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Araştırma Makalesi / Research Article

Constructions Of Traveling Wave Solutions Of The Fractional Nonlinear Model Of The Low-Pass Electrical Transmission Lines

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Abstract

In this article, the structure of the improved $tan(\varphi/2)$-expansion method and the simplest equation method are applied. The fractional nonlinear model of the low-pass electrical transmission lines via Atangana-Baleanu derivative operator is taken into consideration and exact solutions have been constructed of this equation using proposed methods. This article explores the applicability and effectiveness of these methods on fractional nonlinear evolution equations.

Alçak Geçiren Elektrik İletim Hatlarının Kesirli Mertebeden Lineer Olmayan Modelinin İlerleyen Dalga Çözümlerinin Oluşturulması

Öz

Bu makalede, geliştirilmiş $tan(\varphi/2)$-açılım yöntemi ve en basit denklem yöntemi uygulanmıştır. Alçak geçiren elektrik iletim hatlarının Atangana-Baleanu türev operatörü aracılığıyla kesirli mertebeden lineer olmayan modeli dikkate alınmış ve önerilen yöntemler kullanılarak bu denklemin tam çözümleri oluşturulmuştur. Bu makale, bu yöntemlerin kesirli doğrusal olmayan evrim denklemleri üzerindeki uygulanabilirliğini ve etkinliğini araştırmaktadır.

Anahtar kelimeler
Atangana-Baleanu türev operatörü; geliştirilmiş $tan(\varphi/2)$-açılım yöntemi ; en basit denklem yöntemi

1. Introduction

Fractional nonlinear evolution equation is one of the branches of science that has attracted attention especially in recent years. Fractional analysis studies that started with the discussion between L'Hospital and Leibniz have attracted the attention of researchers for many years. In addition to mathematics, it has a very deep physical application area where it can formulate many different phenomena in different fields such as physics, engineering sciences, economics, chemistry, signal processing, rheology, diffusion processes, food supplements, semi-chaotic dynamic systems, mechanics-mechatronics, seismology, hydrodynamics. Due to its wide scope and diverse applications in different disciplines, the importance of exact (analytical) and numerical solutions of fractional differential equations has increased. Many methods such as the direct algebraic method (Rezazadeh *et al*. 2017), the sinh-Gordon function method (Yokuş *et al.* 2020), the decomposition method (Ray 2006), the discrete homotopy perturbation method (Özpınar 2020), the finite forward difference method (Yokuş 2020), the modified homotopy analysis transform method

(Morales-Delgado *et al.* 2018), the $(m+1/G')$-expansion method (Durur *et al.* 2020), the sub-equation method (Tasbozan *et al.* 2019, Yokuş *et al.* 2020), the (G'/G) method (Durur 2020, Shang and Zheng 2013), fractional sub-equation method (Yaşar and Yıldırım 2018), (1/G')-expansion method (Durur ve Yokuş 2019, Durur ve Yokuş 2020, Yokuş *et al.* 2020, Yokuş *et al.* 2020), the generalization exponential rational method (Khater *et al.* 2020), the modified auxiliary equation method (Alderremy *et al.* 2019) etc. to be used to reach solutions have been proposed.

The most popular definitions in the fractional mathematics world are the Riemann-Liouville, Grünwald-Letnikov and Caputo (Podlubny 1999, Caputo 1967, Caputo and Fabrizio 2015) definitions. Atangana-Baleanu (Atangana and Baleanu 2016) fractional derivative and integral due to Caputo and Riemann-Liouville fractional derivatives have played an important role in mathematical modeling these days.

In this study, we construct the solutions for the fractional nonlinear model of the low-pass electrical transmission lines which is given by (Abdou and Soliman 2018)

$$\mathcal{D}_{tt}^{2v}W-\alpha\mathcal{D}_{tt}^{2v}W^2+\sigma\mathcal{D}_{tt}^{2v}W^3-\lambda^2\mathcal{D}_{xx}^{2v}W-\frac{\lambda^4}{12}\mathcal{D}_{xxxx}^{4v}W=0, \tag{1}$$

where $W=W(x,t)$ is the function that is used to describe the dynamical behavior (the voltage) of the nonlinear wave processes low-pass electrical transmission lines. Additionally, $\alpha,\ \lambda,\ \sigma$ are arbitrary constants while $0<v<1$. The variable x is interpreted as the propagation distance and t is the slow time. The physical details of the derivation of Eq. (1) using the Kirchhoffs laws are given in (Abdou and Soliman 2018).

Applying the following definition of ABR fractional operator (Atangana and Gomez-Aguilar 2018) to Eq. (1);

Definition 1 *It is given by (Fernandez et al. 2019)*

$$^{\mathrm{ABR}}\mathcal{D}_{a+}^{v}\mathcal{F}(t)=\frac{\mathcal{B}(v)}{1-v}\frac{d}{dt}\int_a^t\mathcal{F}(x)\mathcal{G}_v\left(\frac{-v(\mathrm{t}-v)^v}{1-v}\right)dx, \tag{2}$$

where $\mathcal{G}_v$ is the Mittag-Leffler function which is defined by

$$\mathcal{G}_v\left(\frac{-v(\mathrm{t}-v)^v}{1-v}\right)=\sum_{n=0}^{\infty}\frac{\left(\frac{-v}{1-v}\right)^n(t-x)^{vn}}{\Gamma(vn+1)}, \tag{3}$$

and $\mathcal{B}(v)$ being a normalisation function.

Thus

$$^{\mathrm{ABR}}\mathcal{D}_{a+}^{v}\mathcal{F}(x)=\frac{\mathcal{B}(v)}{1-v}\sum_{n=0}^{\infty}\left(\frac{-v}{1-v}\right)^n\ {}^{RL}\mathcal{J}_a^{vn}\mathcal{F}(x), \tag{4}$$

leads to $W(x,t)=u(\zeta)$,

$$\zeta=(1-v)(x^{-vn}+kt^{-vn})\big(B(v)\big)^{-1}\left(\sum_{n=0}^{\infty}\left(-\frac{v}{1-v}\right)^n\Gamma(1-vn)\right)^{-1}, \tag{5}$$

where k is the speed of the traveling wave.

