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RESEARCH ARTICLE

STABILIZED FEM SOLUTION of MAGNETOHYDRODYNAMIC FLOW in DIFFERENT GEOMETRIES

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ABSTRACT

In this study, the stable numerical solution of the magnetohydrodynamic (MHD) flow in different geometries is presented using the stabilized finite element method (FEM). Numerical solution of coupled convection-diffusion type MHD equations have been acquired for the different Hartmann numbers (M_i) and different angles of the MHD flows. The resultant matrix-vector system has been solved as a whole with the reciprocal MHD flow and boundary conditions. We have observed from the solution of reciprocal MHD flow when the Hartmann number increases the velocity and the induced magnetic field of the flows decrease. We have been acquired the stable numerical solution for the $M_i = 10^2$ Hartmann number. The obtained stable numerical results are displayed by graphics.

Keywords: *Magnetohydrodynamic, Stabilized-FEM, Different Geometries*

1. INTRODUCTION

Magnetohydrodynamic flow is popular among researchers, due to the fluid is under the influence of the magnetic field, such that engineers, medical scientists, etc. Therefore there is a wide range of theoretical, experimental like in [1–3] and numerical studies about MHD flow. Because of the coupled nature of the problem the theoretical results can obtain special cases of the problem. Therefore the numerical methods have been used to obtain the numerical solution of MHD equations. The finite element solution of fully developed MHD flow in channels has been obtained by Singh and Lal in [4] for the steady-state form of the problem in the different geometries. The boundary element method (BEM) solution of MHD flow problems for the high values of Hartmann number has been obtained in [5]. In [6], numerical solution of the MHD flow problems is obtained using the dual reciprocity boundary element method(drbem) with the external electrically conducting medium. Aydın and Selvitopi [7] have been solved the MHD flow problems in an unbounded conducting medium using stabilized FEM in the pipe and BEM for the exterior region considering rectangular and circular pipe with the different angle of the induced magnetic field and high values of Reynolds number, magnetic pressure and magnetic Reynolds numbers. The BEM solution of magnetohydrodynamic channel flows has been obtained in [8] for the high Hartmann numbers. The numerical methods, FEM, finite difference method (FDM), BEM and meshless methods, etc., have been applied to obtain the

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numerical solution of MHD flow problems in different geometries i.e. rectangular and circular duct case until this time.

In general, it has been considered in the studies that there is only one MHD flow in the pipe up to now. In this study, we have considered two MHD flow in the T-junction. Therefore, this study has two important novelty according to literature, the first one is the geometry of the pipe and the other is it has two MHD flow equations in junction. Tezer-Sezgin and Aydın [9] have obtained the numerical solution of the MHD flow in one, two and three parallel ducts which are separated by conducting walls and there is only one MHD flow in the all ducts using stabilized-FEM for the high Hartmann numbers. There are many studies in the literature about fluid flow in T-junction. The flow is organized in T-junction as division or the combination of a fluid flow. Vimmr and Jon´aˇsov´a [10] have considered the coronary and femoral bypasses model and they solved the non-newtonian blood flow using finite volume method with the fourth order Runge-Kutta algorithm. In [11] the numerical modeling and Piv measurement comparison were presented for the division and the combined fluid flow in T-junction with two inlets and only one outlet. Moshkin and Yambangwai [12] have solved the pressure-driven startup laminar flows in T-junction using the finite volume method. Beneˇs, et al. [13] have been obtained the numerical simulation of the laminar and turbulent flows of Newtonian and non-Newtonian fluids in T-junction with one inlet and two outlets that the mathematical model of the flows is Reynolds averaged Navier-Stokes equations using finite volume method with the artificial compressibility method. Matos and Oliveria [14] have presented the numerical solution of the steady and unsteady non-Newtonian inelastic flows using the finite volume method in a planar T-junction for the high values of Reynolds numbers. The volume of the fluid solution of the ferrofluid microdroplets in T-junction with an asymmetric magnetic field has been given in [15]. In [16] the numerical solution of the Newtonian, incompressible and thermostatic flow in T-junction has been investigated using the volume of fluid method. The application of the FEM for the linear and non-linear physical problems is presented in [17,19].

