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PROPOSAL OF A SOLUTION TO FUZZY TRANSPORTATION PROBLEM

USING FUZZY SET APPROACH

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Özet

Gerçek problemlerde, miktarlarının kesin olarak bilinmediği taşıma problemleri ile sık sık karşılaşılır. Mevcut stok ve talep miktarları bazı kontrol edilemeyen etmenlerden dolayı belirsiz olabilir.

Bu çalışmada, birim taşıma maliyetleri ile stok ve talep miktarları bulanık sayılar olduğunda, bulanık sayıların üyelik fonksiyonlarını kullanarak bulanık taşıma problemini çözen bir algoritma sunduk. Önerilen bu çözüm algoritmasında, taşıma probleminin optimal uygun çözümleri elde edildi. Önerilen çözümün etkinliğini göstermek için, sayısal örnek verildi. Verilen örnek WINQSB[16] optimizasyon programı ile çözüldü.

Abstract

In the real world applications, frequently may be faced up with transportation problems that these quantities may not be known in precise manner. The supplies and demands may be uncertain due to some uncontrollable factors.

In this study, we have presented an algorithm solving fuzzy transportation problem using membership functions of these fuzzy numbers when the unit shipping costs, the supply quantities and the demand quantities are fuzzy numbers. The proposed solution algorithm to fuzzy transportation problem yields optimal compromise solutions. To show the ability the proposed solution, the numerical example has been presented. The given example is solved using optimization software WINQSB [16].

1. INTRODUCTION

The conventional transportation problem is a special form of linear programming problem. In a transportation problem a product is to be transported from m sources to n destinations and their capacities are $s_1, s_2, ..., s_m$ and $d_1, d_2, ..., d_n$, respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from source *i* th to destination *j* th. This penalty may be cost or time or safety of delivery, etc. A variable x_{ij} represents the unknown quantity to be transported from source *i* th to destination *j* th.

Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and supply and demand quantities are known exactly. However, there are transportation problems that these quantities may not be known in precise manner. For example; the unit transport cost may vary in a time. The supplies and demands may be uncertain due to some uncontrollable factors. After Bellmann and Zadeh [1] introduced the concept of fuzziness [2], a lot of techniques have been developed that apply the existing fuzzy linear programming to the fuzzy transportation problem [2-13] in the literature. Shiang-Tai Liu and Chiang Kao [3] develop a solution procedure that is able to calculate the fuzzy objective value of fuzzy transportation problem, where at least one of the parameters are fuzzy numbers: using Zadeh's extension principle. Waid F.Abd El-Wahed [4] presented a fuzzy programming approach to find an optimal compromise solution of a classical transportation problem with multi objectives. J.L.Ringuest and D.B Rinks [6] presented two interactive algorithms which take advantage of the special form of the multiple objective transportation problems.

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In this study, we have presented an algorithm solving fuzzy transportation problem using membership functions of these fuzzy numbers when the unit shipping costs, the supply quantities and the demand quantities are fuzzy numbers.

2. FUZZY TRANSPORTATION PROBLEM

Consider a transportation problem with m supply and n demand, in that $s_i > 0$ units are supplied by supply i *th* and $d_j > 0$ units are required by demand j *th*. There is a unit shipping cost c_{ij} for transportation. Let x_{ij} denote the number of units to be transported from supply *ith* to demand j *th*. The mathematical model of the conventional transportation problem is written as follow:

$$Z = \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$
s.t
$$\sum_{j=1}^{n} x_{ij} \leq s_{i} \qquad i = 1,...,m$$

$$\sum_{i=1}^{m} x_{ij} \geq d_{j}, \qquad j = 1,...,n$$

$$x_{ij} \geq 0, \forall i, j$$

$$(1)$$

In (1) if only at one least of parameters c_{ij} , s_i , or d_j is fuzzy, the total transportation cost Z becomes fuzzy as well. Let \tilde{C}_{ij} , \tilde{S}_i , \tilde{D}_j denote the convex fuzzy sets of parameters c_{ij} , s_i , or d_j , respectively. The fuzzy transportation problem is of the following form:

$$Z = \min \sum_{i \neq 1}^{m} \sum_{j=1}^{n} \widetilde{C}_{ij} x_{ij}$$
s.t
$$\sum_{j=1}^{n} x_{ij} \leq \widetilde{S}_{i} \qquad i = 1, ..., m$$