Using wave transformation given in (5), Eq. (1) can be converted to ordinary differential equation (ODE). Twice integration of the obtained ODE with zero constant of the integration, gives

$$(k^2-\lambda^2)u(\zeta)-\alpha\,k^2\big(u(\zeta)\big)^2+\sigma\,k^2\big(u(\zeta)\big)^3-1/12\,\lambda^4\frac{d^2}{d\ \zeta^2}u(\zeta)=0. \tag{6}$$

In the next sections, we will examine an ordinary differential equation (ODE) obtained above. The remainder of this paper is divided into five sections. In Sects. 2 and 3, methods are described briefly. In Sects. 4 and 5, proposed methods are applied to the model equation. In Sect. 6, results and discussions are given. In Sect. 7, conclusions and recommendations for future study are presented.

2. The improved $\boldsymbol{tan(\varphi/2)}$-expansion method

In this section, the mathematical architecture (Manafian *et al.* 2016) is used to product exact traveling wave solutions. Consider the general nonlinear partial differential equation (NLPDE) for $q(x,t)$ is given by,

$$P(q,q_x,q_t,q_{xx},q_{tt},\dots)=0. \tag{7}$$

Taking into account wave transformation $\zeta = kx + vt$, one can gain

$$Q(q, kq', vq', k^2q'', v^2q'', \dots) = 0. \quad (8)$$

The solution of Eq. (8) can be articulated as

$$q(\zeta) = S(\varphi) = \sum_{j=-M}^{M} A_j[\rho + tan(\varphi/2)]^j, \quad (9)$$

where A_j $(0 \le j \le M)$ and $A_{-j} = B_j$ $(1 \le j \le M)$ are constants and ρ is arbitrary constant, such that $A_j \neq 0$, $B_j \neq 0$ and $\varphi = \varphi(\zeta)$ is the solution of the following first order differential equation:

$$\varphi'(\zeta) = \gamma\sin(\varphi(\zeta)) + \beta\cos(\varphi(\zeta)) + \theta. \quad (10)$$

If we try to find the solution of the (10), then we obtain special solutions that vary according to the state of the coefficients:

Family 1. When $\Delta = \gamma^2 + \beta^2 - \theta^2 < 0$ and $\beta - \theta \neq 0$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{\gamma}{\beta-\theta} - \frac{\sqrt{-\Delta}}{\beta-\theta}\tan(\frac{\sqrt{-\Delta}}{2}\bar{\zeta})\right]$.

Family 2. When $\Delta = \gamma^2 + \beta^2 - \theta^2 > 0$ and $\beta - \theta \neq 0$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{\gamma}{\beta-\theta} + \frac{\sqrt{\Delta}}{\beta-\theta}\tanh(\frac{\sqrt{\Delta}}{2}\bar{\zeta})\right]$.

Family 3. When $\Delta = \gamma^2 + \beta^2 - \theta^2 > 0$, $\beta \neq 0$ and $\theta = 0$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{\gamma}{\beta} + \frac{\sqrt{\beta^2+\gamma^2}}{\beta}\tanh\left(\frac{\sqrt{\beta^2+\gamma^2}}{2}\bar{\zeta}\right)\right]$.

Family 4. When $\Delta = \gamma^2 + \beta^2 - \theta^2 < 0$, $\theta \neq 0$ and $\beta = 0$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{-\gamma}{\theta} + \frac{\sqrt{\theta^2-\gamma^2}}{\theta}\tan\left(\frac{\sqrt{\theta^2-\gamma^2}}{2}\bar{\zeta}\right)\right]$.

Family 5. When $\Delta = \gamma^2 + \beta^2 - \theta^2 > 0$, $\beta - \theta \neq 0$ and $\gamma = 0$, then $\varphi(\zeta) = 2\tan^{-1}\left[\sqrt{\frac{\beta+\theta}{\beta-\theta}}\tanh\left(\frac{\sqrt{\beta^2-\theta^2}}{2}\bar{\zeta}\right)\right]$.

Family 6. When $\gamma = 0$ and $\theta = 0$, then $\varphi(\zeta) = \tan^{-1}\left[\frac{e^{2\beta\bar{\zeta}}-1}{e^{2\beta\bar{\zeta}}+1}, \frac{e^{2\beta\bar{\zeta}}}{e^{2\beta\bar{\zeta}}+1}\right]$.

Family 7. When $\beta = 0$ and $\theta = 0$, then $\varphi(\zeta) = \tan^{-1}\left[\frac{e^{2\gamma\bar{\zeta}}}{e^{2\gamma\bar{\zeta}}+1}, \frac{e^{2\gamma\bar{\zeta}}-1}{e^{2\gamma\bar{\zeta}}+1}\right]$.

Family 8. When $\gamma^2 + \beta^2 = \theta^2$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{\gamma\bar{\zeta}+2}{(\beta-\theta)\bar{\zeta}}\right]$.

Family 9. When $\gamma = \beta = \theta = r\gamma$, then $\varphi(\zeta) = 2\tan^{-1}\left[e^{r\gamma\bar{\zeta}} - 1\right]$.

Family 10. When $\gamma = \theta = r\gamma$ and $\beta = -r\gamma$, then $\varphi(\zeta) = -2\tan^{-1}\left[\frac{e^{r\gamma\bar{\zeta}}}{e^{r\gamma\bar{\zeta}}-1}\right]$.

Family 11. When $\theta = \gamma$, then $\varphi(\zeta) = -2\tan^{-1}\left[\frac{(\gamma+\beta)e^{\beta\bar{\zeta}}-1}{(\gamma-\beta)e^{\beta\bar{\zeta}}-1}\right]$.