In the present study, we have obtained the stable numerical solution of the MHD flow in a T-junction using streamline upwind Petrov–Galerkin (SUPG) type stabilized FEM for the different values of Hartmann number.

The paper is introduced as follows. In Section 2 physical problem and the mathematical model of the problem are given. The stabilized finite element formulation of the considered problem is presented in Section 3. The obtained numerical results using stabilized FEM are given and discussed in Section 4. The conclusion is given in the last section.

2. MATERIAL AND METHOD

2.1. Physical Problem And Mathematical Model

The steady, reciprocal MHD flow has been considered in a T-junction. In the T-junction, there are two different flows under the influence of the magnetic field with different angle. The length of the channel has been assumed enough long and the problem dimension is reduced to two-dimension.

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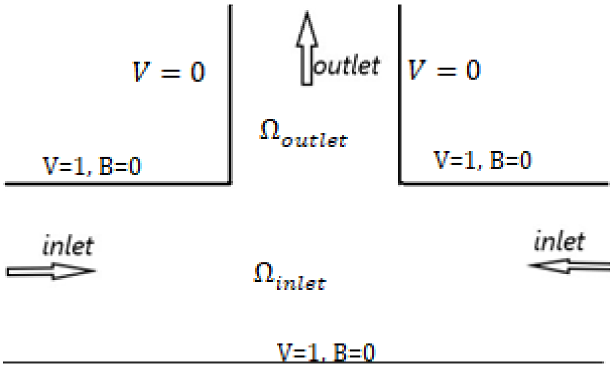

Figure 1. Problem domain.

The mathematical model of the considered problem is obtained by Navier-Stokes and Maxwell equations:

$$\nabla^2 V_i + \overline{M_{x_i}}\frac{\partial B_i}{\partial x} + \overline{M_{y_i}}\frac{\partial B_i}{\partial y} = \Delta P_i$$
$$\nabla^2 B_i + \overline{M_{x_i}}\frac{\partial V_i}{\partial x} + \overline{M_{y_i}}\frac{\partial V_i}{\partial y} = 0 \qquad i=1,2 \qquad (1)$$

where $\overline{M_{x_i}} = M_i sin\theta_i$, $\overline{M_{y_i}} = M_i cos\theta_i$, $\Delta P_1 = -1$ and $\Delta P_2 = 1$. M_i is the nondimensional parameter (Hartmann Numbers), describing $M_i = B_0 L_0 \sqrt{\sigma}/\sqrt{\mu}$. Here θ_i is the angle between applied magnetic field and y −axis and L_0, σ, μ are the characteristic length, electrical conductivity and viscosity of the fluid respectively. In the solution domain we have been considered the boundary consitiond as:

$$\text{V=0, B=1 in } \Omega_{inlet}$$
$$\frac{\partial V}{\partial n} = 0 \quad \text{in} \quad \Omega_{outlet} \qquad (2)$$

2.2. Stabilized Fem Formulation

Applyiing the standard finite element method to the coupled equations in (1), we get weak formulation using lineer function space $L = (H_0^1(\Omega))^2$ as: Find $\{V_i, B_i\} \in \{L \times L\}$ such that

$$B(V_i; B_i, w_1; w_2) = (-\Delta P_i, w_1), \ \ \forall\{w_1; w_2\} \in \{L \times L\} \qquad (3)$$

where

$$B(V_i; B_i, w_1; w_2) = (\nabla \mathrm{V_i}, \nabla \mathrm{w_1}) - \left(\overline{M_x}\frac{\partial B_i}{\partial x}, w_1\right) - (\overline{M_y}\frac{\partial B_i}{\partial y}, w_1) + (\nabla \mathrm{B_i}, \nabla \mathrm{w_2}) - \left(\overline{M_x}\frac{\partial V_i}{\partial x}, w_2\right) - (\overline{M_y}\frac{\partial V_i}{\partial y}, w_2)$$

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Then, the variational formulation is written by the choice of finite dimensional subspaces $L_h \subset L$, defined by triangulation of the domain. Specifying a finite element discretisation, the weak formulation becomes: Find $\{V_{i_h}; B_{i_h}\} \in \{L_h \times L_h\}$ such that

$$B(V_{i_h}; B_{i_h}, w_{1_h}; w_{2_h}) = (-\Delta P_{i_h}, w_{1_h}), \quad \forall\{w_{1_h}; w_{2_h}\} \in \{L_h \times L_h\}. \tag{4}$$

