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CROSS-SECTION ANALYSIS OF A COMPOSITE BOX BEAM

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ABSTRACT

Cross-sectional analysis of a composite box-beam is performed by an analytical method and a variational-asymptotical method.

The beam stiffness coefficients are obtained and compared for both analytical and variational-asymptotical methods. The parameters of box-beam width and height, wall thicknesses, initial twist and composite orientation angle are selected for the sensitivity of box-beam stiffnesses.

The sensitivity results give the most important parameters affecting the stiffnesses of the box-beam.

Key Words: Box Beam, Cross-Section Analysis, Variational-Asymptotic Method

BİR KOMPOZİT KUTU KİRİŞİN KESİT ANALİZİ

ÖZET

Bir kompozit kutu kirişin kesit analizi analitik ve variasyonel asimptotik yöntemlerle yapılmıstır.

Kiriş katılık katsayıları hem sayısal hem de variasyonel asimptotik yöntemle bulunmuş ve karşılaştırılmıştır. Kutu kiriş katılık hassasiyetleri için kutu kiriş genişliği, yüksekliği, duvar kalınlıkları, ön-burulma ve kompozit yönlenme açısı parametreleri seçilmiştir.

Hassasiyet değerleri her bir kutu kiriş katılığı etkileyen en önemli parametreleri vermektedir.

Anahtar Kelimeler: Kutu Kiriş, Kesit Analizi, Variasyonel Asimptotik Yöntem

1. INTRODUCTION

Modern rotor blades are frequently made built of composites. Important parameters of such a model consist of transverse shear deformation, cross-sectional warping, elastic coupling, and geometric nonlinearities. A review of literature on the modeling of composite rotor blades is given by Hodges [1]. One widely acceptable approach is to calculate cross-sectional warping and elastic constants by the use of a linear theory, assuming smallness of the warping. So the geometrically nonlinear three-dimensional (3-D) problem is divided into two independent subproblems: the first one deals with a nonlinear one-dimensional (1-D) beam analysis, and second one deals with a linear two-dimensional (2-D) cross-sectional analysis.

Early studies of the cross-sectional properties were done by Mansfield and Sobey [2]. In the study, analytical expressions of the cross-section stiffnesses of a simplified helicopter blade model were obtained. Worndle [3] formulated a 2-D, finite element based procedure for determination of the shear center and warping functions. Later Rehfield [4] presented a method for determination of cross-section properties of a general rotor blade. The cross section was approximated as a single cell composite box-beam whose torsional warping function was determined analytically.

Giavotto et al. [5] formulated a general 2-D finite element model for determining the cross section properties including warping function, shear center location and stiffness. This analysis yielded a very general FORTRAN code (ANBA-Anisotropic Beam Analysis and an America version called NABSA-Nonhomogenous Anisotropic Beam Section Analysis).

Hodges, Nixon and Rehfield [6] compared the Rehfield's approach [4] with a NASTRAN finite element model for a beam having a single-closed cell. Smith and Chopra [7] developed an analytical beam formulation for predicting the effective elastic stiffness and load deformation behavior of composite box-beams. Deformation of the beam is described by extension, bending, torsion, transverse shearing and torsion-related warping. The results were compared with experimental results.

Later Berdichevsky et al. [8] obtained analytical formulas for the stiffness of thin-walled closed-section beams. The extension, torsion and bending behaviors of nonhomogenoues, anisotropic beam were governed by 4×4 stiffness matrix. The theory was based on the asymptotic reduction of 2-D shell theory. The analysis of [8] gives a good approximation for thin-walled closed cells.

Recently, Cesnik et al. [9] applied the variational-asymptotic method to obtain stiffness properties of the nonhomogeneous, anisotropic beam. The theory yielded a general purpose FORTRAN code (VABS-Variational Beam Sectional Analysis) [10].

In this study, stiffness coefficients of a box beam are obtained by both analytical and variational-asymptotic methods. The results are given for a cross-section of extension-torsion coupled composite beam. The sensitivities of stiffnesses to design variables such as box width, height, wall thicknesses and pretwist are presented.