Family 12. When $\gamma = \theta$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{(\theta+\beta)e^{\beta\bar{\zeta}}+1}{(\beta-\theta)e^{\beta\bar{\zeta}}-1}\right]$.

Family 13. When $\theta = -\gamma$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{e^{\beta\bar{\zeta}}+\beta-\gamma}{e^{\beta\bar{\zeta}}-\beta-\gamma}\right]$.

Family 14. When $\beta = -\theta$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{\gamma e^{\gamma\bar{\zeta}}}{1-de^{\gamma\bar{\zeta}}}\right]$.

Family 15. When $\beta = 0$ and $\gamma = \theta$, then $\varphi(\zeta) = -2\tan^{-1}\left[\frac{\theta\bar{\zeta}+2}{\theta\bar{\zeta}}\right]$.

Family 16. When $\gamma = 0$ and $\beta = \theta$, then $\varphi(\zeta) = 2\tan^{-1}[\theta\bar{\zeta}]$.

Family 17. When $\gamma = 0$ and $\beta = -\theta$, then $\varphi(\zeta) = -2\tan^{-1}\left[\frac{1}{\theta\bar{\zeta}}\right]$.

Family 18. When $\gamma = 0$ and $\beta = 0$, then $\varphi(\zeta) = \theta\zeta + C$.

Family 19. When $\beta = \theta$, then $\varphi(\zeta) = 2\tan^{-1}\left[\frac{e^{\gamma\bar{\zeta}}-\theta}{\gamma}\right]$, where $\bar{\zeta} = \zeta + C$, $\rho, A_0, A_i, B_i (i = 1,2,\dots,M), \gamma, \beta$ and θ are constants to be determined later.

As is often done in similar methods, balancing the highest order derivatives with the highest order nonlinear terms in Eq. (8), one can acquire j. Following determining j, if Eq. (9) is substitued into reduced equation (8), an algebraic equations set which contains $tan(\varphi/2)^i$, $cot(\varphi/2)^i$, $(i = 0,1,2,\dots)$ is obtained. Then setting each coefficient of $tan(\varphi/2)^i$, $cot(\varphi/2)^i$ to zero, we can get a set of over-determined equations for $A_0, A_i, B_i(i = 1,2,\dots,M), \gamma, \beta, \theta$ and ρ. Using computer programming such as Maple, and Mathematica obtained system can be solved. Finally, $A_0, A_1, B_1, \dots, A_M, B_M, \rho$ are replaced in the Eq. (9).

3. The Simplest Equation Method

In this section we will outline the simplest equation method proposed by Kudryashov (Kudryashov 2005, Kudryashov 2005) in 2005. Consider a NLPDE given in Eq. (7) and (8). Suppose that Eq. (8) have solutions in the following form:

$$q(\zeta) = \sum_{i=0}^{M} A_i\left(P(\zeta)\right)^i, \tag{11}$$

where $P(\zeta)$ satisfies the well-known Bernoulli and Riccati equations. Using of the balancing principle, the number M can be calculated here. The coefficients $A_0, A_1, \dots, A_M$ are constants.

If these well-known equations are used respectively, the form the solutions will have is given below.

For the Bernoulli equation:

$$P'(\zeta) = AP(\zeta)^2 + BP(\zeta), \tag{12}$$

where A and B are arbitrary constants. This equation is a well-known nonlinear ODE. The solution is represented as follows

$$P(\zeta) = \frac{B(\cosh[B(\zeta+C)] + \sinh[B(\zeta+C)])}{1 - A\cosh[B(\zeta+C)] - A\sinh[B(\zeta+C)]}.$$

For the Riccati equation

$$P'(\zeta) = AP(\zeta)^2 + BP(\zeta) + D, \tag{13}$$

the solutions are represented as follows

$$P(\zeta) = -\frac{B + \theta\tanh\left(\frac{1}{2}\theta(\zeta+C)\right)}{2A},$$

and

$$P(\zeta) = -\frac{B + \theta\tanh\left(\frac{1}{2}\theta\zeta\right)}{2A} + \frac{\operatorname{sech}\left(\frac{\theta}{2}\zeta\right)}{C\cosh\left(\frac{\theta}{2}\zeta\right) - \frac{2A}{\theta}\sinh\left(\frac{\theta}{2}\zeta\right)},$$

where $\theta^2 = B^2 - 4\mathrm{AD} > 0$.

4. Application of ITEM

Now, the ITEM will be explained for constructing traveling wave solutions to Eq. (6). With the help of the balancing principle between the $\frac{d^2}{d\zeta^2}u(\zeta)$ and $u^3(\zeta)$ in Eq. (6), we obtain $3M = M + 2$, $M = 1$. Therefore, Eq. (9) is given as

$$u(\zeta) = A_0 + A_1\tan\left(\frac{\varphi(\zeta)}{2}\right) + B_1\left(\tan\left(\frac{\varphi(\zeta)}{2}\right)\right)^{-1}.$$

Imposing the above equation into (6) and collect all terms with the same order $\tan(\varphi(\zeta)/2)$ together and comparing, we obtain a set of algebraic equations of $\gamma, \beta, \theta, A_0, A_1, B_1$ as

$$-\frac{1}{12}\lambda^4 B_1\beta\theta - \frac{1}{24}\lambda^4 B_1\theta^2 - \frac{1}{24}\lambda^4 B_1\beta^2 + \sigma k^2 {B_1}^3 = 0,$$

$$-\alpha k^2 {B_1}^2 + 3\sigma k^2 A_0 {B_1}^2 - \frac{1}{8}\lambda^4 B_1\gamma\theta - \frac{1}{8}\lambda^4 B_1\gamma\beta = 0,$$

$$3\sigma k^2 {A_0}^2 B_1 + 3\sigma k^2 A_1 {B_1}^2 - \frac{1}{24}\lambda^4 B_1\theta^2 - 2\alpha k^2 A_0 B_1 + \frac{1}{24}\lambda^4 B_1\beta^2 - \lambda^2 B_1 - \frac{1}{12}\lambda^4 B_1\gamma^2 + k^2 B_1 = 0,$$