$$\sum_{i=1}^{m} x_{ij} \geq \widetilde{D}_{j}, \qquad j = 1, ..., n$$

$$x_{ij} \geq 0, \quad \forall i, j$$

$$(2)$$

Let $\mu(\tilde{C}_{ij}), \mu(\tilde{S}_i)$ and $\mu(\tilde{D}_j)$ denote membership functions of $\tilde{C}_{ij}, \tilde{S}_i$ and \tilde{D}_j . We have

$$\begin{split} \widetilde{C}_{ij} &= \left\{ (c_{ij}, \mu(c_{ij})) / c_{ij} \in S(\widetilde{C}_{ij}) \right\} \\ \widetilde{S}_i &= \left\{ (s_i, \mu(s_i)) / s_i \in S(\widetilde{S}_i) \right\} \\ \widetilde{D}_j &= \left\{ (d_j, \mu(d_j)) / d_j \in S(\widetilde{D}_j) \right\} \end{split}$$
(3) In our study, we

proposed the algorithm of the solution to the fuzzy transportation problem. In our proposed algorithm to fuzzy transportation model, we consider problem (2) involving a trapezoidal fuzzy numbers of form

 $\tilde{C}_{ij}(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4), \quad \tilde{S}_i(s_i^1, s_i^2, s_i^3, s_i^4) \quad and \quad \tilde{D}_j(d_j^1, d_j^2, d_j^3, d_j^4).$

3. AN ALGORITHM

Under assumption that is given above these steps of algorithm to solution of the fuzzy transportation problem (2) can be combined as follow:

Step1. Calculate the total fuzzy supply and the total fuzzy demand

$$\sum \tilde{S}_{i} = (\sum s_{i}^{1}, \sum s_{i}^{2}, \sum s_{i}^{3}, \sum s_{i}^{4})$$

$$\sum \tilde{D}_{j} = (\sum d_{j}^{1}, \sum d_{j}^{2}, \sum d_{j}^{3}, \sum d_{j}^{4})$$
(4)

Step2. Calculate to $\alpha = \overline{\alpha}_k$, k = 1, 2, ... from the following equations

$$\sum s_i^1 + \alpha \sum s_i^2 = \sum d_j^1 + \alpha \sum d_j^2,$$

$$\sum s_i^1 + \alpha \sum s_i^2 = \sum d_j^4 - \alpha \sum d_j^3,$$

$$\sum s_i^4 - \alpha \sum s_i^3 = \sum d_j^1 + \alpha \sum d_j^2,$$

$$\sum s_i^4 - \alpha \sum s_i^3 = \sum d_j^4 - \alpha \sum d_j^3.$$
(5)

Step3. Get $\overline{\alpha}^* = Max \{ \overline{\alpha}_k \}$, k=1,2,...

Step4. Get following constraint (6) as crisp constraints of the transport problem If $\overline{\alpha}^*$ is obtained from equation $\sum s_i^1 + \alpha \sum s_i^2 = \sum d_i^1 + \alpha \sum d_i^2$.

$$\sum_{j=1}^{n} x_{ij} = s_{i}^{1} + \overline{\alpha}^{*} s_{i}^{2}, \qquad i = 1,...,m$$

$$\sum_{i=1}^{m} x_{ij} = d_{j}^{1} + \overline{\alpha}^{*} d_{j}^{2}, \qquad j = 1,...,n$$

$$x_{ii} \ge 0, \quad \forall i, j.$$
(6)

Go to the step 8.

Step 5. Get following constraint (7) as crisp constraints of the transport problem If $\overline{\alpha}^*$ is obtained from equation $\sum s_i^1 + \alpha \sum s_i^2 = \sum d_i^4 - \alpha \sum d_i^3$.

$$\sum_{j=1}^{n} x_{ij} = s_{i}^{1} + \overline{\alpha}^{*} s_{i}^{2}, \qquad i = 1,...,m$$

$$\sum_{i=1}^{m} x_{ij} = d_{j}^{4} - \overline{\alpha}^{*} d_{j}^{3}, \qquad j = 1,...,n$$

$$x_{ij} \ge 0, \quad \forall i, j.$$
(7)

Go to the step 8.

Step6. Get following constraint (8) as crisp constraints of the transport problem If $\overline{\alpha}^*$ is obtained from equation $\sum s_i^4 - \alpha \sum s_i^3 = \sum d_j^1 + \alpha \sum d_j^2$.

$$\sum_{j=1}^{n} x_{ij} = s_{i}^{4} - \overline{\alpha}^{*} s_{i}^{3}, \qquad i = 1,...,m$$

$$\sum_{i=1}^{m} x_{ij} = d_{j}^{1} + \overline{\alpha}^{*} d_{j}^{2}, \qquad j = 1,...,n$$

$$x_{ij} \ge 0, \quad \forall i, j.$$
(8)

Go to the step 8.

