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On the new travelling wave solution of a neural communication model

Gülnur YEL*

Faculty of Educational Sciences, Final International University, Kyrenia Mersin 10, Turkey.

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Abstract

The aim of this study is to present some new travelling wave solutions of conformable time-fractional Fitzhugh–Nagumo equation that model the transmission of nerve impulses. For this purpose, the improved Bernoulli sub-equation function method has been used. The obtained results are shown by way of the the 3D-2D graphs and contour surfaces for the suitable values.

Keywords: *Time-Fractional Fitzhugh–Nagumo equation, conformable fractional derivative, travelling wave solutions.*

Bir sinirsel iletişim modelinin yeni salınımlı dalga çözümleri üzerinde

Özet

Bu çalışmanın amacı, sinir uyarılarının iletişimini modelleyen uyumlu zaman-kesirli türevli Fitzhugh–Nagumo denkleminin bazı yeni salınımlı dalga çözümlerini sunmaktır. Bu amaçla, geliştirilmiş Bernoulli alt denklem fonksiyon metodu kullanılmıştır. Elde edilen çözümler uygun değerler için 2-3 boyutlu grafikler ve kontur yüzeyleri ile gösterilmiştir.

Anahtar Kelimeler: *Zaman-kesirli türevli Fitzhugh–Nagumo denklemi, uyumlu kesirli türev, salınımlı dalga çözümleri.*

* Gülnur YEL, gulnur.yel@final.edu.tr, https://orcid.org/0000-00012-5134-4431

1. Introduction

The reaction–diffusion equations have an important role to describe mathematical models. These equations are widely used in many branches such as ecology, spreading of biological populations, combustion theory, physiology, chemistry, geology, physics and engineering [1-6].

Neural communications have a significant role in mathematical biology. Understanding of electrical connections in neural networks several cognitive brain activities as memory, attention, sleep, and in various pathologies as Parkinson's disease, epilepsy, gives reliable information to make diagnosis and treatment in medicine and physiology. In recent years, a considerable amount of models have appeared to chaotic neurons systems. One of a good mathematical model is the Hodgkin-Huxley (HH) which describes the transmission of nerve impulses through an axon [7]. The FitzHugh–Nagumo (FHN) model which characterise neural network by nerve cells via electrical signalling is a simpler model of HH model [8,9]. This model has been studied extensively in mathematical biology and neurobiology [10-12]. In [13], a strongly connective model has been considered and its numerical simulations of neural networks with neurons have been shown. The dynamical behaviours of the FitzHugh–Nagumo model with fractional order have been addressed [14]. The author has concluded that the fractional order models can be used to control of abnormal stimulations. The approximate solutions of the fractional-order multiple chaotic FitzHugh–Nagumo neurons model under external electrical stimulation have been submitted by the multistep generalized differential transform method [15].

The space-fractional FitzHugh–Nagumo model in two-dimensional space has investigated the effect of implementation performance and scalability under such factors in [16]. Armanyos et. al. have discussed different order of fractional Fitzhugh-Nagumo and Izhikevich
models and concluded that fractional orders enable broader scope understanding of the neuron behaviors [17]. To receive further studies on the fractional-order Fitzhugh-Nagumo model, it can be seen in [18-22].

Many scientists, especially in medicine, biology and engineering fields, have studied on mathematical models of real-life phenomena to present numerical simulation [23- 33]. However, analytical solutions are crucial to test accuracy of approximate solutions. Thus, so many analytical methods have been improved such as sine-Gordon expansion method [34, 35],the modified simple equation method [36], the extended sinh-Gordon equation expansion method [37-39] in recent years.

In the last century, fractional-order derivatives have been approved because of their advantages both closer to real-life circumstance and have higher precision by numbers of researchers [40-46]. Many novels on definitions and theories of fractional calculus have been submitted to the literature. As known Riemann Liouville and Caputo fractional derivatives have singularity resulting from the power kernel functions cause some computational difficulties. Whereas, Caputo and Fabrizio have submitted a new fractional operator without singular kernel [47]. Another non-singular derivative operator with the

Mittag-Leffler kernel function is defined by Atangana and Baleanu [48]. The conformable fractional derivative operator which is compatible to many real-world problems and provides some properties of classical calculus such as derivative of the quotient of two functions, the chain rule, the product of two functions has been submitted to the literature by Khalila et.al [49]. In this study, the new travelling waves will be analysed of the time-fractional Fitzhugh–Nagumo equation in conformable sense by the improved Bernoulli sub-equation function method (IBSEFM).