$$-\lambda^2 A_0 - \frac{1}{24}\lambda^4 A_1\gamma\theta + \frac{1}{24}\lambda^4 B_1\gamma\beta - \frac{1}{24}\lambda^4 B_1\gamma\theta + \sigma k^2 {A_0}^3 - \frac{1}{24}\lambda^4 A_1\gamma\beta - \alpha k^2 {A_0}^2 + k^2 A_0 + 6\sigma k^2 A_0 A_1 B_1 - 2\alpha k^2 A_1 B_1 = 0,$$

$$k^2 A_1 - \frac{1}{12}\lambda^4 A_1\gamma^2 + 3\sigma k^2 {A_1}^2 B_1 - \frac{1}{24}\lambda^4 A_1\theta^2 - \lambda^2 A_1 + \frac{1}{24}\lambda^4 A_1\beta^2 + 3\sigma k^2 {A_0}^2 A_1 - 2\alpha k^2 A_0 A_1 = 0,$$

$$-\frac{1}{8}\lambda^4 A_1\gamma\theta + 3\sigma k^2 A_0 {A_1}^2 + \frac{1}{8}\lambda^4 A_1\gamma\beta - \alpha k^2 {A_1}^2 = 0,$$

$$\sigma k^2 {A_1}^3 + \frac{1}{12}\lambda^4 A_1\beta\theta - \frac{1}{24}\lambda^4 A_1\beta^2 - \frac{1}{24}\lambda^4 A_1\theta^2 = 0.$$

Solving the above algebraic equations with the help of Maple, we get three sets of coefficients for the solutions.

Case I

We have the desired constants as

$$k=\frac{\sqrt{36+3\,\Delta\,\lambda^2}\lambda}{6},\lambda=\lambda,\sigma=\frac{2(12+\Delta\,\lambda^2)\alpha^2}{9\Delta\,\lambda^2},A_0=-\frac{3\lambda^2(-\Delta+\gamma\sqrt{\Delta})}{2(12+\Delta\,\lambda^2)\alpha},A_1=\frac{3\sqrt{\Delta}(-\theta+\beta)\lambda^2}{2(12+\Delta\,\lambda^2)\alpha},B_1=0. \tag{14}$$

By using *Family 1*, (6) becomes

$$u_1=-\frac{3}{2}\frac{\lambda^2\left(-\Delta+\sqrt{\Delta}\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}{\alpha\,(\lambda^2\Delta+12)}, \tag{15}$$

where $\Delta=\beta^{\,2}+\gamma^{\,2}-\theta^{\,2}$ and ζ is given in (5).

By using *Family 2*, (6) becomes

$$u_2=\frac{3}{2}\frac{\lambda^2\left(\Delta+\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\Delta\right)}{\alpha\,(\lambda^2\Delta+12)}, \tag{16}$$

where $\Delta=\beta^{\,2}+\gamma^{\,2}-\theta^{\,2}$ and ζ is given in (5).

By using *Family 3*, (6) one gets

$$u_3=\frac{3}{2}\frac{\lambda^2\left(\beta^3+\beta\gamma^2+(\gamma^2+\beta^2)\beta\tanh\left(\frac{1}{2}\sqrt{\gamma^2+\beta^2}\zeta\right)\right)}{\beta\alpha\,(\,\lambda^2(\gamma^2+\beta^2)+12)}, \tag{17}$$

where ζ is given in (5).

By using *Family 4*, (6) can be written as

$$u_4=\frac{3}{2}\frac{\lambda^2\left(\theta\gamma^2-\theta^3-i(\theta^2-\,\gamma^2)\theta\tan\left(\frac{1}{2}\sqrt{\theta^2-\gamma^2}\zeta\right)\right)}{\theta\alpha\,(\lambda^2(\gamma^2-\theta^2)+12)}, \tag{18}$$

where ζ is given in (5).

By using *Family 5*, (6) can be written as

$$u_5=\frac{3}{2}\frac{\lambda^2\,(\beta-\theta)\left(\beta+\theta+|\beta+\theta|\tanh\left(\frac{1}{2}\sqrt{\beta^2-\theta^2}\zeta\right)\right)}{\alpha(\lambda^2\beta^2-\lambda^2\theta^2+12)}, \tag{19}$$

where ζ is given in (5).

By using *Family 13*, (6) can be written as

$$u_6=\frac{3\lambda^2\beta^2\mathrm{e}^{\beta\,\zeta}}{(\mathrm{e}^{\beta\,\zeta}-\beta-\gamma)\alpha\,(\lambda^2\beta^2+12)}, \tag{20}$$

where $\beta>0$ and ζ is given in (5).

By using *Family 18*, (6) can be written as

$$u_7=\frac{3\theta\lambda^2\left(\theta+\sqrt{-\theta^2}\tan\left(\frac{1}{2}\theta\zeta+\frac{1}{2}C\right)\right)}{2\alpha\,(-12+\lambda^2\theta^2)}, \tag{21}$$

where ζ is given in (5) and C is arbitrary constant.

Case II

We have the desired constants as

$$k=\frac{\sqrt{36+3\,\Delta\,\lambda^2}}{6}\lambda,\lambda=\lambda,\sigma=\frac{2(12+\Delta\,\lambda^2)\alpha^2}{9\Delta\,\lambda^2},A_0=\frac{3\lambda^2(\Delta+\gamma\sqrt{\Delta})}{2(12+\Delta\,\lambda^2)\alpha},A_1=0,B_1=\frac{3\sqrt{\Delta}\lambda^2(\theta+\beta)}{2(12+\Delta\,\lambda^2)\alpha}, \tag{22}$$

By using *Family 1*, (6) becomes

$$u_8=\frac{3}{2}\frac{\lambda^2\left(-\gamma\Delta-\Delta\sqrt{\Delta}+\sqrt{-\Delta}(\Delta+\gamma\,\sqrt{\Delta})\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}{(\lambda^2\Delta+12)\,\alpha\left(-\gamma+\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}, \tag{23}$$

where ζ is given in (5) and $\Delta=\beta^{\,2}-\theta^{\,2}+\gamma^{\,2}$.