Step 7. Get following constraint (9) as crisp constraints of the transport problem If $\overline{\alpha}^*$ is obtained from equation $\sum s_i^4 - \alpha \sum s_i^3 = \sum d_j^4 - \alpha \sum d_j^3$.

$$\sum_{j=1}^{n} x_{ij} = s_{i}^{4} - \overline{\alpha}^{*} s_{i}^{3}, \qquad i = 1,...,m$$

$$\sum_{i=1}^{m} x_{ij} = d_{j}^{4} - \overline{\alpha}^{*} d_{j}^{3}, \qquad j = 1,...,n$$

$$x_{ij} \ge 0, \quad \forall i, j.$$
(9)

Step 8. Calculate the lower bound objective $Z_{\overline{\alpha}^*}^L = \sum_{i=1}^m \sum_{j=1}^n (c_{ij}^1 + \overline{\alpha}^* (c_{ij}^2 - c_{ij}^1)) x_{ij}$ and the upper bound

objective $Z_{\overline{\alpha}^*}^U = \sum_{i=1}^m \sum_{j=1}^n (c_{ij}^4 - \overline{\alpha}^* (c_{ij}^3 - c_{ij}^4)) x_{ij}$, that is, a fuzzy objective function become two crisp objective

functions.

Step 9. Determine solution vectors $x_{\overline{\alpha}^*}^{L^*}$, $x_{\overline{\alpha}^*}^{U^*}$ optimised objectives $Z_{\overline{\alpha}^*}^L = \sum_{i=1}^m \sum_{j=1}^n (c_{ij}^1 + \overline{\alpha}^* (c_{ij}^2 - c_{ij}^1)) x_{ij}$ and m

$$Z_{\overline{\alpha}^{*}}^{U} = \sum_{i=1}^{m} \sum_{j=1}^{m} (c_{ij}^{4} - \overline{\alpha}^{*} (c_{ij}^{3} - c_{ij}^{4})) x_{ij}$$

subject to the above calculated crisp constraints. Step 10. Calculate $Z_{\overline{\alpha}^*}^L(x_{\overline{\alpha}^*}^{L^*}) \leq Z_{\overline{\alpha}^*}^L \leq Z_{\overline{\alpha}^*}^L(x_{\overline{\alpha}^*}^{U^*})$ and $Z_{\overline{\alpha}^*}^U(x_{\overline{\alpha}^*}^{U^*}) \leq Z_{\overline{\alpha}^*}^U \leq Z_{\overline{\alpha}^*}^U(x_{\overline{\alpha}^*}^{L^*})$ using non dominant solution set.

Step11. Define the membership function as following equations

$$\mu(Z_{\overline{\alpha}^{\star}}^{L}) = \begin{cases}
1 & Z_{\overline{\alpha}^{\star}}^{L} \leq Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{L^{*}}) \\
\frac{Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{U^{*}}) - Z_{\overline{\alpha}^{\star}}^{L}}{Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{L^{*}}) - Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{L^{*}})} & Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{L^{*}}) \leq Z_{\overline{\alpha}^{\star}}^{L} \leq Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{U^{*}}) \\
0 & Z_{\overline{\alpha}^{\star}}^{L} \geq Z_{\overline{\alpha}^{\star}}^{L}(x_{\overline{\alpha}^{\star}}^{U^{*}})
\end{cases} (10)$$

$$\mu(Z_{\overline{\alpha}^{*}}^{U}) = \begin{cases}
1 & Z_{\overline{\alpha}^{*}}^{U} \leq Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{U^{*}}) \\
\frac{Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{L^{*}}) - Z_{\overline{\alpha}^{*}}^{U}}{Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{U^{*}})} & Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{U^{*}}) \leq Z_{\overline{\alpha}^{*}}^{U} \leq Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{L^{*}}) \\
0 & Z_{\overline{\alpha}^{*}}^{U} \geq Z_{\overline{\alpha}^{*}}^{U}(x_{\overline{\alpha}^{*}}^{L^{*}})
\end{cases}$$
(11)

Step 12. Using Zimmermann's min operator the fuzzy model (2) transformed to the crisp model as:

 $Max \lambda$ $\mu(Z_{\overline{\alpha}}^{L}) \geq \lambda,$ $\mu(Z_{\overline{\alpha}}^{U}) \geq \lambda,$ $\sum_{i=1}^{m} x_{ij} = s_{i} \in (S_{i})_{\alpha},$ $\sum_{j=1}^{n} x_{ij} = d_{j} \in (D_{j})_{\alpha},$ $x_{ij} \geq 0, \forall i, j.$ (12)

Solve the problem (12).

