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Numerical Solution of the Neutron Transport Equation Using S_N Method with the First Kind of Chebyshev Polynomials

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Abstract. The numerical solution of the neutron transport equation for one-speed neutrons in a finite homogeneous slab is investigated. The neutrons are assumed to be scattered isotropically through the medium involving constant isotropic source. The stationary transport equation is first written in the form of discrete ordinates and then it is solved for the eigenvalue spectrum of the neutrons using the Chebyshev polynomials of first kind. The eigenvalues are calculated for various values of the c_0, the mean number of secondary neutrons per collision, using the Gauss-Chebyshev quadrature set and they are given in the tables.

Keywords: Neutron transport equation, eigenvalues, S_N method, Chebyshev polynomials.

Birinci Tip Chebyshev Polinomlarıyla S_N metodu kullanılarak Nötron Transport Denkleminin Nümerik Çözümü

Özet. Nötron transport denkleminin, sonlu ve homojen bir dilimde tek-gruplu nötronlar için nümerik çözümü incelenmiştir. Nötronların sabit ve izotropik bir kaynak bulunan ortam boyunca izotropik olarak saçıldıkları varsayılmıştır. Kararlı durum transport denklemi, önce diskret ordinatlar formunda yazılmış ve daha sonra bu denklem birinci tip Chebyshev polinomları kullanılarak nötronların özdeğer spektrumu için çözülmüştür. c_0'ın, çarpışma başına ortalama ikincil nötron sayısı, farklı değerleri için Gauss-Chebyshev kuadratür seti kullanılarak özdeğerler hesaplanmış ve bunlar tablolarda verilmiştir.

Anahtar Kelimeler: Nötron Transport Denklemi, özdeğerler, S_N metodu, Chebyshev polinomları

1. INTRODUCTION

As well known, in order to maintain the fission chain reaction and thus to produce continuous power generation in a reactor system, it is important to protect the number of neutrons travelling throughout the system. The conservation of the neutron population or the constant power production can be perceived as isotropic scattering. The neutron transport equation which explains the distribution and the conservation of the neutrons in the system can be solved for the isotropic scattering.

The deterministic and stochastic methods are developed for the solution of the transport equation. Although a stochastic method of the Monte Carlo (MCNP) is one of the most effective and first methods used in the solution of the transport equation and the results obtained from it can be accepted as the benchmark for some cases, the deterministic methods such as spherical harmonics or discrete ordinates (S_N) are commonly used for the solution of the transport equation because of their fast iterations and accurate results obtained from easily derivable equations with less computational efforts [1-3].

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In this study, the first step for numerically solving the S_N transport equation is taken using the first kind of Chebyshev polynomials in the integral transform of the neutron angular flux. Therefore, the one-dimensional transport equation is converted into a discrete ordinates form in order to specify the eigenvalue spectrum of the monoenergetic neutrons traveling in a finite homogeneous slab using the Gauss-Chebyshev quadrature set.

2. THEORY AND EQUATIONS

With conventional notation, the neutron transport equation for monoenergetic neutrons in finite homogeneous slab can be written [4],

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \sigma_T \psi(x,\mu) = \frac{\sigma_{S0}}{2} \int_{-1}^{1} \psi(x,\mu')\,\mathrm{d}\mu' + \frac{Q_0}{2}, \tag{1}$$

where $\psi(x,\mu)$ is the neutron angular flux at position x and in direction μ, cosine of the angle between the neutron velocity vector and the positive x-axis. While σ_T is the total macroscopic cross-section, σ_{S0} is the differential scattering cross-section in the case of isotropic scattering and Q_0 is the internal source. The integrand in Eq. (1) can be evaluated as the integral transform with Chebyshev quadrature;

$$\int_{-1}^{1} \psi(x,\mu')\,\mathrm{d}\mu' = \int_{-1}^{1} \sqrt{1-\mu'^2}\,\psi(x,\mu') \frac{\mathrm{d}\mu'}{\sqrt{1-\mu'^2}} \cong \sum_{n=1}^{N} \sqrt{1-\mu_n^2}\,\psi_n(x)\omega_n . \tag{2}$$

Here ω_n is the Gauss-Chebyshev quadrature weight or weighting factor for direction μ_n, i.e. the roots of the Nth order Chebyshev polynomials of first kind. Then the roots and the weighting factors of the Chebyshev polynomials of first kind can be given as, respectively [5];

$$\mu_m = \cos\left(\frac{(2m-1)\pi}{2N}\right), \tag{3}$$

$$\omega_m = \frac{\pi}{N}, \qquad m = 1,2,\ldots,N. \tag{4}$$

When the evaluated integrand in Eq. (2) is replaced in Eq. (1), the discrete ordinates S_N equations can be obtained for the numerical solution,

$$\mu_m \frac{\mathrm{d}\psi_m(x)}{\mathrm{d}x} + \sigma_T \psi_m(x) = \frac{\sigma_{S0}}{2} \sum_{n=1}^{N} \sqrt{1-\mu_n^2}\,\psi_n(x)\omega_n + \frac{Q_0(x)}{2} . \tag{5}$$