The rest of the paper is organized as follows; in the second section, the definition of conformable fractional derivative and some theorems corresponding to the definition are submitted. In the third section, analysis of the mentioned method with four steps are given. The application of the method to the time-fractional Fitzhugh–Nagumo equation in conformable sense and the figures of the valid solutions under the suitable values are presented in section 4. Some conclusions are given in the last section 5.

2. The facts of conformable derivative

Definition: Let $f:[0,\infty)\rightarrow\mathbb{R}$ be a function, the γ -order conformable derivative of f is defined as,

$$T_{\gamma}(f)(t)=\lim_{\upsilon\rightarrow 0}\frac{f\left(t+\upsilon t^{1-\gamma}\right)-f(t)}{\upsilon}$$

for all $t>0,\ 0<\gamma<1$ [49].

Theorem: Suppose that T_{γ} is fractional derivative operator with order γ and $\gamma\in(0,1]$, f,g be γ-differentiable at point $t>0$. Then [49,50],

i. $T_{\gamma}(af+bg)=aT_{\gamma}(f)+bT_{\gamma}(g),\forall a,b\in\mathbb{R}.$

ii. $T_{\gamma}(t^{p})=pt^{p-\gamma},\forall p\in\mathbb{R}.$

iii. $T_{\gamma}(fg)=fT_{\gamma}(g)+gT_{\gamma}(f).$

iv. $T_{\gamma}\left(\frac{f}{g}\right)=\frac{gT_{\gamma}(f)-fT_{\gamma}(g)}{g^{2}}.$

v. $T_{\gamma}(\lambda)=0,$ for all constant functions $f(t)=\lambda.$

vi. If f is differentiable then $T_{\gamma}(f)(t)=t^{1-\gamma}\frac{df}{dt}(t).$

Theorem: Let $f,g:(0,\infty)\rightarrow\mathbb{R}$ be differentiable functions, the following rule holds;
$T_{\gamma}(f\circ g)(t)=t^{1-\gamma}g'(t)f'(g(t)).$

3. Analysis of the method

The improved Bernoulli sub-equation function method has the following four steps.

Step 1. Suppose the following partial differential equation;

$$G\left(D_t^{\gamma}u,u_x,u_t,u_{xt},...\right)=0, \tag{1}$$

and take the wave transformation;

$$u(x,t)=U(\zeta),\quad \zeta=\sigma x-\frac{\mu t^{\gamma}}{\gamma}, \tag{2}$$

where σ,μ are constants and can be determined later. Substituting partial derivatives of Eq.(2) with respect to x and t into Eq.(1), we obtain the following nonlinear ordinary differential equation;

$$N\left(U,U',U'',U''',\cdots\right)=0. \tag{3}$$

Step 2. Assume that the following trial equation is solution of Eq.(3),

$$U(\zeta)=\frac{\sum_{i=0}^{n}a_iB^i(\zeta)}{\sum_{j=0}^{m}b_jB^j(\zeta)}=\frac{a_0+a_1B(\zeta)+a_2B^2(\zeta)+\cdots+a_nB^n(\zeta)}{b_0+b_1B(\zeta)+b_2B^2(\zeta)+\cdots+b_mB^m(\zeta)}. \tag{4}$$

Using the Bernoulli theory, we can write the general form of Bernoulli differential equation for B' as follows;

$$B'=wB+dB^M,w\neq 0,\ d\neq 0,\ M\in\mathbb{R}-\{0,1,2\}, \tag{5}$$

where $B=B(\zeta)$ is Bernoulli differential polynomial. Substituting above relations in Eq.(3), it yields us an equation of polynomial $\varpi(B)$ of B as follows;

$$\varpi(B)=\rho_sB^s+\cdots+\rho_1B+\rho_0. \tag{6}$$

According to the homogenous balance principle, we can determine the relationship between n,m and M .

Step 3. The coefficients of $\varpi(B)$ all be zero will yield us an algebraic system of equations;

$$\rho_i=0,i=0,\ldots,s. \tag{7}$$

The solution of this system will give the values of $a_0,\ldots,a_n$ and $b_0,\ldots,b_m$.

