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Topological Hoarded Graphs

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Abstract
In this paper, we first introduced the steps that need to be taken to get the set-family that goes with a hoarded graph, as well as an example of how these steps could be used. Then, we explained what a topological hoarded graph is and showed when a set-family induced by a topological hoarded graph is a topology on a set. We also presented some useful facts about topological hoarded graphs.

1. Introduction

A subfamily $\mathcal{S}_X^{(n)}$ (orshortly$\mathcal{S}^{(n)}$) of n-times-iterated power set of a set X is called a *n-set-family on X*. In particular, we use the convention that the 0-set-family $\mathcal{S}^{(0)}$ is a subset of X. We denote m-times generalized union of a family $\mathcal{S}^{(n)}$ by $\coprod^m \mathcal{S}^{(n)}$, that is,

$$\coprod^m \mathcal{S}^{(n)} = \underbrace{\bigcup \cdots \bigcup}_{m \text{ times}} \mathcal{S}^{(n)} \tag{1}$$

where $1 \leq m \leq n$. For simplicity, we adopt the convention $\coprod^0 \mathcal{F}^{(n)} = \mathcal{F}^{(n)}$. Let I be a partially ordered set with the least element. An indexed family $\{A_i | i \in I\}$ whose the least-indexed element is empty, *i.e.*, in which $A_{i_0} = \emptyset$ where $i_0 = \min I$ is said to be *first-empty*. We denote the set of all integers $\geq k$ and $\leq n$ where $k, n \in \mathbb{Z}$ by I_n^k.

Given a digraph $G = (V, A)$. The sets of heads and tails of all arcs in G is denoted by $V_h(G)$ and $V_t(G)$, respectively. Hence the set $V(G)$ of its all endpoints is union of $V_t(G)$ and $V_h(G)$. Furthermore, we denote the set of all heads of all v-tailed arcs in G by $V_h(G; v)$, or in short $V_h(v)$; and similarly the sets of all tails of all v-headed arcs in G by $V_t(G; v)$, or in short $V_t(v)$. A path in G whose the first and last vertices are in V' and V'', respectively, where $V', V'' \subseteq V$, is denoted by $p_{V' \to V''}$. Especially, we prefer to use the element of that set in the notation if V' or V'' is a singleton, and the dot symbol is used instead of unknown sets in the notation $p_{V' \to V''}$. The set of last vertices of all directed paths $p_{v \to W}$ in G where $W \subseteq V$ is denoted by $V_l(v \to W; G)$, or in short $V_l(v \to W)$, and similarly the set of first vertices of all directed paths $p_{W \to v}$ in G by $V_f(W \to v; G)$, or in short $V_f(W \to v)$. We prefer to use the notation $V_l(v)$ and $V_f(v)$ instead if W is not particular. *The length of a directed path* in G is the number of arcs on it. A directed path with length n in G is called a *n-directed path*. Let $G[G']$ denote a subgraph G' of G. Also, we denote a *vertex-induced subgraph* by $V' \subseteq V$ of G by $G[V', \cdot]$, and denote an *edge-induced subgraph* by $A' \subseteq A$ of G by $G[\cdot, A']$ (for detailed information, see [1-3, 6-11]). The pair v, w of vertices in G is called *semiconnected* if G contains a directed path from v to w or vice versa; the pair is called *non-semiconnected* if they are not semiconnected (see [5]).

We introduced the notion of cumulative graph as a subclass of acyclic digraphs [4]. We recall that a n-cumulative graph $G = (\mathcal{V}, \mathcal{A}, \mathcal{B})$ with first-empty indexed families $\mathcal{V} = \{V_i\}_{i \in I_n^0}$, $\mathcal{A} = \{A_i\}_{i \in I_n^1}$ and $\mathcal{B} = \{B_i\}_{i \in I_n^1}$ is an acyclic digraph $G = (\bigcup \mathcal{V}, \bigcup(\mathcal{A} \cup \mathcal{B}))$ satisfying the following : (i) $V_n = V(G[\cdot, A_n]) \cup V_t(G[\cdot, B_n])$, and for every integer $1 \leq i < n$, $V_i = V(G[\cdot, A_i]) \cup V_t(G[\cdot, B_i]) \cup V_h(G[\cdot, B_{i+1}])$, (ii) for every $1 \leq i \leq n$, $vw \in A_i$ and $ws \in A_i \Rightarrow vs \notin A_i$, (iii) for every $1 \leq i \leq n$, $vw \in A_i$ and $ws \in B_i \Rightarrow vs \notin B_i$.