We have been decoupled the equations in (1), using the transformations $U_i = V_i + B_i$ and $Z_i = V_i - B_i$ to be able to apply the SUPG type stabilized FEM technique. Using the transformations $U_i = V_i + B_i$ and $Z_i = V_i - B_i$, we obtain the decoupled convection-diffusion type equations as:

$$\begin{aligned} \nabla^2 U_i + \overline{M_x}\frac{\partial U_i}{\partial x} + \overline{M_y}\frac{\partial U_i}{\partial y} &= \Delta P_i \\ \nabla^2 Z_i - \overline{M_x}\frac{\partial Z_i}{\partial x} - \overline{M_y}\frac{\partial Z_i}{\partial y} &= \Delta P_i \end{aligned} \qquad \text{i=1,2} \tag{5}$$

We have been considered the SUPG type stabilization technique to obtain the smooth behaviour of the induced magnetic field and the velocity for the large values of Ha which contained in the convection-dominated convection-duffusion type equations in (4). That is; find $\{U_{i_h}, Z_{i_h}\} \in \{L_h \times L_h\}$ such that

$$\begin{aligned} &(\nabla U_{i_h}, \nabla v_{1_h}) - \left(\overline{M_x}\frac{\partial U_{i_h}}{\partial x}, v_{1_h}\right) - \left(\overline{M_y}\frac{\partial U_{i_h}}{\partial y}, v_{1_h}\right) + \\ &\quad (\nabla Z_{i_h}, \nabla v_{2_h}) + \left(\overline{M_x}\frac{\partial Z_{i_h}}{\partial x}, v_{2_h}\right) + \left(\overline{M_y}\frac{\partial Z_{i_h}}{\partial y}, v_{2_h}\right) \\ &\quad + \tau_K\left\{\left(-\overline{M_x}\frac{\partial U_{i_h}}{\partial x} - \overline{M_y}\frac{\partial U_{i_h}}{\partial y} + \Delta P_{i_h}, -\overline{M_x}\frac{\partial v_{1_h}}{\partial x} - \overline{M_y}\frac{\partial v_{1_h}}{\partial y}\right)\right. \\ &\quad \left. + \left(\overline{M_x}\frac{\partial Z_{i_h}}{\partial x} + \overline{M_y}\frac{\partial Z_{i_h}}{\partial y} + \Delta P_{i_h}, \overline{M_x}\frac{\partial v_{2_h}}{\partial x} + \overline{M_y}\frac{\partial v_{2_h}}{\partial y}\right)\right\} \\ &\quad = (-\Delta P_{i_h}, v_{1_h}) + (-\Delta P_{i_h}, v_{2_h}) \end{aligned} \tag{6}$$

$\forall\{v_{1_h}, v_{2_h}\} \in \{L_h \times L_h\}$, $v_{1_h} = w_{1_h} + w_{2_h}$, $v_{2_h} = w_{1_h} - w_{2_h}$ with the stabilization parameter [20]

$$\tau_K = \begin{cases} \dfrac{h_K}{2Ha} & if \quad Pe_K \geq 1 \\ \dfrac{h_K^2}{12} & if \quad Pe_K < 1 \end{cases} \tag{7}$$

Here, h_k is the diameter of the eşlement K and $Pe_K = h_K \frac{Ha}{6}$ is the Peclet number.

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Eventually, using the inverse transformations $V_i = (U_i + Z_i)/2$ and $Z_i = (U_i - Z_i)/2$ one can get the final system of the stabilized FEM discrete formulation for the induced magnetic field and the velocity:

$$\begin{aligned}(\nabla \mathrm{V}_{\mathrm{i_h}}, \nabla \mathrm{w}_{1_\mathrm{h}}) &- \left(\overline{M_x}\frac{\partial B_{i_h}}{\partial x}, w_{1_h}\right) - \left(\overline{M_y}\frac{\partial B_{i_h}}{\partial y}, w_{1_h}\right) + (\nabla \mathrm{B}_{\mathrm{i_h}}, \nabla \mathrm{w}_{2_\mathrm{h}}) - \left(\overline{M_x}\frac{\partial V_{i_h}}{\partial x}, w_{2_h}\right)\\ &- \left(\overline{M_y}\frac{\partial V_{i_h}}{\partial y}, w_{2_h}\right) + \tau_K\left(\overline{M_x}\frac{\partial V_{i_h}}{\partial x} + \overline{M_y}\frac{\partial V_{i_h}}{\partial y}, \overline{M_x}\frac{\partial w_{1_h}}{\partial x} + \overline{M_y}\frac{\partial w_{1_h}}{\partial y}\right)\\ &+ +\tau_K\left(\overline{M_x}\frac{\partial B_{i_h}}{\partial x} + \overline{M_y}\frac{\partial B_{i_h}}{\partial y}, \overline{M_x}\frac{\partial w_{2_h}}{\partial x} + \overline{M_y}\frac{\partial w_2}{\partial y}\right)\\ &= (-\Delta P_{i_h}, w_{1_h}) + \tau_K\left(\Delta P_{i_h}, \overline{M_x}\frac{\partial w_{2_h}}{\partial x} + \overline{M_y}\frac{\partial w_{2_h}}{\partial y}\right)\end{aligned} \tag{8}$$

$\forall\{w_1, w_2\} \in \{L_h \times L_h\}$. Then, the solution of the system (8) has been given the induced currents and the velocity of the fluid.

3. NUMERICAL RESULTS

In this section, we have given the stabilized FEM solution of MHD equations (1) in a T-Junction for the different Hartmann numbers and different angles. We have been considered the $Ha = 1, 10$ and 100. The angles between MHD flow and y −axis $\theta_{1,2} = \pi/4$ and $\pi/2$.

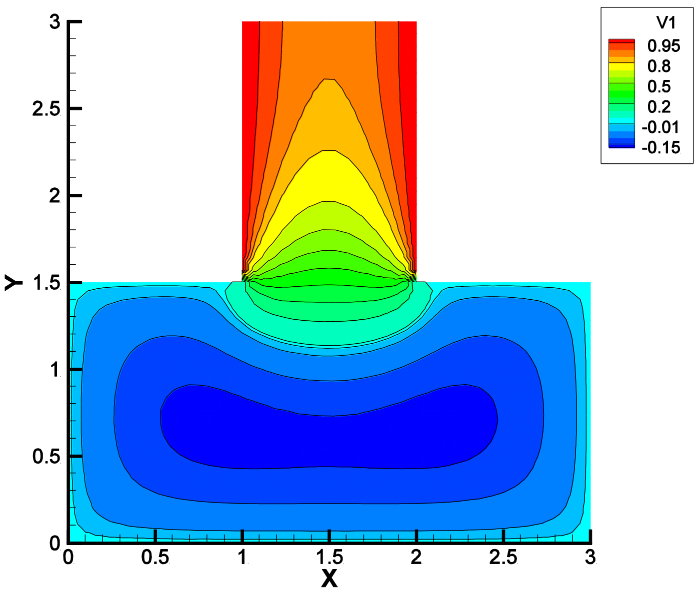

Figure. 1. Velocity for $M_1 = 1$ and $\theta_1 = \pi/2$.

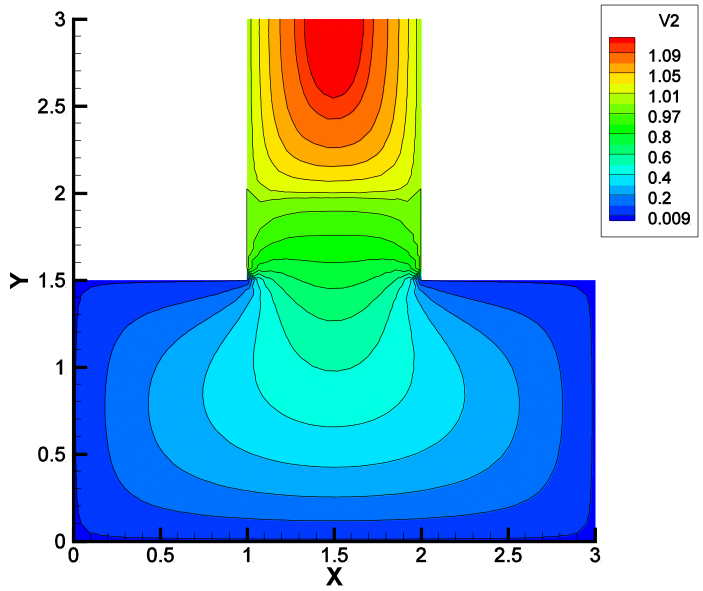

Figure. 2. Velocity for $M_2 = 1$ and $\theta_2 = \pi/2$.

