ec by the coefficient f_c which is the function of five different coefficients as expressed in CEB Recommendations [4].

The expression of ε_o which is a function of time is given below.

$$\varepsilon_o = \varepsilon_u[0.05(\log^2 t + 0.5\log t + 18)] \tag{6}$$

where ε_u is the ultimate strain.

The parameter D is introduced in the σ-ε expression in order to control the slope of the descending branch. Many of the expressions of σ-ε curve for concrete previously proposed assume either D = 0 or D = 1.0. In this paper, D is taken as 0.20 as a more realistic value.

Thrust - Moment - Curvature Relationship (N-M-K)

Thrust-moment-curvature (N-M-K) relationships are obtained by solving a set of equations which are dictated by equilibrium, stress-strain relationship and compatibility conditions. It is possible to obtain time-dependent N-M-K relationships by using time-dependent σ-ε relationships, since the other two conditions to be satisfied are independent of time [5].

In order to obtain the N-M-K relationship, the cross-section is divided into finite areas and a piecewise numerical integration is performed since the σ-ε relationships are non-linear. The governing equations are presented in Fig.2, where b is the width of the beam, dh the differential height of the finite area, A_s' the area of compression steel, A_s the area of tension steel, E_s the modulus of elasticity of steel, C_s the force in the compression steel, C_c the compression force on concrete, T_c the tension force on concrete, T_s the force in the tension steel. The maximum concrete strain in compression is ε_4 and the maximum concrete strain in tension is ε_1, while ε_3 and ε_1 are the strains in the compression and tension steel, respectively. K is the resulting curvature of the cross-section.

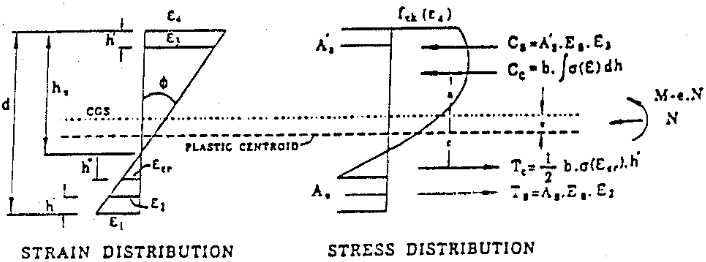


Fig.2. Governing equations for P-M-K relationship.

Equilibrium of forces:

$$N = b \, . \int_{o}^{h_x} \sigma(\varepsilon) dh - \frac{b}{2}\sigma(\varepsilon_{cr})h'' + A_s{}'.E_s.\varepsilon_3 - A_s.E_s.\varepsilon_2 \tag{7}$$

Moment about geometric centroid:

$$M = \left(\frac{D}{2} - h^1\right).(C_s + T_s) + C_c.a + T_c.c + N.e \tag{8}$$

Compatibility of strains:

$$\frac{\varepsilon_4}{h_x} = \frac{\varepsilon_3}{h_x - h'} = \frac{\varepsilon_2}{d - h_x - h'} = \frac{\varepsilon_1}{d - h_x} = K \tag{9}$$

EQUIVALENT FLEXURAL RIGIDITY OF COLUMNS

Using the developed computer program, N-M-K relationships were obtained to bring out the effect of the variables under study. Attention was paid to keep all other variables constant except the variable whose effect was searched. A typical set of N-M-K curves is shown in Figs.3. On the N-M-K curves, the secant flexural rigidity was taken as the basis of comparison. The secant rigidity was drawn at a point which corresponds to 0.85 M_u where M_u is the peak moment of the N-M-K curves.

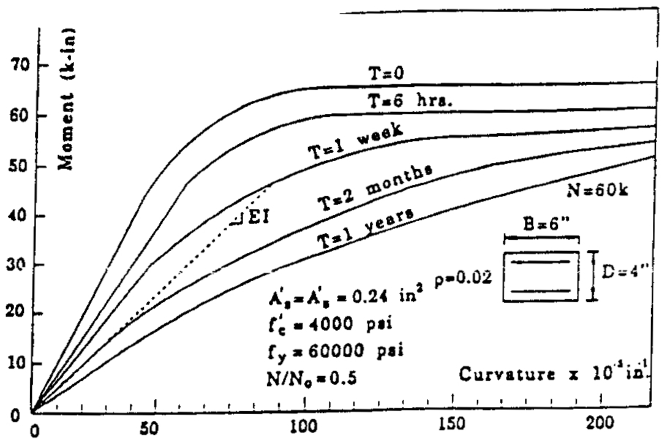


Fig.3. Time-dependent moment-curvature at $N/N_0 = 0.50$.

Fig.4 shows a typical relationship between the flexural rigidity and time as a function of N/N_O. Such relationships are developed for various cross-sections to bring out the effect of the parameters studied.

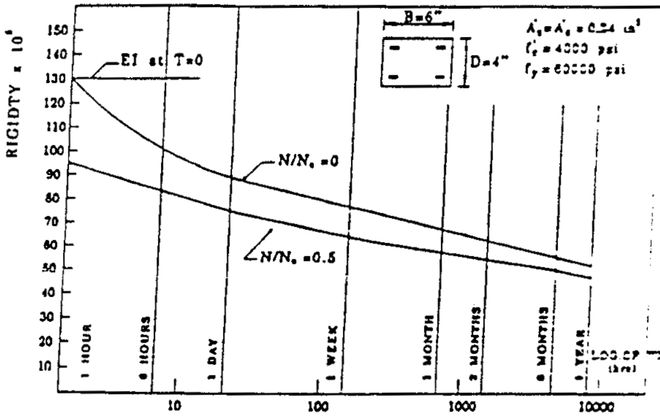


Fig.4. Variation of flexural rigidity as a function of time.