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2. A Set-family Corresponding to A Hoarded Graph

We introduced the definition of a cumulative graph in our previous paper [4]. The main motivation for this definition was to specify a particular class of graphs that would correspond to a n-set-family. It is natural to ask for which class of graphs there is a set-family corresponding to any graph of that class. To answer this question, we give the following definition.

Definition 1. A *n-hoarded graph* $G = (\mathcal{V}, \mathcal{A}, \mathcal{B})$ with pairwise disjoint families $\mathcal{V} = \{V_i\}_{i\in I_n^1}$, $\mathcal{A} = \{A_i\}_{i\in I_n^2}$ *and* $\mathcal{B} = \{B_i\}_{i\in I_n^2}$ is an acyclic digraph $\mathrm{G} = \big(\cup\mathcal{V}, \cup(\mathcal{A}\cup\mathcal{B})\big)$ which satisfies the following conditions:

(1) For every $\boldsymbol{2 \le i \le n}$, the endpoints of every arc in $\boldsymbol{A_i}$ belong to $\boldsymbol{V_i}$ while tails of every arc in $\boldsymbol{B_i}$ belong to $\boldsymbol{V_i}$ and the set of heads of all arcs in $\boldsymbol{B_i}$ equals to $\boldsymbol{V_{i-1}}$.

(2) If a vertex in $\boldsymbol{V_i}$ precedes that in $\boldsymbol{V_j}$ on some directed path in $\boldsymbol{G}$, then $\boldsymbol{i \ge j}$.

(3) If $\boldsymbol{u_1 u_2 \dots u_m}$ with $\boldsymbol{m \ge 3}$ is a directed path in $\boldsymbol{G}$ every arc of which belongs to $\boldsymbol{A_i}$ for some $\boldsymbol{2 \le i \le n}$, then $\boldsymbol{u_1 u_m \notin A_i}$.

(4) For every $\boldsymbol{2 \le i \le n}$, $\boldsymbol{vw \in A_i}$ and $\boldsymbol{ws \in B_i \Rightarrow vs \notin B_i}$.

For every distinct pair $\boldsymbol{u, v}$ of vertices in some $\boldsymbol{V_i}$ with $\boldsymbol{1 \le i \le n}$, there exists a vertex $\boldsymbol{w}$ such that $\boldsymbol{w}$ is the last vertex of some directed path with the first vertex $\boldsymbol{u}$ but not that of any directed path with the first vertex $\boldsymbol{v}$.

In the paper [4], we have shown the steps to obtain the $(\boldsymbol{n+1})$-cumulative graph induced by a $\boldsymbol{n}$-set-family. Now we introduce the steps to be taken to get the $(\boldsymbol{n-1})$-set-family corresponding to a $\boldsymbol{n}$-hoarded graph $\boldsymbol{G = (\mathcal{V}, \mathcal{A}, \mathcal{B})}$.

Step 1 We set $\boldsymbol{\mathcal{F} = V_n}$.

Step 2 We perform the following steps from $\boldsymbol{i = n}$ to $\boldsymbol{i = 2}$,

Step 2.1 We substitute the set $\boldsymbol{v \cup \cup V_h(G[\cdot, A_i]; v)}$ for each vertex $\boldsymbol{v}$ occurring in $\boldsymbol{\mathcal{F}}$.

Step 2.2 We substitute the set $\boldsymbol{V_h(G[\cdot, B_i]; v)}$ for each vertex $\boldsymbol{v}$ occuring in $\boldsymbol{\mathcal{F}}$.

After performing the above steps, the resulting $\boldsymbol{\mathcal{F}}$ is the set-family corresponding to the hoarded graph $\boldsymbol{G}$.