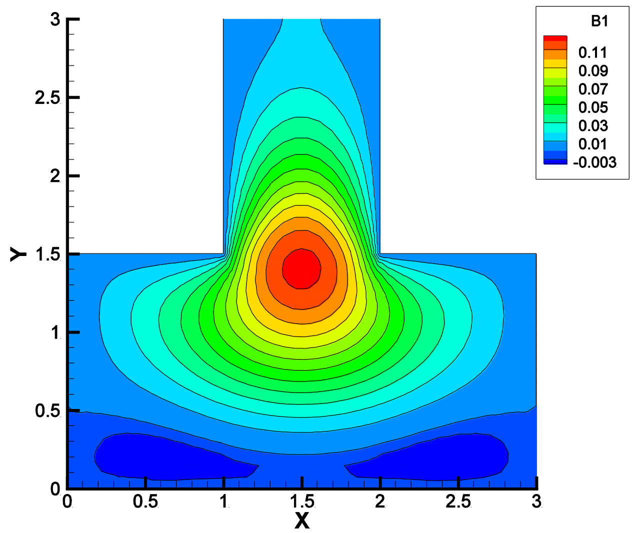

Figure. 3. Magnetic induction for $M_1 = 1$ and $\theta_1 = \pi/2$.

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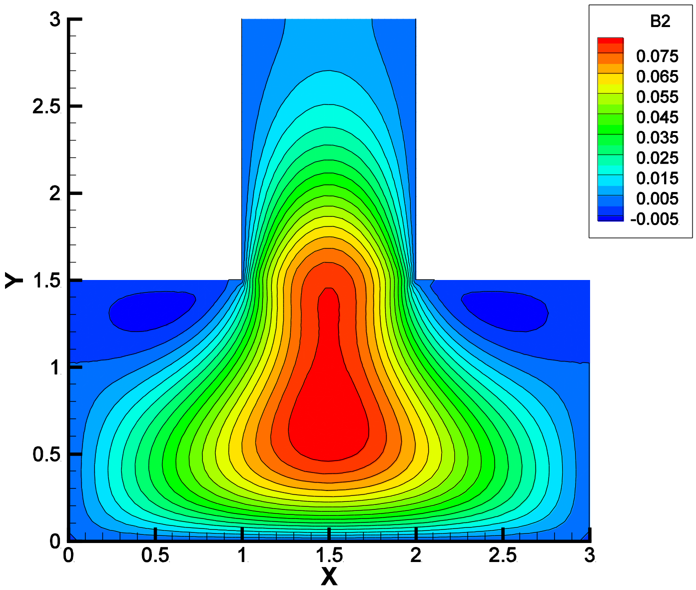

Figure. 4. Magnetic induction for $M_2 = 1$ and $\theta_2 = \pi/2$.

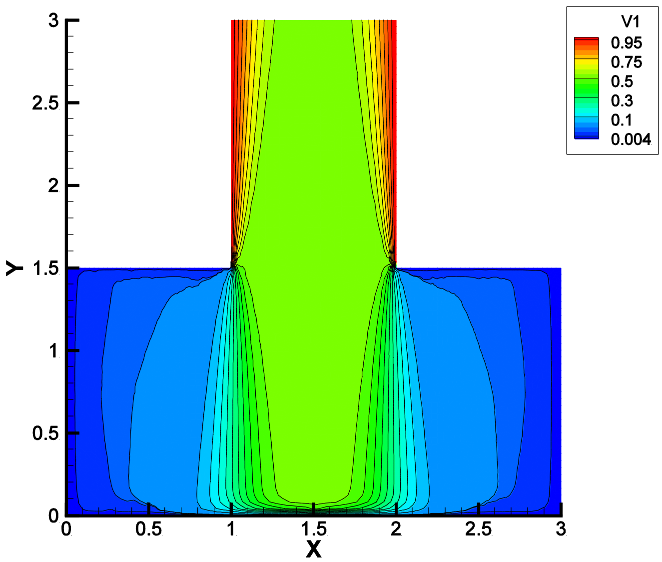

Figure. 5. Velocity for $M_1 = 100$ and $\theta_1 = \pi/2$.

