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# Hyperbolic Traveling Wave Solutions for Sawada–Kotera Equation Using $(1/G')$-Expansion Method

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***Keywords***
$(1/G')$-expansion method ; Sawada–Kotera equation ; Nonlinear partial differential equation; Hyperbolic traveling wave solution

**Abstract**

In this study, we obtain hyperbolic traveling wave solutions of the Sawada–Kotera equation (S-K), using $(1/G')$-expansion methods. Special values are given to the parameters in the solutions obtained and graphs are drawn. These graphs are presented using a computer package program. In this paper, $(1/G')$-expansion method is applied to reach the goals set. $(1/G')$-expansion method is an effective and powerful method to obtain the traveling wave solutions of nonlinear partial differential equations.

# $(1/G')$-Açılım Metodunu Kullanarak Sawada–Kotera Denkleminin Hiperbolik Yürüyen Dalga Çözümleri

***Anahtar kelimeler***
$(1/G')$-açılım metodu; Sawada–Kotera denklemi; Lineer olmayan kısmi diferansiyel denklem; Hiperbolik yürüyen dalga çözümü

**Öz**

Bu çalışmada, $(1/G')$-açılım metodunu kullanarak Sawada- Kotera denkleminin (S-K) hiperbolik yürüyen dalga çözümleri elde edildi. Elde edilen çözümlerdeki parametrelere özel değerler verilerek, grafikler çizildi. Bu grafikler bilgisayar paket programı kullanılarak sunuldu. Bu makalede, belirlenen hedefe ulaşmak için $(1/G')$-açılım metodu uygulandı. $(1/G')$-açılım metodu lineer olmayan kısmi diferansiyel denklemlerin yürüyen dalga çözümlerini elde etmede etkili ve güçlü bir metottur.



## 1. Introduction

Recently, many studies have been performed in various fields such as in the field of physics, engineering, fluid dynamics, and chemistry. Thus, some of the methods to obtain the exact solutions are the $(G'/G)$-expansion method (Wang et al. 2008), Homotopy analysis method (Liao 2004), Extended trial equation method (Gurefe et al. 2013), Generalized auxiliary equation method (Zhang and Xia 2007), The first integral method (Raslan 2008), The functional variable method (Liu and Chen 2013), $(1/G')$-expansion method (Yokuş 2015), $(G'/G, 1/$

$G$)-expansion and $(1/G')$-expansion methods, (Daghan and Donmez 2016), Numerical solution (Aziz and Ahmad 2015), Optical solitons (Esen et al. 2018), inverse Laplace homotopy technique (Yavuz and Ozdemir 2018), solutions of partial differential equations (Yavuz et al. 2018), the modified $\exp(-\Omega(\xi))$-expansion function method (Çelik et al. 2018, Sulaiman et al. 2019 ) and so on.

The Sawada–Kotera equation appears in variety study, such as a new supersymmetric equation (Tian and Liu 2009), new exact solutions of generalized Riccati equation obtained using $(G'/G)$-expansion method (Saba et al. 2015), some solutions have been found using the projective Riccati equation method (Salas 2008), with aid Hirota bilinear method, for exact soliton solutions of the fifth-order (S–K) Eq. have been studied (Liu and Dai 2008), by means of scaling, have investigated new solutions to general Sawada–Kotera equation (Gómez and Salas 2010), exact traveling wave solutions of Nonlinear Evolution equations (NLEEs) have been obtained using the extended simplest equation method (Bilige and Chaolu 2010), have been obtained approximate analytical solutions of the (S–K) and Lax's fifth-order KdV Eqs. using (HAM) (Dinarvand et al. 2008), with aid the simplified Hirota's method, have been the constructed couplings of the fifth-order nonlinear integrable (S-K) Eq. and Lax Eq. (Wazwaz and Ebaid 2014), exact traveling wave solutions of the fifth-order standard (S-K) equation have been obtained using generalized $\exp(-\phi(\xi))$-expansion method (Ali et al. 2016) and so on.