Example 2. Let $\boldsymbol{G = (\mathcal{V}, \mathcal{A}, \mathcal{B})}$ be a $\boldsymbol{4}$-hoarded graph with $\boldsymbol{\mathcal{V} = \{V_i\}_{i\in I_4^1}}$, $\boldsymbol{\mathcal{A} = \{A_i\}_{i\in I_4^2}}$ and $\boldsymbol{\mathcal{B} = \{B_i\}_{i\in I_4^2}}$ where

$$\boldsymbol{V_1 = \{v_1, \dots, v_6\}, V_2 = \{v_7, \dots, v_{10}\},}$$
$$\boldsymbol{V_3 = \{v_{11}, \dots, v_{14}\}, V_4 = \{v_{15}, v_{16}, v_{17}\},}$$
$$\boldsymbol{A_2 = \{v_8v_7, v_9v_7, v_{10}v_8\},}$$
$$\boldsymbol{A_3 = \{v_{13}v_{11}, v_{14}v_{11}, v_{14}v_{12}\},}$$
$$\boldsymbol{A_4 = \{v_{16}v_{15}, v_{17}v_{16}\},}$$
$$\boldsymbol{B_2 = \{v_8v_1, v_8v_3, v_8v_4, v_9v_1, v_9v_2, v_9v_3, v_9v_6, v_{10}v_5\},}$$
$$\boldsymbol{B_3 = \{v_{11}v_7, v_{12}v_{10}, v_{13}v_9, v_{14}v_8\},}$$
$$\boldsymbol{B_4 = \{v_{16}v_{11}, v_{16}v_{13}, v_{17}v_{12}, v_{17}v_{14}\}}$$

as Figure 1.

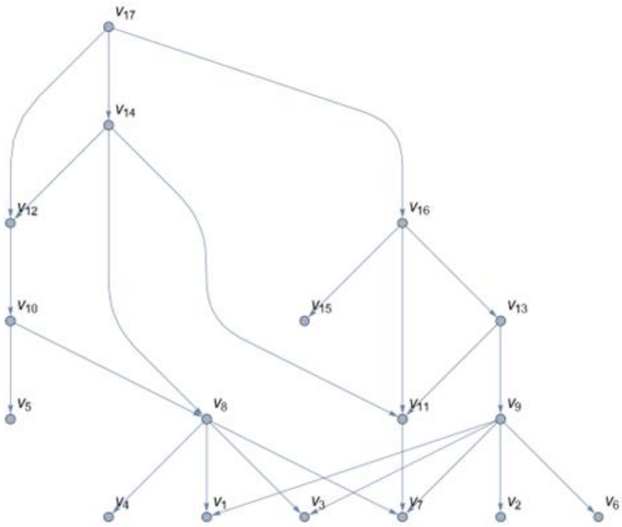

Figure 1. An example of a hoarded graph.

We first set $\mathcal{F} = V_4 = \{v_{15}, v_{16}, v_{17}\}$. For $i = 4$, we write $\mathcal{F} = \{v_{15}, v_{16} \cup v_{15}, v_{17} \cup v_{16} \cup v_{15}\}$ since

$$V_h(G[\cdot, A_4]; v_{15}) = v_{15} \cup \emptyset = v_{15},$$
$$V_h(G[\cdot, A_4]; v_{16}) = v_{16} \cup \bigcup \{v_{15}\} = v_{16} \cup v_{15},$$
$$V_h(G[\cdot, A_4]; v_{17}) = v_{17} \cup \bigcup \{v_{16}\}$$
$$= v_{17} \cup v_{16} \cup v_{15}.$$

And since

$$V_h(G[\cdot, B_4]; v_{15}) = \emptyset,$$
$$V_h(G[\cdot, B_4]; v_{16}) = \{v_{11}, v_{13}\},$$
$$V_h(G[\cdot, B_4]; v_{17}) = \{v_{12}, v_{14}\},$$

we get $\mathcal{F} = \big\{\emptyset, \{v_{11}, v_{13}\}, \{v_{11}, v_{12}, v_{13}, v_{14}\}\big\}$. Then by performing Step 2 for $n = 3$, we get

$$V_h(G[\cdot, A_3]; v_{11}) = v_{11} \cup \emptyset = v_{11},$$
$$V_h(G[\cdot, A_3]; v_{12}) = v_{12} \cup \emptyset = v_{12},$$
$$V_h(G[\cdot, A_3]; v_{13}) = v_{13} \cup \bigcup \{v_{11}\} = v_{13} \cup v_{11},$$
$$V_h(G[\cdot, A_3]; v_{14}) = v_{14} \cup \bigcup \{v_{11}, v_{12}\}$$
$$= v_{14} \cup v_{12} \cup v_{11}.$$