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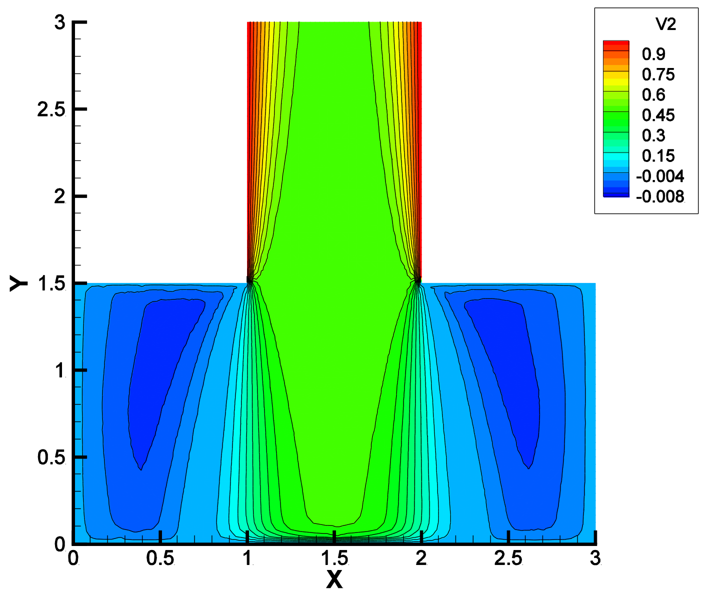

Figure. 6. Velocity for $M_2 = 100$ and $\theta_2 = \pi/2$.

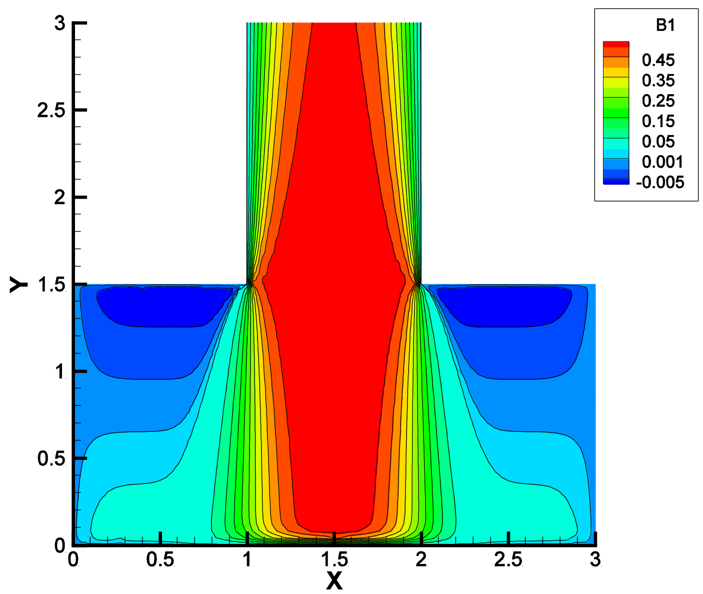

Figure. 7. Magnetic induction for $M_1 = 100$ and $\theta_1 = \pi/2$.