Step 4. When we solve nonlinear Bernoulli differential equation Eq.(5), we obtain the following two situations according to w and d;

$$B(\zeta)=\left[\frac{-d}{w}+\frac{E}{e^{w(M-1)\zeta}}\right]^{\frac{1}{1-M}},\ \ w\neq d, \tag{8}$$

$$B(\zeta)=\left[\frac{(E-1)+(E+1)\tanh\left(w(1-M)\zeta/2\right)}{1-\tanh\left(w(1-M)\zeta/2\right)}\right]^{\frac{1}{1-M}},\ \ w=d,\ \ E\in\mathbb{R}. \tag{9}$$

Using a complete discrimination system for polynomial of $B(\zeta)$, we obtain the analytical solutions to the Eq.(3) with the help of computational program. For a better explanation of the valid solutions in this way, the 2D-3D figures and contour surfaces can be plotted.

4. Application

This section presents the application of mention method to the time-fractional Fitzhugh–Nagumo equation with conformable sense.
The nonlinear time-fractional partial differential equation is expressed as [51-53]

$$u_t^{\gamma}-u_{xx}-u(1-u)(u-\theta)=0,\ 0<\gamma\leq 1, 0<\theta<1, \tag{10}$$

where γ is order of conformable time-fractional derivative and θ represents an arbitrary constant.

When $\theta=-1$, the Eq.(10) converts to the Newell–Whitehead equation.

Consider the fractional travelling wave transformation as

$$u(x,t)=U(\zeta),\quad \zeta=kx-\frac{ct^{\gamma}}{\gamma}, \tag{11}$$

where k,c are nonzero arbitrary constants. Putting Eq.(11) into Eq.(10), we obtain the following nonlinear ordinary differential equation,

$$-cU'-k^2U''+\theta U-(1+\theta)U^2+U^3=0. \tag{12}$$

According to homogeneous balance principle between U'' and U^3, we get a relationship as $M+m=n+1$.

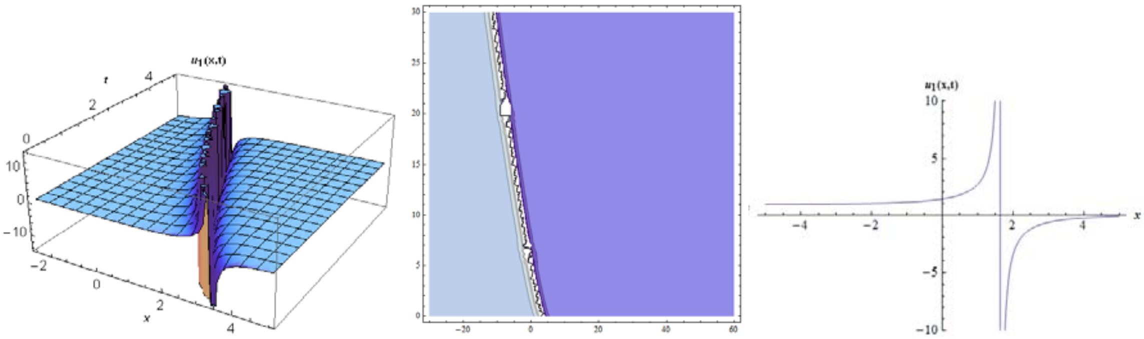
For the values $M=3, n=3$ and $m=1$, we obtain the following travelling wave solutions.

Set 1: For $w \neq d$

$$a_0 = \theta b_0, a_1 = \frac{wa_3}{d}, a_2 = \frac{d\theta b_0}{w}, b_1 = \frac{wa_3}{d\theta}, k = \frac{\theta}{2\sqrt{2w}}, c = \frac{(-2+\theta)\theta}{4w}, \tag{13}$$

When the Eq.(8) and Eq.(4) are used for Eq.(13), we have

$$u_1(x,t) = \frac{Ew\theta}{-de^{\frac{x\theta}{\sqrt{2}} - \frac{t^{\alpha}(-2+\theta)\theta}{2\alpha}} + Ew}. \tag{14}$$



(a) (b) (c)