So, we obtain

$$\mathcal{F} = \{\emptyset, \{v_{11}, v_{13} \cup v_{11}\}, \{v_{11}, v_{12}, v_{13} \cup v_{11}, v_{14} \cup v_{12} \cup v_{11}\}\}.$$

Then we write

$$V_h(G[\cdot, B_3]; v_{11}) = \{v_7\},$$
$$V_h(G[\cdot, B_3]; v_{12}) = \{v_{10}\},$$
$$V_h(G[\cdot, B_3]; v_{13}) = \{v_9\},$$
$$V_h(G[\cdot, B_3]; v_{14}) = \{v_8\}$$

which yield

$$\mathcal{F} = \{\emptyset, \{\{v_7\}, \{v_7, v_9\}\}, \{\{v_7\}, \{v_{10}\}, \{v_7, v_9\}, \{v_7, v_8, v_{10}\}\}\}$$

Continuing Step 2, we rewrite

$$\mathcal{F} = \{\emptyset, \{\{v_7\}, \{v_7, v_9 \cup v_7\}\}, \{\{v_7\}, \{v_{10} \cup v_8 \cup v_7\}, \{v_7, v_9 \cup v_7\}, \{v_7, v_8 \cup v_7, v_{10} \cup v_8 \cup v_7\}\}\}$$

because

$$V_h(G[\cdot, A_2]; v_7) = v_7 \cup \emptyset = v_7,$$

$$V_h(G[\cdot, A_2]; v_8) = v_8 \cup \bigcup \{v_7\} = v_8 \cup v_7,$$

$$V_h(G[\cdot, A_2]; v_9) = v_9 \cup \bigcup \{v_7\} = v_9 \cup v_7,$$

$$V_h(G[\cdot, A_2]; v_{10}) = v_{10} \cup \bigcup \{v_8\} = v_{10} \cup v_8 \cup v_7.$$

In the sequel, we find as

$$V_h(G[\cdot, B_2]; v_7) = \emptyset,$$
$$V_h(G[\cdot, B_2]; v_8) = \{v_1, v_3, v_4\},$$
$$V_h(G[\cdot, B_2]; v_9) = \{v_1, v_2, v_3, v_6\},$$
$$V_h(G[\cdot, B_2]; v_{10}) = \{v_5\}$$

and hence we finally get

$$\mathcal{F} = \Big\{\emptyset, \Big\{\{\emptyset\}, \{\emptyset, \{v_1, v_2, v_3, v_6\}\}\Big\}, \Big\{\{\emptyset\}, \{\{v_1, v_3, v_4, v_5\}\}, \{\emptyset, \{v_1, v_2, v_3, v_6\}\}, \{\emptyset, \{v_1, v_3, v_4\}, \{v_1, v_3, v_4, v_5\}\}\Big\}\Big\}.$$

3. Topological Hoarded Graphs

We first introduce the definition of topological hoarded graph:

Definition 3. A 2-hoarded graph $G = (\mathcal{V}, \mathcal{A}, \mathcal{B})$ with $\mathcal{V} = \{V_1, V_2\}$, $\mathcal{A} = \{A_2\}$ and $\mathcal{B} = \{B_2\}$ is called a *topological hoarded graph* and denoted by $G = (V_1, V_2, A_2, B_2)$ if it satisfies the following conditions:
(1) There exists a vertex in V_2 that is the tail of no arc in G.
(2) For every vertex v in V_1, there exists a vertex $u \in V_2$ in which a directed path from itself to v exists.
(3) For any subset S of mutually two non-semiconnected vertices in V_2, there exists a vertex v in V_2 such that G contains a dipath from v to s for each vertex $s \in S$.
For any non-semiconnected pair u, w of vertices in V_2, if G contains pairs of directed paths with the first vertices u, w and the same last vertices in V_1, then there exists a vertex $v \in V_2$ such that G contains pairs of v-headed arcs with the tails u, w on these directed paths.