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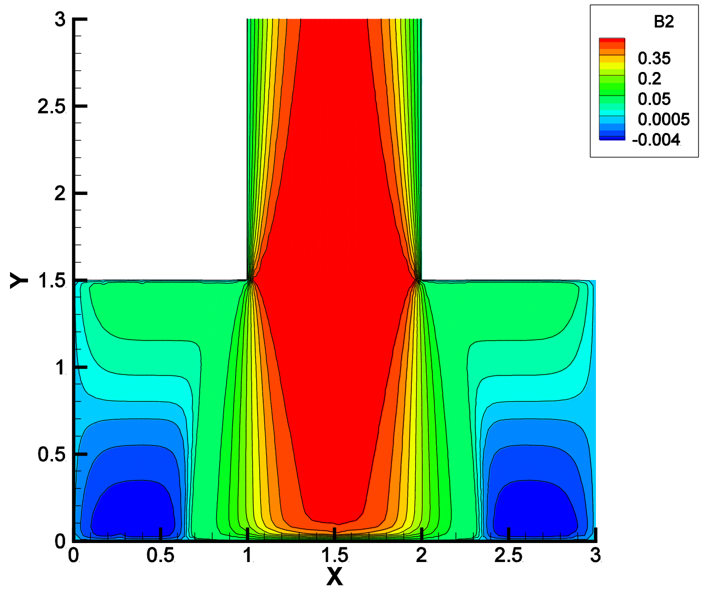

Figure. 8. Magnetic induction for $M_2 = 100$ and $\theta_2 = \pi/2$.

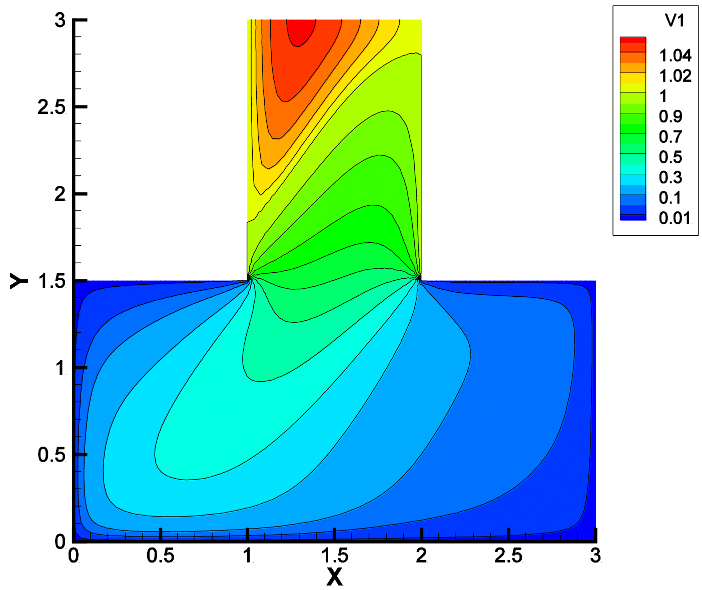

Figure. 9. Velocity for $M_1 = 10$ and $\theta_1 = \pi/4$.

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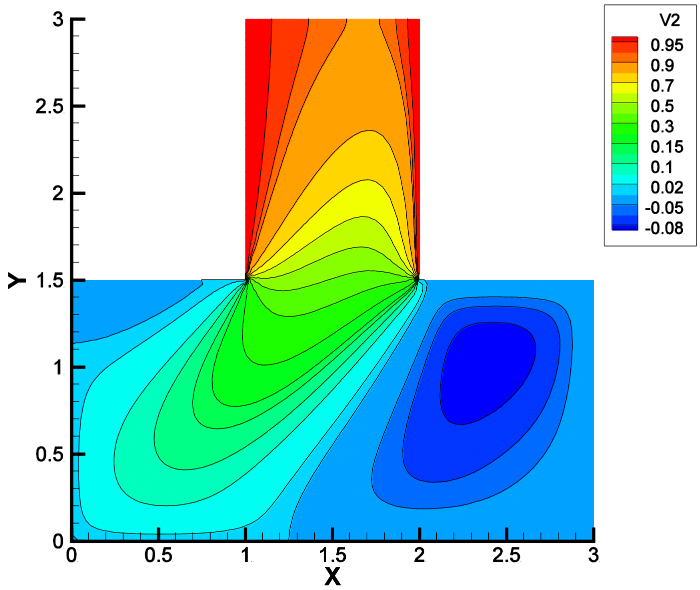

Figure. 10. Velocity for $M_2 = 10$ and $\theta_2 = \pi/4$.