A study of the results has shown that the most influencing parameters are steel percentage, level of axial load and creep. These effects must be reflected in the flexural rigidity of a column if it is to be realistic. The general expression of effective EI for columns is taken as given by the American Concrete Institute (ACI) Building Code [6].

$$EI = \frac{\frac{E_c I_g}{\alpha} + E_c I_c}{1+\beta} \tag{10}$$

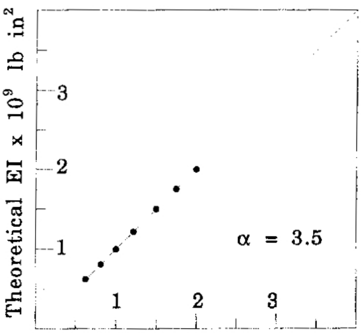
$E_c I_g$ = gross flexural rigidity

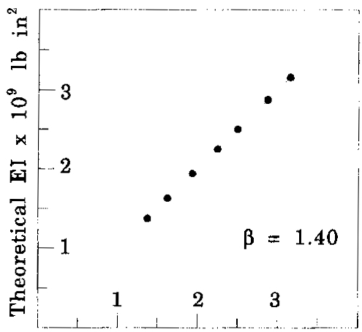
$E_e I_e$ = rigidity contributed by steel

α = factor to represent the effect of material non-linearity and cracking

β = factor to represent the effect of creep

The factors a and b are studied as a function of N/N_0, as presented in Fig.5, and Fig.6, for the selected value of N/N_0 = 0.50 and at time of 1 year after initial loading. The values of a and b are determined which make the theoretical computer rigidities agree with Eq[7].





$N/N_0 = 0.50$

$$\frac{\frac{E_c I_g}{\alpha} + E_c I_c}{1+\beta}$$

$$\left(\frac{E\ I_g}{\alpha} + E_c I_c\right) x\, 10^9$$

$N/N_0 = 0.50$, $\alpha = 3.5$ T=1 Year

Fig.5. Determination of α from the theoretical flexural rigidity.

Fig.6. Determination of a from the theoretical flexural rigidity.

The above described procedure is extended to cover a wider ranges of N/N_O and time intervals.

Table 1 is prepared for values of a and Table 2 for values of β. As can be observed from Table 1, α values range from 3.5 to 5.5 for low and high values of N/N_O, respectively. The ACI value for α is 5.0.

ACI defines β as the ratio of dead load moment to total load moment. This definition obviously yields again a constant value for β. However, the duration of sustained load and level of axial load N/N_O have a very significant effect on the value of β, as presented in Table 2.

Table 1. Variation of α as a function of N/N_O

N/N_O	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
a	3.5	3.5	3.5	3.5	3.5	3.5	4.0	4.6	5.5

Table 2. Variation of β as a function of time and N/N_O.

N/N_O	0.40			0.45			0.50		
Time	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
b	0.50	0.90	1.10	0.50	0.90	1.20	0.50	1.0	1.30

N/N_O	0.55			0.60			0.65		
Time	T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
b	0.60	1.10	1.40	0.70	1.20	1.50	1.00	1.30	1.80

If a simple comparison between calculated and ACI proposed values of α is to be made, assume α = 0.7 as a typical value for ratio of dead moment to total load moment. This yields a constant value of 1.43 for β. Contrary to this constant value, computer results show a variation between 0.5 - 2.9, depending on the level of axial load, N/N_O.

COMPARISON OF PROPOSED EI WITH EXPERIMENTAL DATA

To test the validity of the developed expression, two sets of frame tests done by Ferguson and Breen [7], and Rad [8] were considered. The experimental momentmagnification factors (F-values) are compared with those obtained by using Eq.7.in conjunction with Table 2. Results obtained by the ACI expression of EI (a = 5.0) are also included. The comparisons are given in Table 3 and Table 4.

Table 3. F-Values of Ferguson and Breen's frames

Frame	Test	From Eq.(7)	%Error	FromACI	%Error	N/No
L1	4.7	5.5	17	4.0	15.0	0.35
L2	2.35	2.18	7.2	3.04	29.4	0.25
L5	1.4	1.36	2.8	1.42	1.4	0.40
L6	2.3	1.60	30.4	1.62	30.4	0.55
L7	3.4	3.0	11.8	4.64	36.5	0.45

Table 4 F-Values of Rad's frames

Frame	Test	FromEq. (7)	%Error	FromACI	%Error	N/No
A1	1.33	1.56	17.3	2.22	66.9	0.60
A2	1.86	1.99	7.0	2.50	34.4	0.50
A3*	1.84	1.76	4.3	2.38	29.3	0.55
A4*	1.50	2.02	34.7	3.09	10.6	0.50
A5	1.87	1.86	0.05	2.83	51.3	0.60

* Frames A3 and A4 have columns of unequal area and moment of inertia.

A careful study of Table 3 and Table 4 shows that the results of the proposed EI (Eq.7) are a definite improvement over ACI (% error is usually much smaller than of ACI). It must be stated here that the validity of β values cannot be tested because experimental results are lacking. However, from the nature of β, engineering judgment will expect a much better correlation if the proposed values of β are used instead of a constant value of β, as ACI proposes.

CONCLUSIONS

It can be concluded that $\alpha = 3.5$ can be used for normal ranges of N/N_0 in evaluating the equivalent flexural rigidity of a column. More experimental data are needed to test the validity of the developed expression under high axial load levels, applied eccentricities and slenderness ratios.

Experimental results are also lacking to test the validity of the proposed values of β. Such experiments are strongly needed, even though they are costly and time taking. However, the proposed values of β seem to be a refinement in reflecting creep effect than merely using the ratio of dead load moment and total load moment as ACI proposes.

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