Theorem 4. If $G = (X, Y, A, B)$ be a topological hoarded graph, then X equipped with the 1-set-family τ corresponding to G is a topological space.

Proof. Let us first show that τ contains the empty set. From Definition 3(3), there exists a vertex y in Y such that y is not the tail of any arc in G. When we first perform Step 1 to obtain 1-set-family τ corresponding to G, we get $\tau = Y$. In Step 2.1, we write

$$y \cup \bigcup V_h(G[\cdot, A]; y) = y \cup \emptyset = y$$

instead of y in τ since y is not the tail of any arc in G. In Step 2.2, since y is not the tail of any arc in G, we replace y in τ with

$$V_h(G[\cdot, B]; y) = \emptyset$$

which means that τ contains $\emptyset$.
Now we show that τ contains the set X. Assume that $X \notin \tau$. It implies that $X \neq V_h(G[\cdot, B]; y)$ for every occurrence y in τ obtained by applying Step 2.1. Then for every occurrence y in τ obtained by applying Step 2.1, there exists a point $x \in X$ such that $x \notin V_h(G[\cdot, B]; y)$ which contradicts Definition 3(3). So $X \in \tau$.
Given a subfamily $\{U_i\}_{i \in I}$ of τ. Let's show that τ contains $\bigcup_{i \in I} U_i$. If $U_{i_0} = X$ for a particular $i_0 \in I$, then $\bigcup_{i \in I} U_i = X \in \tau$. If there exists a subset $J \subseteq I$ such that there exists an index $j \in J$ such that $U_i \subseteq U_j$ for every $i \in I \backslash J$, then $\bigcup_{i \in I} U_i = \bigcup_{i \in J} U_i$. In such a case, we show that $\bigcup_{i \in J} U_i$. In that case, $\{U_i\}_{i \in J}$ is a subfamily of τ such that U_i is neither a subset nor a superset U_j for every distinct indices $i, j \in J$. For each $i \in J$, U_i corresponding some vertex $v_i \in Y$ is obtained by performing Step 2.1 and Step 2.2. From Definition 3(3), there exists a vertex w in Y such that G contains a dipath from w to v_i for every $i \in J$. Just after applying Step 2.1 and Step 2.2, we obtain a set, say W, that corresponds $w \in Y$. Furthermore, $\bigcup_{i \in J} U_i = W \in \tau$.
Let U and V be members of τ. Finally, if we show that $U \cap V \in \tau$, then we complete the proof. If U does not intersect V, then $U \cap V = \emptyset \in \tau$. If $U \subseteq V$ or $V \subseteq U$, then it is clear that $U \cap V = U \in \tau$ or $U \cap V = V \in \tau$. In the other case, U and V corresponding some vertices $u, v \in Y$, respectively, are obtained by performing Step 2.1 and Step 2.2. Since $U \cap V \neq \emptyset$ and $U \nsubseteq V$ and $V \nsubseteq U$, G contains pair of directed paths with the first vertices u, v and the same last vertex w_p in X that corresponds to each point $p \in U \cap V$. From Definition 3(3), there exists a vertex w in Y such that G contains pairs of w_p-headed arcs with the tails u, v on these directed paths. Just after performing Step 2.1 and Step 2.2, we obtain a set, say W, that corresponds $w \in Y$. Furthermore, $U \cap V = W \in \tau$.

Example 5. Let $G = (X, Y, A, B)$ be a topological hoarded graph where

$X = \{v_1, \dots, v_6\}, Y = \{v_7, \dots, v_{19}\}$,
$A = \{v_8v_7, v_9v_7, v_{10}v_8, v_{10}v_9, v_{11}v_9, v_{12}v_9, v_{13}v_{10}, v_{13}v_{11}, v_{14}v_{10}, v_{14}v_{12}, v_{15}v_{11}, v_{15}v_{12}, v_{16}v_{13}, v_{16}v_{14}, v_{16}v_{15}, v_{17}v_{14}, v_{18}v_{16}, v_{18}v_{17}, v_{19}v_{18}\}$,
$B = \{v_8v_1, v_9v_2, v_{11}v_3, v_{12}v_5, v_{17}v_6, v_{19}v_4\}$
as Figure 5.