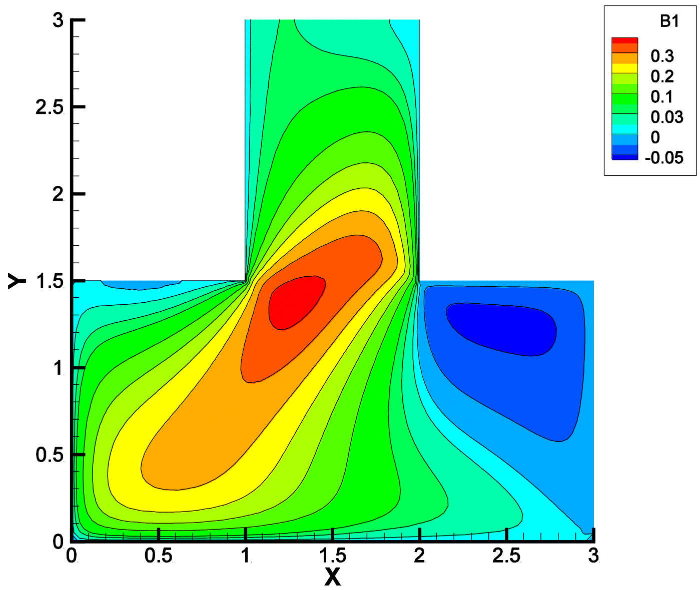

Figure. 11. Magnetic induction for $M_1 = 10$ and $\theta_1 = \pi/4$.

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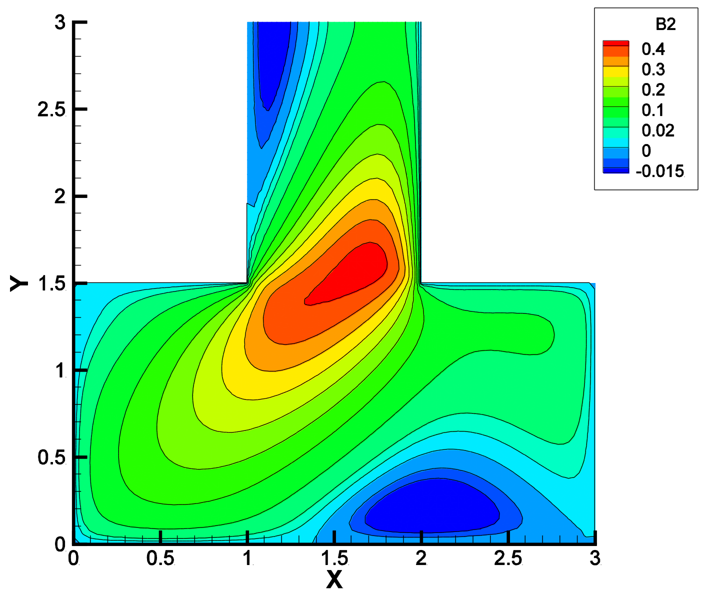

Figure. 12. Magnetic induction for $M_2 = 10$ and $\theta_2 = \pi/4$.

4. DISCUSSION AND CONCLUSION

The stabilized FEM solution of the MHD equations has been acquired for different Hartman numbers and different angles between induced magnetic fields and the $y-$axis in a T-Junction. In **Fig. 1,2,5,6,9** and **10,** one can observe the velocity contour of the fluid and in **Fig. 3,4,7,8,11** and **12** one can observe the contour of the magnetic induction.

We have been considered the angles $\theta_1 = \theta_2 = \pi/2$ with $M_1 = M_2 = 1$ and 100 for **Fig. 1.** to **Fig. 8.** and we have also been considered the angles $\theta_1 = \theta_2 = \pi/4$ with $M_1 = M_2 = 10$ for **Fig. 9.** to **Fig. 12.** We can see from the figures that the layers occur with changing of the angles. We can observe from **Fig. 3,4,7,8** the boundary layer in the inlet walls occurs for the magnetic induction when the Harmann numbers increases. We can also observe from **Fig. 1,2,5,6** the flow approaches the walls for the large number of the Hartmann numbers.

One can also say that, the velocity of the first flow is dominant in the inlet and the velocity of the second flow is dominant in the outlet of the channels. The induced magnetic field is dominant in the all domain for the $\theta_1 = \theta_2 = \pi/2$.

As a result, in this work we have focused on the investigation of the MHD flow in different geometries numerically using SUPG type stabilized FEM. We have determined the dominant velocity and induced magnetic field of the flows according to the different values of Hartmann numbers and different angles.

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