Step 13. Ask the decision maker solution of the problem(12).

Step 14. If the decision maker is satisfied, STOP. If the decision maker is not satisfied, get other maximum $\overline{\alpha}^*$, continue with step 3.

4. AN EXAMPLE

We consider a fuzzy transportation problem with fuzzy shipping costs, fuzzy supplies and fuzzy demands. All of these fuzzy costs, fuzzy supplies and fuzzy demands are trapezoidal fuzzy numbers or triangular numbers. The problem has the following fuzzy transportation problem form:

| $\tilde{c}_{11} = (60, 70, 80, 90)$ | $\widetilde{c}_{12} = (50, 55, 60, 65)$ | $\widetilde{S}_1 = (70, 90, 100)$ |
|--------------------------------------|-----------------------------------------|-----------------------------------|
| $\tilde{c}_{21} = (50, 70, 90, 110)$ | $\tilde{c}_{22} = (40, 50, 70, 100)$ | $\tilde{S}_2 = (40,60,70,80)$ |
| $\widetilde{D}_1 = (30, 40, 50, 70)$ | $\tilde{D}_2 = (60, 80, 90, 130)$ | |

The given fuzzy transportation problem can be written as the following fuzzy linear programming problem:

 $\widetilde{Z} = \min\{(60, 70, 80, 90)x_{11} + (50, 55, 60, 65)x_{12} + (50, 70, 90, 110)x_{21} + (40, 50, 70, 100)x_{22}\}$ s.t.

$$\begin{aligned} x_{11} + x_{12} &\leq (70,90,100) \\ x_{21} + x_{22} &\leq (40,60,70,80) \\ x_{11} + x_{21} &\geq (30,40,50,70) \\ x_{12} + x_{22} &\geq (60,80,90,130) \\ x_{11}, x_{12}, x_{21}, x_{22} &\geq 0. \end{aligned}$$
(13)

The total supply and the total demand are

$$\widetilde{S}_{1} + \widetilde{S}_{2} = (110, 150, 160, 180) = [110 + 40\alpha; 180 - 20\alpha],$$

$$\widetilde{D}_{1} + \widetilde{D}_{2} = (90, 120, 140, 200) = [90 + 30\alpha; 200 - 60\alpha].$$
(14)

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From bounds in intervals of the total supply and the total demand

$$110 + 40\alpha = 90 + 30\alpha$$

$$110 + 40\alpha = 200 - 60\alpha$$

$$90 + 30\alpha = 180 - 20\alpha$$

$$180 - 20\alpha = 200 - 60\alpha$$

(15)

are calculated as $\overline{\alpha} = 0.9$ and $\overline{\alpha} = 0.5$. $\overline{\alpha} = 0.9$ is obtained from the lower bound of the total fuzzy supply and the upper bound of the total fuzzy demand. That is, the total supply is equal the total demand at these satisfactory levels of α . The fuzzy transportation problem is balanced for $\overline{\alpha} = 0.9$ and $\overline{\alpha} = 0.5$. We have the following fuzzy transportation problem:

$$Z_{\alpha}^{L} = \min\{(60+10\alpha)x_{11} + (50+5\alpha)x_{12} + (50+20\alpha)x_{21} + (40+10\alpha)x_{22}\}$$

$$x_{11} + x_{12} = 70 + 20\alpha,$$

$$x_{21} + x_{22} = 40 + 20\alpha,$$

$$x_{11} + x_{21} = 70 - 20\alpha,$$

$$x_{11} + x_{22} = 130 - 40\alpha$$

$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0.$$

$$Z_{\alpha}^{U} = \min\{(90-10\alpha)x_{11} + (65-5\alpha)x_{12} + (110-20\alpha)x_{21} + (100-30\alpha)x_{22}\}$$