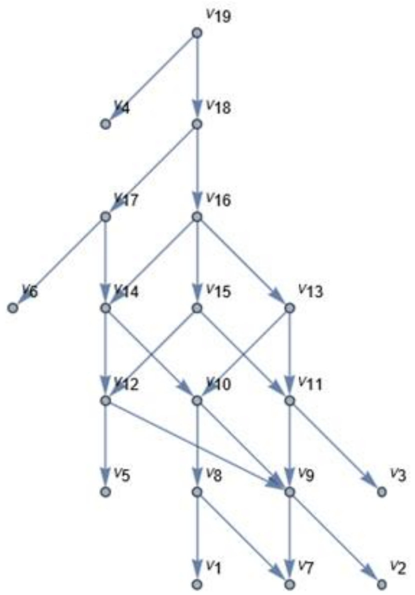

Figure 2. An example of a topological hoarded graph.

Indeed, it can be easily verified that G satisfies the conditions in Definition 3. We first set $\mathcal{F} = Y = \{v_7, \dots, v_{19}\}$. For $i = 2$, we write

$$\begin{aligned}\mathcal{F} = \{&v_7, v_8 \cup v_7, v_9 \cup v_7, v_{10} \cup \cdots \cup v_7,\\ &v_{11} \cup v_9 \cup v_7, v_{12} \cup v_9 \cup v_7,\\ &v_{13} \cup v_{11} \cup \cdots \cup v_7,\\ &v_{14} \cup v_{12} \cup v_{10} \cup \cdots \cup v_7,\\ &v_{15} \cup v_{12} \cup v_{11} \cup v_9 \cup v_7,\\ &v_{16} \cup \cdots \cup v_7,\\ &v_{17} \cup v_{14} \cup v_{12} \cup v_{10} \cup \cdots \cup v_7,\\ &v_{18} \cup \cdots \cup v_7, v_{19} \cup \cdots \cup v_7\}\end{aligned}$$

since

$$\begin{aligned}
V_h(G[\cdot, A_2]; v_7) &= v_7 \cup \emptyset = v_7,\\
V_h(G[\cdot, A_2]; v_8) &= v_8 \cup \bigcup \{v_7\} = v_8 \cup v_7,\\
V_h(G[\cdot, A_2]; v_9) &= v_9 \cup \bigcup \{v_7\} = v_9 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{10}) &= v_{10} \cup \bigcup \{v_8, v_9\}\\
&= v_{10} \cup \cdots \cup v_7,\\
V_h(G[\cdot, A_2]; v_{11}) &= v_{11} \cup \bigcup \{v_9\} = v_{11} \cup v_9 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{12}) &= v_{12} \cup \bigcup \{v_9\} = v_{12} \cup v_9 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{13}) &= v_{13} \cup \bigcup \{v_{10}, v_{11}\}\\
&= v_{13} \cup v_{11} \cup v_{10} \cup v_9 \cup v_8 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{14}) &= v_{14} \cup \bigcup \{v_{10}, v_{12}\}\\
&= v_{14} \cup v_{12} \cup v_{10} \cup v_9 \cup v_8 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{15}) &= v_{15} \cup \bigcup \{v_{11}, v_{12}\}\\
&= v_{15} \cup v_{12} \cup v_{11} \cup v_9 \cup v_7,\\
V_h(G[\cdot, A_2]; v_{16}) &= v_{16} \cup \bigcup \{v_{13}, v_{14}, v_{15}\}\\
&= v_{16} \cup \cdots \cup v_7,\\
V_h(G[\cdot, A_2]; v_{17}) &= v_{17} \cup \bigcup \{v_{14}\}\\
&= v_{17} \cup v_{14} \cup v_{12} \cup v_{10} \cup v_9 \cup v_8\\
&\cup v_7,\\
V_h(G[\cdot, A_2]; v_{18}) &= v_{18} \cup \bigcup \{v_{16}, v_{17}\}\\
&= v_{18} \cup \cdots \cup v_7,\\
V_h(G[\cdot, A_2]; v_{19}) &= v_{19} \cup \bigcup \{v_{18}\}\\
&= v_{19} \cup \cdots \cup v_7.
\end{aligned}$$