A general solution of Eq. (5) can be written as the sum of the particular ($\psi_m^p(x)$) and the homogeneous ($\psi_m^h(x)$) solutions of Eq. (5),

$$\psi_m(x) = \psi_m^p(x) + \psi_m^h(x) . \tag{6}$$

A spatially constant particular solution of Eq. (5) can easily be given by

$$\psi_m^p(x) = \frac{Q_0}{\sigma_T(2 - c_0\alpha_N)}, \qquad 0 \le x \le a, \qquad 1 \le m \le N, \tag{7}$$

where

$$\alpha_N = \sum_{n=1}^{N} \sqrt{1-\mu_n^2}\,\omega_n, \tag{8}$$

and $c_0 = \sigma_{S0}/\sigma_T$. It is customary to use the method of separation of variables in order to obtain the homogeneous solution $\psi_m^h(x)$ of Eq. (5). Therefore, the homogeneous solution can be written in the form of [6],

$$\psi_m^h(x) = H_m(\nu)\exp(\sigma_T x/\nu), \quad 0 \le a \le x, \qquad 1 \le m \le N. \tag{9}$$

An expression for the angular part of the neutron angular flux can be obtained by substituting Eq. (9) into Eq. (5),

$$H_m(\nu) = \frac{\nu c_0}{2(\nu+\mu_m)} \sum_{n=1}^{N} \sqrt{1-\mu_n^2}\, H_n(\nu)\omega_n, \tag{10}$$

where the function $H_m(\nu)$ is normalized by,

$$\sum_{n=1}^{N} \sqrt{1-\mu_n^2}\, H_n(\nu)\omega_n = 1. \tag{11}$$

When Eq. (10) is multiplied by $\sqrt{1-\mu_m^2}\,\omega_m$ from both sides and then summed over all *m*, an equation for the ν eigenvalues can be obtained as,

$$\sum_{m=1}^{N} \frac{\nu c_0\sqrt{1-\mu_m^2}}{2(\nu+\mu_m)}\omega_m = 1, \qquad \nu \ne -\mu_m. \tag{12}$$

Since the determination of the eigenvalues is accepted as the first step in most of the transport studies, in this study, the eigenvalue spectrum of the monoenergetic neutrons is investigated and an expression for this purpose is derived and given in Eq. (12). In other words, Eq. (12) is referred to as the dispersion relation and the roots $\nu_k, 1 \le k \le N$, of Eq. (12) are the eigenvalues of the S_N equations. These eigenvalues are symmetric about the origin for any c_0 satisfying $0 \le c_0 \le 1$ due to the symmetry of Gauss-Chebyshev quadrature set.

3. NUMERICAL RESULTS AND DISCUSSION

The eigenvalue spectrum as a first step for the solution of the neutron transport equation in one dimensional slab geometry is studied for monoenergetic neutrons. The transport equation is written in the form of discrete ordinates (S_N) by applying integral transform and then it solved using Gauss-Chebyshev quadrature set. Then, an analytic expression for the eigenvalues is obtained and given in Eq.

(12) by solving the homogeneous part of Eq. (5). Thus, the roots $\nu_k, 1 \le k \le N$ of Eq. (12) are the eigenvalues of the S_N equations, i.e. Eq. (5).

The eigenvalues are calculated from Eq. (12) and given in Table 1 for various values of the $c_0 < 1$ and $c_0 > 1$. These can be used in the studies of transport theory.

Table 1. Eigenvalue spectrum.

N	$c_0 = 0.40$	$c_0 = 0.80$	$c_0 = 0.98$	$c_0 = 0.99$	$c_0 = 1.20$	$c_0 = 2.00$
2	±0.9485500	±2.1183454	±2.3768285i^*	±2.2404014i	±1.2256063i	±0.6398065i
4	±0.5485379	±0.6293344i	±0.4829046i	±0.4773492i	±0.3913600i	±0.2591057i
	±1.5231626	±0.7012099	±0.7280382	±0.7291494	±0.7469371	±0.7744063
8	±0.6089840i	±0.2463427i	±0.2036999i	±0.2018929i	±0.1720590i	±0.1194641i
	±0.9508914	±0.9562871	±0.9570950	±0.9571302	±0.9577187	±0.9587589
	±0.7063372	±0.7338907	±0.7381875	±0.7383745	±0.7415029	±0.7470087
	±0.2766079	±0.3430797	±0.3578635	±0.3585199	±0.3695826	±0.3890247

$^* i = \sqrt{-1}$

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