$$x_{11} + x_{12} = 70 + 20\alpha,$$

$$x_{21} + x_{22} = 40 + 20\alpha,$$

$$x_{21} + x_{22} = 40 + 20\alpha,$$

$$x_{11} + x_{21} = 70 - 20\alpha,$$

$$(17)$$

$$x_{12} + x_{22} = 130 - 40\alpha$$

$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0.$$

Since maximum value of $\overline{\alpha}$'s is $\overline{\alpha} = 0.9$, for $\overline{\alpha} = 0.9$, we have the following crisp transportation problems from problem (16) and (17):

$$Z_{\alpha}^{L} = \min\{69x_{11} + 54.5x_{12} + 68x_{21} + 49x_{22}\}$$

$$x_{11} + x_{12} = 88,$$

$$x_{21} + x_{22} = 58,$$

$$x_{11} + x_{21} = 52,$$

$$x_{12} + x_{22} = 94,$$

$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0.$$

$$Z_{\alpha}^{U} = \min\{81x_{11} + 60.5x_{12} + 92x_{21} + 73x_{22}\}$$

$$x_{11} + x_{12} = 88,$$

$$x_{21} + x_{22} = 58,$$

$$x_{11} + x_{21} = 52,$$

$$x_{12} + x_{22} = 94,$$

$$x_{12} + x_{22} = 94,$$

$$x_{11}, x_{12}, x_{21}, x_{22} \ge 0.$$
(19)

 $Min Z_{0.9}^{L}(52, 36, 0, 58) = 8392, Min Z_{0.9}^{U}(0, 88, 52, 6) = 10546.$ For the $\overline{\alpha} = 0.9$ cut of \widetilde{Z} , the lower bound of $Z^* = 8392$ occurs at $x_{11}^* = 52$, $x_{11}^* = 36$, $x_{11}^* = 0$, $x_{11}^* = 58$ with $s_1 = 88$, $s_2 = 58$ and $d_1 = 52$, $d_2 = 94$ and the upper bound of $Z^* = 10546$ occurs at

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13. Sayı Haziran 2007 N. G $x_{11}^* = 0, x_{11}^* = 88, x_{11}^* = 52, x_{11}^* = 6$ with $s_1 = 88, s_2 = 58$ and $d_1 = 52, d_2 = 94$. From step 10 is calculated $8392 \le Z_{0.9}^L \le 8626, 10546 \le Z_{0.9}^U \le 10624$.

Using the membership functions defined in (11) and (12) for $\overline{\alpha} = 0.9$, the above fuzzy model (MTP) reduces to the crisp problem (12) as follows:

$\begin{aligned} &Max\,\lambda\\ & 69x_{11} + 54.5x_{12} + 68x_{21} + 49x_{22} + 234\lambda \leq 8626,\\ & 81x_{11} + 60.5x_{12} + 92x_{21} + 73x_{22} + 78\lambda \leq 10624,\\ & x_{11} + x_{12} = 88,\\ & x_{21} + x_{22} = 58,\\ & x_{11} + x_{21} = 52,\\ & x_{12} + x_{22} = 94,\\ & x_{11}, x_{12}, x_{21}, x_{22}, \lambda \geq 0. \end{aligned}$

Solving the above LP model (20), solution is obtained as $x_{11}^* = 26$, $x_{12}^* = 62$, $x_{21}^* = 26$, $x_{22}^* = 32$ and $\lambda = 0.5$. Therefore is calculated as $\mu(Z_{0.9}^L(x^*)) = 0.5$ and $\mu(Z_{0.9}^L(x^*)) = 0.5$. The calculated results are presented the decision maker. If the decision maker is satisfied, stop, if the decision maker is not satisfied, continue the algorithm for $\overline{\alpha} = 0.5$.

5.CONCLUSION

We have presented a solution algorithm that finds an optimal compromise solution of a fuzzy transportation problem with fuzzy supplies, fuzzy demands and fuzzy costs where fuzzy quantities can be trapezoidal and triangular fuzzy numbers.

In this study firstly are calculated values of α , -cuts that total supply is equal total demand from the mathematical form of the membership functions of total fuzzy supply and demand directly. Thus, constraints of fuzzy transportation problem are transformed to crisp constraints of the transportation problem. So the solution procedures are easier. After the constraints become the crisp constraints, objective function with the fuzzy costs becomes two objective function with crisp costs obtained lower and upper bounds for the calculated α . Using Zimmermann min operator and adding these membership functions of two objective functions calculated for lower and upper bounds optimization model of the fuzzy transport is written as problem (12). The obtained solution from problem (12) is presented to decision maker. If decision maker is not accepted these results, for other α – cuts, transportation problem is solved again.

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