By using *Family 2*, (6) becomes

$$u_9=\frac{3}{2}\frac{\lambda^2\Delta(\gamma+\sqrt{\Delta})\left(1+\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\right)}{(\lambda^2\Delta+12)\,\alpha\left(\gamma+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\right)}, \tag{24}$$

where ζ is given in (5) and $\Delta=\beta^{\,2}-\theta^{\,2}+\gamma^{\,2}$.

By using *Family 3*, (6) becomes

$$u_{10}=\frac{3}{2}\frac{\lambda^2(\gamma^2+\beta^2)\left(\gamma+\sqrt{\gamma^2+\,\beta^2}\right)\left(\tanh\left(\frac{1}{2}\sqrt{\gamma^2+\beta^2}\zeta\right)+1\right)}{\alpha\,(\lambda^2(\beta^2+\gamma^2)+12)\left(\gamma+\sqrt{\gamma^2+\beta^2}\tanh\left(\frac{1}{2}\sqrt{\gamma^2+\beta^2}\zeta\right)\right)}, \tag{25}$$

where ζ is given in (5).

By using *Family 4*, (6) becomes

$$u_{11}=\frac{3}{2}\frac{\lambda^2\left(-\Delta(\gamma+\sqrt{\Delta})+(\Delta+\gamma\sqrt{\Delta})\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}{(\lambda^2\Delta+12)\,\alpha\left(-\gamma+\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}, \tag{26}$$

where ζ is given in (5) and $\Delta=\gamma^{\,2}-\theta^{\,2}$.

By using *Family 5*, (6) becomes

$$u_{12}=\frac{3}{2}\lambda^2\sqrt{\frac{\beta-\theta}{\beta+\theta}}\,\frac{\left(\sqrt{\Delta}(\beta+\theta)+\sqrt{\frac{\beta+\theta}{\beta-\theta}}\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\Delta\right)}{(\lambda^2\Delta+12)\,\alpha\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)}, \tag{27}$$

where ζ is given in (5) and $\Delta=\beta^{\,2}-\theta^{\,2}$.

By using *Family 13*, (6) becomes

$$u_{13}=\frac{3\lambda^2\beta^2\mathrm{e}^{\beta\,\zeta}}{(\lambda^2\beta^2+12)\,\alpha\,(\mathrm{e}^{\beta\,\zeta}+\beta-\gamma)}, \tag{28}$$

where $\beta > 0$ and ζ is given in (5).

By using *Family 18*, (6) becomes

$$u_{14} = \frac{3\lambda^2\theta\left(\tan\left(\frac{1}{2}\theta\zeta+\frac{1}{2}C\right)\theta-\sqrt{-\theta^2}\right)}{2(-12+\lambda^2\theta^2)\,\alpha\tan\left(\frac{1}{2}\theta\zeta+\frac{1}{2}C\right)} \quad (29)$$

where ζ is given in (5).

By using *Family 19*, (6) becomes

$$u_{15} = -\frac{3\lambda^2\gamma^2\theta}{(\lambda^2\gamma^2+12)\,\alpha\,(e^{\gamma\zeta}-\theta)}, \quad (30)$$

where $\gamma < 0$ and ζ is given in (5).

Case III

We have the desired constants as

$$k = \frac{\sqrt{36+3\Delta\lambda^2}\lambda}{6}, \lambda = \lambda, \sigma = \frac{2(12+\Delta\lambda^2)\alpha^2}{9\lambda^2\gamma^2}, A_0 = \frac{3\lambda^2\gamma^2}{(12+\Delta\lambda^2)\alpha}, A_1 = -\frac{3\lambda^2\gamma(-\theta+\beta)}{2(12+\Delta\lambda^2)\alpha}, B_1 = \frac{3\lambda^2\gamma(\theta+\beta)}{2(12+\Delta\lambda^2)\alpha}. \quad (31)$$

By using *Family 1*, (6) becomes

$$u_{16} = \frac{-3\lambda^2\gamma\left(\Delta+\Delta\left(\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)^2\right)}{2(\lambda^2\Delta+12)\,\alpha\left(-\gamma+\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\zeta\right)\right)}, \quad (32)$$

where ζ is given in (5) and $\Delta = \beta^2 - \theta^2 + \gamma^2$.

By using *Family 2*, (6) becomes

$$u_{17} = \frac{-3\lambda^2\Delta\gamma\left(-1+\left(\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\right)^2\right)}{2(\lambda^2\Delta+12)\alpha\left(\gamma+\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\zeta\right)\right)}, \quad (33)$$

where ζ is given in (5) and $\Delta = \beta^2 - \theta^2 + \gamma^2$.

By using *Family 3*, (6) becomes

$$u_{18} = \frac{3\lambda^2\gamma\beta(\gamma^2+\beta^2)\left(1-\left(\tanh\left(\frac{1}{2}\sqrt{\gamma^2+\beta^2}\zeta\right)\right)^2\right)}{2(\lambda^2(\beta^2+\gamma^2)+12)\,\alpha\,\beta\left(\gamma+\sqrt{\gamma^2+\beta^2}\tanh\left(\frac{1}{2}\sqrt{\gamma^2+\beta^2}\zeta\right)\right)}, \quad (34)$$

where ζ is given in (5).

By using *Family 4*, (6) becomes

$$u_{19} = \frac{3\lambda^2\gamma(-\gamma^2+\theta^2)\theta\left(1+\left(\tan\left(\frac{1}{2}\sqrt{\theta^2-\gamma^2}\zeta\right)\right)^2\right)}{2\left(12+\lambda^2(\gamma^2-\theta^2)\right)\alpha\,\theta\left(-\gamma+\sqrt{\theta^2-\gamma^2}\tan\left(\frac{1}{2}\sqrt{\theta^2-\gamma^2}\zeta\right)\right)}, \quad (35)$$

where ζ is given in (5).