In this work, our aim is to find the exact traveling wave solution of the (S-K) equation by using $(1/G')$-expansion methods. The fifth-order (S-K) equation can be shown in the form of

$$u_t + 45u^2u_x + 15u_xu_{xx} + 15uu_{xxx} + u_{5x} = 0. \quad (1)$$

## 2. $(1/G')$-Expansion Method

Firstly, in order to apply this method, consider the two-variable general form of NLPDE

$$H\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0, \quad (2)$$

in the general form. Here, let

$$u = u(x,t) = u(\xi), \quad \xi = \eta(kx + vt), v \neq 0, \eta = 1$$

and where $v$ is a constant. Then, we can be converted into following nonlinear ODE for $u(\xi)$:

$$\Omega(u', u'', u''', \dots) = 0. \quad (3)$$

The solution of Eq. (3) is assumed to have the form

$$u(\xi) = a_0 + \sum_{i=1}^{m} a_i \left(\frac{1}{G'}\right)^i, \quad (4)$$

where $a_i$ are constants, $m$ is a positive integer which is the equilibrium term in Eq. (3) and $G = G(\xi)$ provides the following second-order Lode

$$G'' + \lambda G' + \mu = 0, \quad (5)$$

where $\lambda$ and $\mu$ are constants to be determined after. To represent the solution of Eq. (5) with $G(\xi)$, the Eq. (4) will include the following equation

$$\frac{1}{G'[\xi]} = \frac{1}{-\frac{\mu}{\lambda} + A\cos h[\xi\lambda] - A\sin h[\xi\lambda]},$$

where $A$ is integral constant. If the desired derivatives of the Eq. (4) are calculated and substituting in the Eq. (3), a polynomial with the argument $(1/G')$ is attained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. This equation system is solved with the help of the Mathematica program and put into place in the default (3) solution function. Consequently, the solutions of the Eq. (1) are found (Yokuş and Durur 2019).

## 3. The solution of Sawada–Kotera Equation

We consider Sawada–Kotera Eq. (1). Therefore, using transmutation $u = u(x,t) = u(\xi)$, $\xi = kx + vt$, $v \neq 0$, taking once the integral of Eq. (1) we obtain

$$vu' + 45ku^2u' + 15k^3u'u'' + 15k^3\mathrm{uu}''' + k^5u^{(5)} = 0, \quad (6)$$

where $v$ represents the speed of the wave. Taking into account the Eq. (6), we find the balancing term

$m=2$ and in Eq. (4), we attain to following the form of the solution

$$u(\xi)=a_0+a_1\left(\frac{1}{G'[\xi]}\right)+a_2\left(\frac{1}{G'[\xi]}\right)^2. \tag{7}$$

If we substitute the Eq. (7) in the Eq. (6) and the coefficients of the algebraic equation are equal to zero, we can establish the following algebraic equation systems

$$\frac{1}{G'[\xi]}=v\lambda a_1+k^3\lambda^5a_1+15k^3\lambda^3a_0a_1+45k\lambda a_0^2a_1=0,$$

$$\frac{1}{G'[\xi]^2}=v\mu a_1+31k^3\lambda^4\mu a_1+105k^3\lambda^2\mu a_0a_1+45k\mu a_0^2a_1+30k^3\lambda^3a_1^2+90k\lambda a_0a_1^2+2v\lambda a_2+32k^3\lambda^5a_2+120k^3\lambda^3a_0a_2+90k\lambda a_0^2a_2=0,$$

$$\frac{1}{G'[\xi]^3}=180k^3\lambda^3\mu^2a_1+180k^3\lambda\mu^2a_0a_1+165k^3\lambda^2\mu a_1^2+90k\mu a_0a_1^2+45k\lambda a_1^3+2v\mu a_2+422k^3\lambda^4\mu a_2+570k^3\lambda^2\mu a_0a_2+90k\mu a_0^2a_2+225k^3\lambda^3a_1a_2+270k\lambda a_0a_1a_2=0,$$

$$\frac{1}{G'[\xi]^4}=390k^3\lambda^2\mu^3a_1+90k^3\mu^3a_0a_1+255k^3\lambda\mu^2a_1^2+45k\mu a_1^3+1710k^3\lambda^3\mu^2a_2+810k^3\lambda\mu^2a_0a_2+1005k^3\lambda^2\mu a_1a_2+270k\mu a_0a_1a_2+180k\lambda a_1^2a_2+240k^3\lambda^3a_2^2+180k\lambda a_0a_2^2=0,$$

$$\frac{1}{G'[\xi]^5}=360k^3\lambda\mu^4a_1+120k^3\mu^3a_1^2+3000k^3\lambda^2\mu^3a_2+360k^3\mu^3a_0a_2+1380k^3\lambda\mu^2a_1a_2+180k\mu a_1^2a_2+990k^3\lambda^2\mu a_2^2+180k\mu a_0a_2^2+225k\lambda a_1a_2^2=0,$$

$$\frac{1}{G'[\xi]^6}=120k^3\mu^5a_1+2400k^3\lambda\mu^4a_2+600k^3\mu^3a_1a_2+1290k^3\lambda\mu^2a_2^2+225k\mu a_1a_2^2+90k\lambda a_2^3=0,$$

$$\frac{1}{G'[\xi]^7}=720k^3\mu^5a_2+540k^3\mu^3a_2^2+90k\mu a_2^3=0, \tag{8}$$

the aim with computer package program, reaching the solutions of system (8) and we obtained the following stations.