And since

$$\begin{aligned}
&V_h(G[\cdot, B_2]; v_7) = \emptyset, V_h(G[\cdot, B_2]; v_{14}) = \emptyset,\\
&V_h(G[\cdot, B_2]; v_8) = \{v_1\}, V_h(G[\cdot, B_2]; v_{15}) = \emptyset,\\
&V_h(G[\cdot, B_2]; v_9) = \{v_2\}, V_h(G[\cdot, B_2]; v_{16}) = \emptyset,\\
&V_h(G[\cdot, B_2]; v_{10}) = \emptyset, V_h(G[\cdot, B_2]; v_{17}) = \{v_6\},\\
&V_h(G[\cdot, B_2]; v_{11}) = \{v_3\}, V_h(G[\cdot, B_2]; v_{18}) = \emptyset,\\
&V_h(G[\cdot, B_2]; v_{12}) = \{v_5\}, V_h(G[\cdot, B_2]; v_{19}) = \{v_4\},\\
&V_h(G[\cdot, B_2]; v_{13}) = \emptyset,
\end{aligned}$$

we get

$$\begin{aligned}\mathcal{F} = \{&\emptyset, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_2, v_5\},\\ &\{v_1, v_2, v_3\}, \{v_1, v_2, v_5\}, \{v_2, v_3, v_5\},\\ &\{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_5, v_6\},\\ &\{v_1, v_2, v_3, v_5, v_6\}, \{v_1, v_2, v_3, v_4, v_5, v_6\}\}\end{aligned}$$

which can easily be proved to be a topology on X.

4. Conclusion and Suggestions

We first give a concept of a n-hoarded graph to which there exists a $(n-1)$-set family corresponding. We present the steps to be performed to get the corresponding set-family, and we have shown the results of these steps in an example. We then introduced the concept of a topological hoarded graph. Above all, we show that X equipped with the 1-set-family τ corresponding to a topological hoarded graph $G = (X, Y, A, B)$ is a topological space. And finally, we have confirmed this fact with an example.

References

[1] B. Bollobas, *Modern Graph Theory.* New York, NY: Springer, 2014.

[2] G. Chartrand, *A First Course in Graph Theory.* Mineola, NY: Dover Publications, 2012.

[3] J. L. Gross, J. Yellen, and P. Zhang, Eds., *Handbook of graph theory,* 2nd ed. London, England: CRC Press, 2018.

[4] K. Polat, “On cumulative graph representations of set-families”, *Karamanoğlu Mehmetbey Üniversitesi Mühendislik ve Doğa Bilimleri Dergisi*, vol. 3, no. 2, pp. 74–78, 2022.

[5] K. S. Htay, K. A. Tint, and N. O. Htike, “Application of connectivity on graph theory”, *International Journal of Scientific Engineering and Technology Research*, vol. 8, no. 1, pp. 525–530, 2019.

[6] K. R. Saoub, *Graph theory: An introduction to proofs, algorithms, and applications.* London, England: CRC Press, 2021.

[7] R. J. Trudeau, *Introduction to graph theory.* Pmapublishing.com, 2017.

[8] L.W. Beineke, “Derived graphs and digraphs”, *In Beitrage zur Graphentheorie*, pp. 17-33, 1968.

[9] G. Chartrand, and L. Lesniak, *Graphs and Digraphs*, 2nd ed., Wadsworth, USA: Springer, 1986.

[10] F.R.K. Chung, “On partitions of graphs into trees”, *Discrete Mathematics*, vol. 23 no. 1, pp. 23-30, 1978.

[11] J. Edmonds, “Paths, trees, and flowers”, *Canadian Journal of Mathematics*, vol. 17, no. 1, pp. 449-467, 1965.