2. CROSS SECTION ANALYSIS

2.1 Analytical Method

Schematic of a composite box-beam is shown in Figure 1. First, the composite box-beam stiffness coefficients are obtained by using an anisotropic, thin-walled, closed-section beam theory [8] analytically. The theory is based on an asymptotic analysis of 2-D shell theory. When driving closed form expressions for the stiffness, major dimensions (width and height) of the box are assumed to be of the same order and thicknesses are assumed to be small compared to the major dimensions. The walls are assumed to be plate-like structures. Initial pretwist and curvature effects are also ignored. Closed-form expressions of the stiffness coefficients are provided through the constituve relations as

$$\begin{Bmatrix} T \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} u_1' \\ \varphi_1 \\ u_3'' \\ u_2'' \end{Bmatrix} \tag{1}$$

where T, M_x, M_y, M_z, are the axial force, torsional moment, and bending moments about the axis y and z; S_{ij} are stiffness coefficients; u_1, u_2 and u_3 are displacements; φ_1 is twist rate. Using an asymptotic analysis reduced axial, coupling and shear stiffnesses are obtained as follows:

$$A(s) = A_{11} - \frac{A_{12}^{\ 2}}{A_{22}}$$
$$B(s) = 2\left[A_{16} - \frac{A_{12}A_{26}}{A_{22}}\right] \quad (2)$$
$$C(s) = 4\left[A_{66} - \frac{A_{26}^{\ 2}}{A_{22}}\right]$$

where A_{ij} are classical lamination theory in-plane stiffnesses.
For an extension-torsion-coupled beam as shown in Figure 2, Eq.(1) reduces to The stiffness coefficients are defined in terms of contour integrals as where A_e is the enclosed area.

$$\begin{Bmatrix} T \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{22} & 0 & 0 \\ 0 & 0 & S_{33} & 0 \\ 0 & 0 & 0 & S_{44} \end{bmatrix} \begin{Bmatrix} u_1' \\ \varphi_1 \\ u_3'' \\ u_2'' \end{Bmatrix} \quad (3)$$

$$S_{11} = \oint\left(A - \frac{B^2}{C}\right)ds + \frac{\left[\oint(B/C)ds\right]^2}{\oint(1/C)ds}$$
$$S_{12} = \frac{\oint(B/C)ds}{\oint(1/C)ds}A_e \quad , \quad S_{22} = \frac{1}{\oint(1/C)ds}A_e^2 \quad (4)$$
$$S_{33} = \oint\left(A - \frac{B^2}{C}\right)z^2ds + \frac{\left[\oint(B/C)zds\right]^2}{\oint(1/C)ds}$$
$$S_{44} = \oint\left(A - \frac{B^2}{C}\right)y^2ds + \frac{\left[\oint(B/C)yds\right]^2}{\oint(1/C)ds}$$

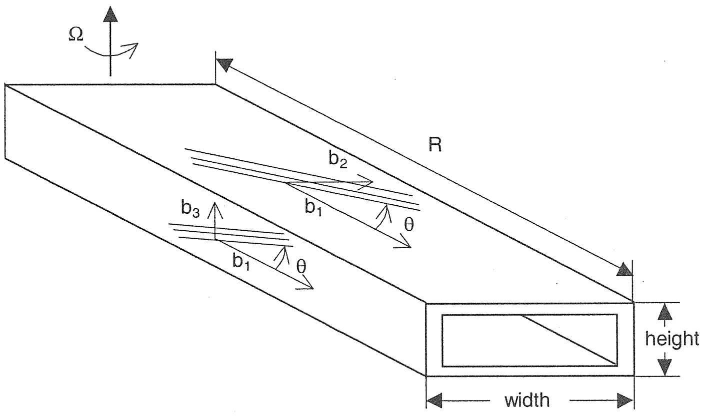

Figure 1. Composite Box Beam

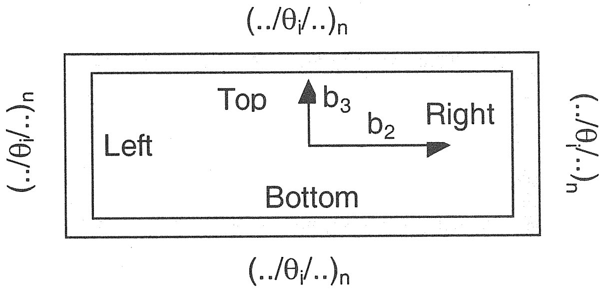

Figure 2. Cross-Section of Extension-Torsion Coupled Box Beam

2.2 Variational-Asymptotic Method

To get more accurate results for the stiffnesses, next variational asymptotic method is used, in which 3-D strain energy is reduced to 1-D strain energy using the method. Three-dimensional warping including in-plane and out-of-plane deformations of the cross section is calculated in terms of 1-D strain measures and strain energy. Formulation is developed for functionals with small parameters. An order assessment is performed based upon the small parameters. For a given 3-D functional $F(\Gamma,h)$ with a small parameter h, the functional is decomposed as (detailed formulation is given in Ref. 9 and Ref. 10)