***Case 1.*** If

$$a_2=-2\mu^2,\ k=\mp1,\ \xi=kx+vt, \tag{9}$$

replacing values Eq. (9) into Eq. (7) and we have the following hyperbolic wave solutions for Eq. (1):

$$u_1(x,t)=-\frac{\lambda^2}{3}-\frac{2\mu^2}{\left(-\frac{\mu}{\lambda}+A\mathrm{Cosh}[\lambda\psi]-A\mathrm{Sinh}[\lambda\psi]\right)^2}-\frac{2\lambda\mu}{-\frac{\mu}{\lambda}+A\mathrm{Cosh}[\lambda\psi]-A\mathrm{Sinh}[\lambda\psi]}, \tag{10}$$

where $\psi=-x+t\lambda^4$.

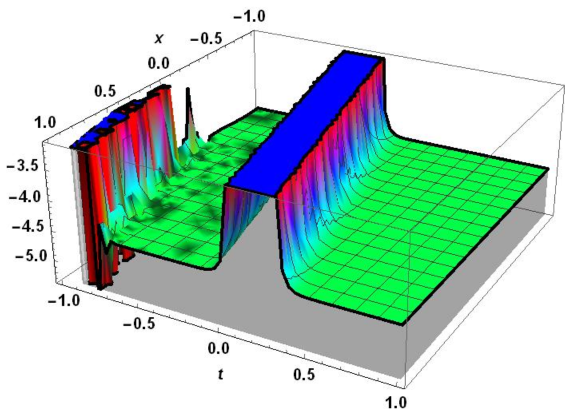


**Figure 1.** The solution representing a new type *hyperbolic traveling wave solution* in the $u_1(x,t)$ obtained of the Eq. (1) for $\mu=-3$, $\lambda=-7$, $a_0=-4.5$, $A=-1$.

***Case 2.*** If

$$v=\lambda^4,\quad a_0=-\frac{\lambda^2}{3},\quad a_1=-4\lambda\mu,\quad a_2=-4\mu^2,\quad k=\mp1,\quad \xi=kx+vt, \tag{11}$$

replacing values reached (11) into (7), attain the following hyperbolic wave solutions for Eq. (1):

$$u_2(x,t)=-\frac{\lambda^2}{3}-\frac{4\mu^2}{\left(-\frac{\mu}{\lambda}+A\mathrm{Cosh}[\lambda\psi]-A\mathrm{Sinh}[\lambda\psi]\right)^2}-\frac{4\lambda\mu}{-\frac{\mu}{\lambda}+A\mathrm{Cosh}[\lambda\psi]-A\mathrm{Sinh}[\lambda\psi]}, \tag{12}$$

where $\psi=-x+t\lambda^4$.

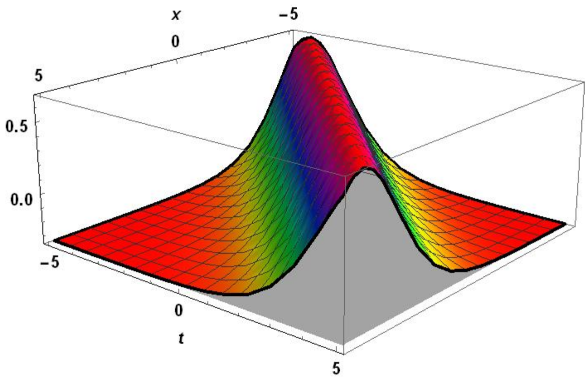


**Figure 2.** The solution representing a new type *hyperbolic traveling wave solution* in the $u_2(x,t)$ obtained of the Eq. (1) for $\mu = -1,\ \lambda = 1,\ A = 2$.

## 4. Conclusion

In this letter, $(1/G')$-expansion method is used to obtain the new hyperbolic traveling wave solutions for (S-K) Eq. .The results obtained here show that $(1/G')$-expansion method is simple, reliable, and may be used to process other NLPDE's. In addition, the computer package program has been used for computations and graphics in this study.

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