By using *Family 8*, (6) becomes

$$u_{20} = \frac{-\gamma\lambda^2}{2\alpha\,\zeta\,(\gamma\zeta+2)}, \quad (36)$$

where ζ is given in (5).

By using *Family 13*, (6) becomes

$$u_{21} = \frac{-6\lambda^2\gamma e^{\beta\zeta}\beta^2}{(\lambda^2\beta^2+12)\,\alpha\,(e^{\beta\zeta}-\beta-\gamma)(e^{\beta\zeta}+\beta-\gamma)}, \quad (37)$$

where $\beta > 0$ and ζ is given in (5).

By using *Family 15*, (6) becomes

$$u_{22} = -\frac{\theta\lambda^2}{2\alpha\,\zeta\,(\theta\zeta+2)}, \quad (38)$$

where ζ is given in (5).

By using *Family 19*, (6) becomes

$$u_{23} = \frac{3\lambda^2\gamma^2 e^{\gamma\zeta}}{(\lambda^2\gamma^2+12)\,\alpha\,(e^{\gamma\zeta}-\theta)}, \quad (39)$$

where ζ is given in (5) and $\gamma < 0$.

5. Application of Simplest Equation Method

In this section, we consider (8). With the help of homogenous balance principle between the $\frac{d^2}{d\zeta^2}u(\zeta)$ and $u^3(\zeta)$ in Eq. (6), we obtain $3M = M + 2$, $M = 1$. Therefore, we get that the trial solution of Eq. (6) can be stated as,

$$u(\zeta) = A_0 + A_1 w(\zeta), \quad (40)$$

where $A_1 \neq 0$, A_0 are constants. For Bernoulli equation, putting u, u'', u^2 in (6) and comparing, we get,

$$k^2A_0 - \lambda^2A_0 + \sigma\,k^2{A_0}^3 - \alpha\,k^2{A_0}^2 = 0,$$

$$k^2A_1 - 2\,\alpha\,k^2A_0\,A_1 - \lambda^2A_1 + 3\,\sigma\,k^2{A_0}^2A_1 - 1/12\,\lambda^4A_1B^2 = 0,$$

$$\sigma\,k^2{A_1}^3 - 1/6\,\lambda^4A_1\,A^2 = 0,$$

$$-\alpha\,k^2{A_1}^2 + 3\,\sigma\,k^2A_0\,{A_1}^2 - 1/4\lambda^4A_1\,BA = 0. \quad (41)$$

Calculation with the aid of Maple software, the solutions of the algebraic equations can be derived.

$$\mathbf{Set1_B}:\left\{\begin{array}{l} A_0=0, A_1=-\frac{2}{3}\frac{A\alpha}{\sigma B}, \lambda=\frac{2\sqrt{6}\alpha}{B\sqrt{9\sigma-2\alpha^2}}, \\ k=\frac{6\sqrt{6}\sqrt{\sigma}\alpha}{(-9\sigma+2\alpha^2)\mathrm{B}}. \end{array}\right\}$$

For Set 1_B, we obtained the desired solutions as

$$u_{24}=\frac{\frac{2}{3}\frac{A\alpha\cosh(B(\zeta+C))}{\sigma}+\frac{2}{3}\frac{A\alpha\sinh(B(\zeta+C))}{\sigma}}{-1+A\cosh(B(\zeta+C))+A\sinh(B(\zeta+C))}, \quad (42)$$

where ζ is given in (5).

$$\mathbf{Set2_B}:\left\{\begin{array}{l} A_0=0, A_1=-\frac{2}{3}\frac{A\alpha}{\sigma B}, \lambda=-\frac{2\sqrt{6}\alpha}{B\sqrt{9\sigma-2\alpha^2}}, \\ k=\frac{6\sqrt{6}\sqrt{\sigma}\alpha}{(-9\sigma+2\alpha^2)\mathrm{B}}. \end{array}\right\}$$

For Set 2_B, we obtained the desired solutions as

$$u_{25}=\frac{\frac{2}{3}\frac{A\alpha\cosh(B(\zeta+C))}{\sigma}+\frac{2}{3}\frac{A\alpha\sinh(B(\zeta+C))}{\sigma}}{-1+A\cosh(B(\zeta+C))+A\sinh(B(\zeta+C))}, \quad (43)$$

where ζ is given in (5).

$$\mathbf{Set3_B}:\left\{\begin{array}{l} A_0=\frac{2}{3}\frac{\alpha}{\sigma}, A_1=\frac{2}{3}\frac{A\alpha}{\sigma B}, \lambda=\frac{2\sqrt{6}\alpha}{B\sqrt{9\sigma-2\alpha^2}}, \\ k=\frac{6\sqrt{6}\sqrt{\sigma}\alpha}{(-9\sigma+2\alpha^2)\mathrm{B}}. \end{array}\right\}$$

For Set 3_B, we obtained the desired solutions as

$$u_{26}=\frac{-2\alpha}{3\sigma\left(-1+A\cosh(B(\zeta+C))+A\sinh(B(\zeta+C))\right)}, \quad (44)$$

where ζ is given in (5).

$$\mathbf{Set4_B}:\left\{\begin{array}{l} A_0=\frac{2}{3}\frac{\alpha}{\sigma}, A_1=\frac{2}{3}\frac{A\alpha}{\sigma B}, \lambda=-\frac{2\sqrt{6}\alpha}{B\sqrt{9\sigma-2\alpha^2}}, \\ k=\frac{6\sqrt{6}\sqrt{\sigma}\alpha}{(-9\sigma+2\alpha^2)\mathrm{B}}. \end{array}\right\}$$

For Set 4_B, we obtained the desired solutions as

$$u_{27}=\frac{2\alpha}{3\sigma\left(-1+A\cosh(B(\zeta+C))+A\sinh(B(\zeta+C))\right)}, \quad (45)$$

where ζ is given in (5).