$$F(\Gamma, h) = \varepsilon_1(\gamma, z_1) + \varepsilon_h(\gamma, z_1, h) \tag{5}$$

where ε_1 is obtained by ignoring smaller contributions to the energy (represented by ε_h); γ is function of x only; z_1 is a 3-D function of all three coordinates. For this problem γ and z_1 correspond generalized strains and warping, respectively. In order to minimize the functional, it can be approximated by the main contributor ε_1

$$\min \ F = \min \varepsilon_1(\gamma, z_1) \tag{6}$$

Solution of the functional via Euler equations can be written as

$$z_1 = y_1(\gamma, \xi_2, \xi_3) \tag{7}$$

which is the first minimizing function only if the second function is higher order. In order to prove this the second approximation needs to be developed. The following minimizing function can be assumed for the second approximation as

$$z_2 = z_1 - y_1(\gamma, \xi_2, \xi_3) \tag{8}$$

The substitution of Eq. 8 into Eq. 5 gives

$$F(\Gamma, h) = F_1(\gamma) + \varepsilon_2(\gamma, z_2, h) + \varepsilon_{h1}(\gamma, z_2, h) \tag{9}$$

where ε_2 is obtained by ignoring all smaller contributions to the energy (represented by ε_{h1}). Now minimum of the functional and then the second minimizing function become

$$\min \ F = F_1(\gamma) + \min \varepsilon_2(\gamma, z_2, h) \tag{10}$$

$$z_2 = y_2(\gamma, \xi_2, \xi_3) \tag{11}$$

When z_2 is higher order than z_1, then z_1 and F_1 gives the first approximation. Otherwise the first and the second approximations must be corrected. z_2 will be the second approximation if the third approximation can be developed in the same manner. After the kth approximation the functional becomes

$$F(\Gamma,h)=F_1(\gamma)+F_2(\gamma)+\ldots+F_k(\gamma) \qquad (12)$$
$$+\varepsilon_{k+1}(\gamma,z_{k+1},h)+\varepsilon_{hk}(\gamma,z_{k+1},h)$$

The small parameter h is the characteristic length of the cross-section of the beam. Then the strain energy density is solved by finite element method in the code VABS. 6-noded elements are used for the meshes of the cross-section. However, calculation of the stiffnesses takes much more computer time than that of the analytical method.

3. RESULTS

The input data range is shown in Table 1. The box-beam is assumed to be made of AS4/3501-6 Graphite/Epoxy. The material properties are $E_1=1.42\times10^{11}$ N/m^2, $E_2=E_3=9.8\times10^9$ N/m^2, $G_{12}=G_{13}=6.0\times10^9$ N/m^2, $G_{23}=4.83\times10^9$ N/m^2, $\rho=1603$ kg/m^3, $\nu_{12}=\nu_{13}=0.42$, $\nu_{23}=0.5$. Each wall used to model the box-beam is made of laminated orthotropic composite plies. The stiffnesses are obtained by the use of both methods. The results of methods are in good agreement for a thin-walled box beam. Nevertheless, the analytical method yields big errors in stiffness for some cases [11]. Sensitivities of stiffnesses to design variables are obtained from VABS, and are given in the following figures: Figure 1 for extension stiffness normalized by $10^8\times$N, Figure 2 for extension-torsion stiffness normalized by $10^6\times$Nm, Figure 3 for torsion stiffness normalized by $10^4\times$Nm^2, Figure 4 for out-of-plane bending stiffness normalized by $10^5\times$Nm^2, Figure 5 for in-of-plane bending stiffness normalized by $10^6\times$Nm^2. It is found that each parameter is of importance on the stiffnesses. The figures show the most important parameters affecting the stiffnesses. Extension stiffness is the most sensitive to width and horizontal wall thickness. Extension-torsion stiffness is the most sensitive to ply angle and height. Width and horizontal wall thickness affect the extension-torsion stiffness less than ply angle and height but more than vertical wall thickness and pretwist. Torsion stiffness is more sensitive to height, width, and ply angle than horizontal wall thickness, vertical wall thickness and pretwist. Out-of-plane bending stiffness is the most sensitive to height whereas in-plane bending stiffness is the most sensitive to width.