For Riccati equation, putting u, u^2, u'', u^3 in (6) and comparing, we get,

$$12\,k^2A_0-\lambda^4A_1\,BD-12\,\lambda^2A_0+12\,\sigma\,k^2{A_0}^3-12\,\alpha\,k^2{A_0}^2=0,$$

$$12\,k^2A_1-12\,\lambda^2A_1+36\,\sigma\,k^2{A_0}^2A_1-24\,\alpha\,k^2A_0\,A_1-\lambda^4A_1\,B^2-2\,\lambda^4A_1\,\mathrm{AD}=0,$$

$$-12\,\alpha\,k^2{A_1}^2-3\,\lambda^4A_1\,BA+36\,\sigma\,k^2A_0\,{A_1}^2=0,$$

$$12\,\sigma\,k^2{A_1}^3-2\,\lambda^4A_1\,A^2=0. \quad (46)$$

Calculation with the aid of Maple software, the solutions of the algebraic equations can be derived.

$$\mathbf{Set1_R}:\left\{\begin{array}{l} A_0=\frac{1}{2}\frac{(B+\sqrt{-4\,AD+B^2})A_1}{A}, A_1=A_1, \\ \alpha=3\,\frac{A}{k^2A_1}\frac{\sqrt{-4\,AD+B^2}}{4\,AD-B^2}\left(2\,\frac{3+\sqrt{9-12\,k^2AD+3\,k^2B^2}}{4\,AD-B^2}-k^2\right), \\ \lambda=\frac{\sqrt{2}\sqrt{(4\,AD-B^2)(3+\sqrt{9-12\,k^2AD+3\,k^2B^2})}}{4\,AD-B^2}, \\ \sigma=2\,A^2\frac{\left(2\,\frac{3+\sqrt{9-12\,k^2AD+3\,k^2B^2}}{4\,AD-B^2}-k^2\right)}{(4\,AD-B^2)k^2{A_1}^2}. \end{array}\right\}$$

For Set 1_R, we obtained the desired solutions as

$$u_{28}=-\frac{A_1\sqrt{-4\,AD+B^2}\left(-1+\tanh\left(\frac{1}{2}\sqrt{-4\,AD+B^2}(\zeta+C)\right)\right)}{2A}, \quad (47)$$

and

$$u_{29}=\frac{1}{2}\frac{(B+\sqrt{B^2-4\,AD})A_1}{A}$$

$$+A_1\left(-\frac{1}{2}\frac{B+\sqrt{B^2-4\,AD}\tanh\left(\frac{1}{2}\sqrt{B^2-4\,AD}\zeta\right)}{A}\right.$$

$$\left.+\frac{sech\left(\frac{1}{2}\sqrt{B^2-4\,AD}\zeta\right)}{C\cosh\left(\frac{1}{2}\sqrt{B^2-4\,AD}\zeta\right)-2\frac{A\sinh\left(\frac{1}{2}\sqrt{B^2-4\,AD}\zeta\right)}{\sqrt{B^2-4\,AD}}}\right), \quad (48)$$

where ζ is given in (5).

$$\mathbf{Set2_R}:\left\{\begin{array}{l} A_0=\frac{1}{2}\frac{(B+\sqrt{-4\,AD+B^2})A_1}{A}, A_1=A_1, \\ \alpha=\frac{3\,A\sqrt{-4\,AD+B^2}}{A_1(4\,AD-B^2)k^2}\left(2\,\frac{3+\sqrt{9-12\,k^2AD+3\,k^2B^2}}{4\,AD-B^2}-k^2\right), \\ \lambda=-\frac{\sqrt{2}\sqrt{(4\,AD-B^2)(3+\sqrt{9-12\,k^2AD+3\,k^2B^2})}}{4\,AD-B^2}, \\ \sigma=\frac{2A^2\left(2\,\frac{3+\sqrt{9-12\,k^2AD+3\,k^2B^2}}{4\,AD-B^2}-k\right)}{(4\,AD-B^2)k^2{A_1}^2}. \end{array}\right\}$$

For Set 2_R, we obtained the desired solutions as

$$u_{30}=\frac{(B+\sqrt{-4\,AD+B^2})A_1}{2A}$$

$$-\frac{BA_1+A_1\sqrt{-4\,AD+B^2}\tanh\left(\frac{1}{2}\sqrt{-4\,AD+B^2}(\zeta+C)\right)}{2A}, \quad (49)$$

and

$$u_{31} = \frac{1}{2}\,\frac{\left(B+\sqrt{-4\,AD+B^2}\right)A_1}{A} + A_1\left(-\frac{1}{2}\,\frac{B+\sqrt{-4\,AD+B^2}\tanh\left(\frac{1}{2}\sqrt{-4\,AD+B^2}\zeta\right)}{A} + \frac{sech\left(\frac{1}{2}\sqrt{-4\,AD+B^2}\zeta\right)}{C\cosh\left(\frac{1}{2}\sqrt{-4\,AD+B^2}\zeta\right)-2\,\frac{A\sinh\left(\frac{1}{2}\sqrt{-4\,AD+B^2}\zeta\right)}{\sqrt{-4\,AD+B^2}}}\right), \qquad (50)$$

where ζ is given in (5).

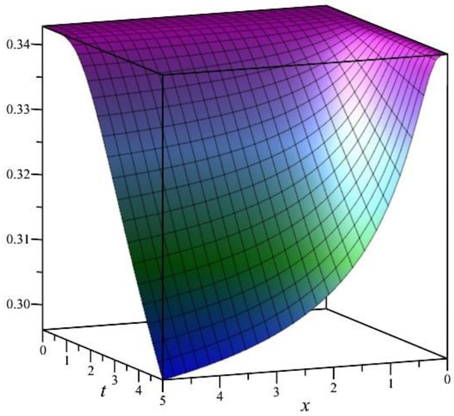

Figure 1. Numerical simulation of u_1 in Eq. (15) in three dimensional plot.