Table 1. Design Data Range

Variables	Minimum	Mid-point	Maximum
Ply angle	-20 deg	-10 deg	0 deg
Width	0.040 m	0.170 m	0.300 m
Height	0.030 m	0.065 m	0.100 m
Horizontal wall thickness	0.002 m	0.006 m	0.010 m
Vertical wall thickness	0.002 m	0.0085 m	0.015 m
Initial pretwist	-0.4 rad/m	-0.2 rad/m	0.0 rad/m

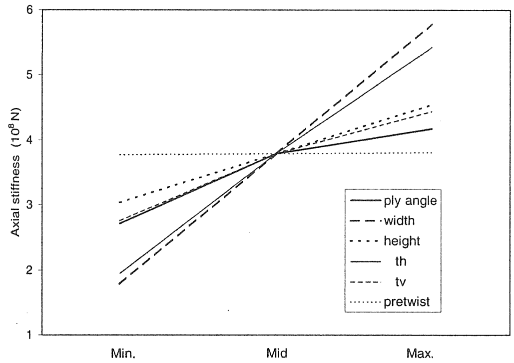

Figure 1. Sensitivity of extension stiffness to design variables

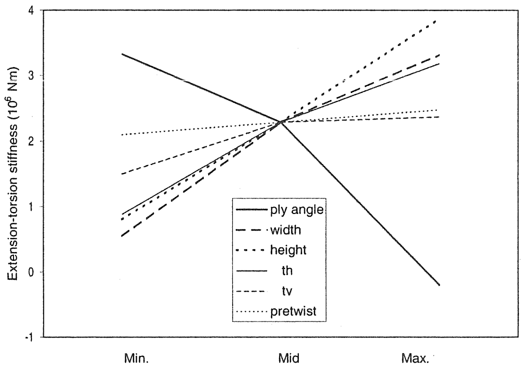

Figure 2. Sensitivity of extension-torsion stiffness to design variables

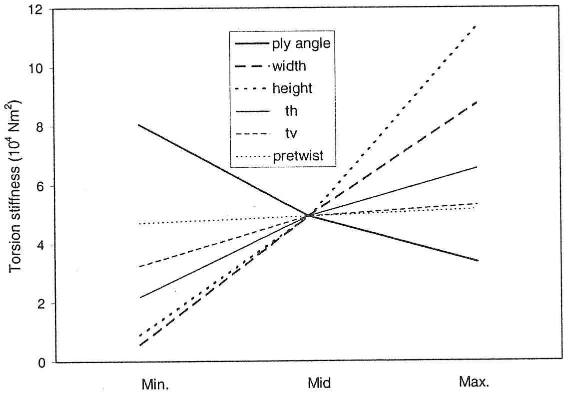

Figure 3. Sensitivity of torsion stiffness to design variables

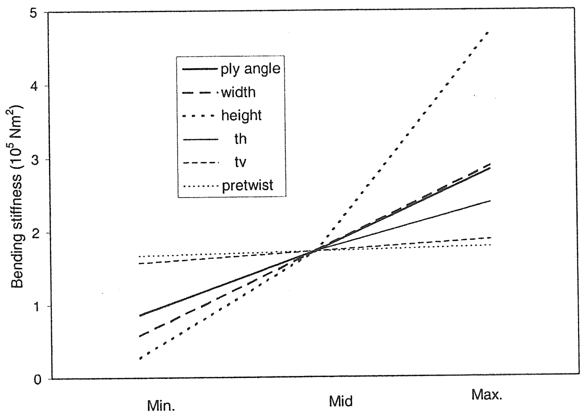

Figure 4. Sensitivity of out-of-plane bending stiffness to design variables

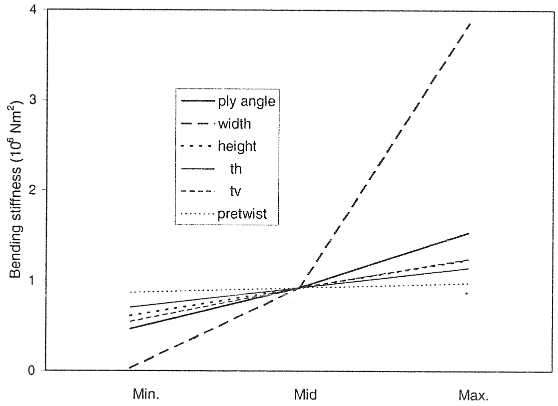

Figure 5. Sensitivity of in-plane bending stiffness to design variables

4. CONCLUSIONS

Cross-sectional analysis of a composite box-beam is carried out by an analytical and a variational asymptotic method. The analytical method is less expensive (computer time), gives accurate results for only a thin-walled box-beam. The variational-asymptotic method is more expensive, but gives more accurate results for both thin and thick-walled box-beams.

For an extension-torsion coupled box-beam the sensitivities of the stiffnesses to design parameters are obtained for the design data range. The most important parameters affecting the stiffnesses are determined.

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