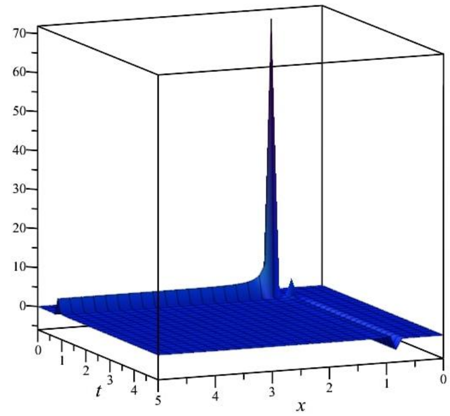

Figure 2. Numerical simulation of u_{18} in Eq. (34) in three dimensional plot.

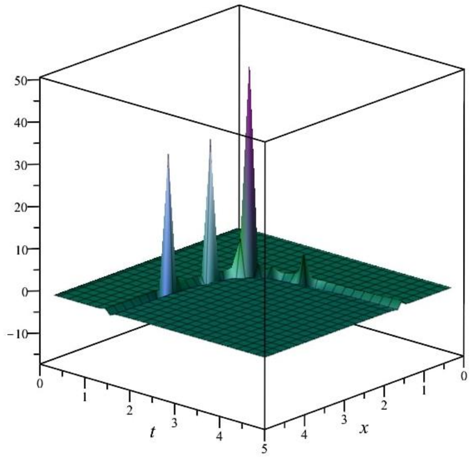

Figure 3. Numerical simulation of u_{24} in Eq. (42) in three dimensional plot.

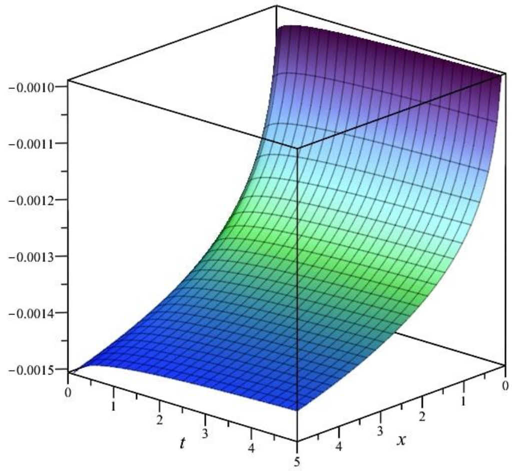

Figure 4. Numerical simulation of u_{26} in Eq. (44) in three dimensional plot.

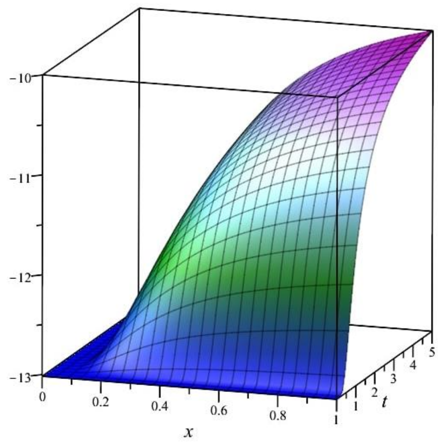

Figure 5. Numerical simulation of u_{28} in Eq. (47) in three dimensional plot.

6 Results and discussion

The model discussed in this study contains the fractional derivative ABR which makes it interesting. The ABR fractional operator is considered as one of the last general fractional operators derived from avoiding the deficiencies and defects of some other fractional operators (Park *et al.* 2020). The techniques applied in this study are effective and powerful techniques for solving partial differential equations. The variety of solutions obtained makes these methods advantageous, which is a scientifically beatiful feature.

The nonlinear integer order PDE describing the low-pass electrical lines has been discussed in (Abdoulkary *et al.* 2013) using an auxiliary equation method an in (Zayed and Alurrfi 2015) using Jacobi elliptic function expansion method. In (Park *et al.* 2020), the fractional nonlinear model of the low–pass electrical transmission lines has been considered and constructed explicit wave solutions using modified Khater method. When our results obtained in this study using two different methods were compared with the results obtained in (Part et al 2020), it was seen that they were different.

The graphical representation of some of obtained solution are plotted by taking suitable values of involved unknown parameters. Here, we give the figure interpretation of the shown figures as following:
Figure 1 explains the periodic wave solution u_1 when $a = 1, b = 1, c = 2, \alpha = 5, \lambda = 4, v = 0.1$.
Figure 2 shows the dark wave solution u_{18} when $a = 2, b = 5, c = 0, \alpha = 2, \lambda = 0.6, v = 0.55$.
Figure 3 shows the graph of hyperbolic function solution u_{24} when $A = 3, B = 2, C = 2, \alpha = -2, \sigma = 5, v = 0.3$.
Figure 4 shows the graph of hyperbolic function solution u_{26} when $A = 1, B = 1, C = 2, \alpha = 0.1, \sigma = 3, v = 0.01$.
Figure 5 indicates the exact soliton wave solution u_{28} when $A = -3, A_1 = 5, B = 1, C = -0.1, D = 5, k = 4.5, v = 0.6$.

7. Conclusions

In this work, the improved $tan(\varphi/2)$-expansion method and the simple equation method are successfully used to obtained exact solutions of the fractional nonlinear model of the low-pass electrical transmission lines. By proposed methods, solitary solutions are established including three types namely, triangular functions solutions, exponential solutions and rational solutions. The variety of complete solutions obtained plays an important role in the interpretation and understanding of the physical model. Some sketches are also depicted for the interpretation physically of the achieved solutions. To our best knowledge, some of the obtained solutions are new and not reported previously. This study shows that the proposed method is quite proficient and practically well organized in finding exact solutions to other physical problems.

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