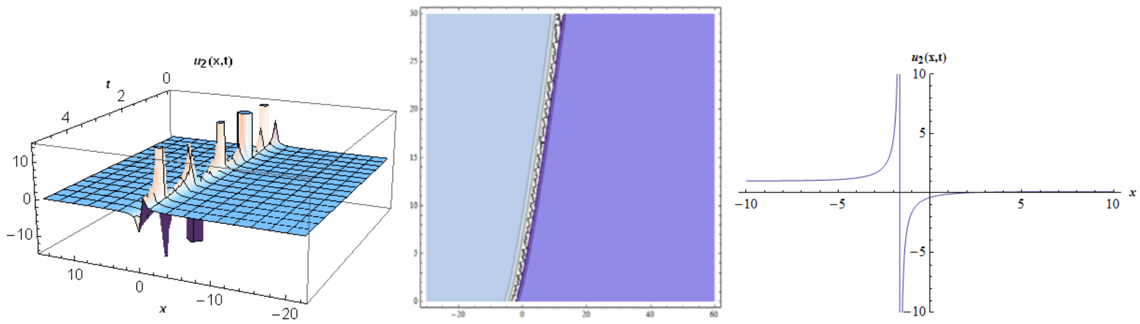
Figure 1. (a) and (b) are the 3D and contour surfaces of Eq.(14) with $\gamma = 0.8$ respectively, and $t = 2$ for the 2D graph (c).

Set 2: For $w \neq d$

$$a_0 = \theta b_0, a_1 = \frac{w\theta a_3}{d(-1+\theta)}, a_2 = \frac{d(-1+\theta)b_0}{w}, b_1 = \frac{wa_3}{d(-1+\theta)}, k = \frac{-1+\theta}{2\sqrt{2w}}, c = \frac{-1+\theta^2}{4w}, \tag{15}$$

When the Eq.(8) and Eq.(4) are used for Eq.(15), we have

$$u_2(x,t) = \frac{d(-1+\theta)}{-d + e^{\frac{(-1+\theta)(-\sqrt{2}x\alpha + t^{\alpha}(1+\theta))}{2\alpha}} Ew} + \theta. \tag{16}$$



(a) (b) (c)

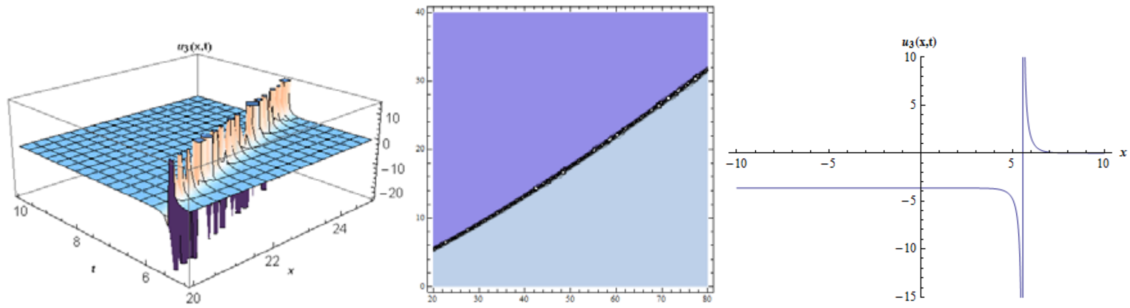
Figure 2. (a) and (b) are the 3D and contour surfaces of Eq.(16) with $\gamma = 0.8$ respectively, and $t = 2$ for the 2D graph (c).

Set 3: For $w \neq d$

$$a_0 = \left(2 - \frac{\sqrt{2}c}{k}\right) b_0, a_1 = \left(2 - \frac{\sqrt{2}c}{k}\right) b_1, a_2 = -2\sqrt{2}dkb_0, a_3 = -2\sqrt{2}dkb_1, \theta = 2 - \frac{\sqrt{2}c}{k}, w = \frac{c - \sqrt{2}k}{2k^2}, \tag{17}$$

When the Eq.(8) and Eq.(4) are used for Eq.(17), we have

$$u_3(x,t) = \frac{E\left(\sqrt{2}c^2 - 4ck + 2\sqrt{2}k^2\right)}{\left(-ckE + k^2\left(\sqrt{2}E + 2de^{\frac{(c-\sqrt{2}k)(kx - \frac{ct^\alpha}{\alpha})}{k^2}}k\right)\right)}. \tag{18}$$



(a) (b) (c)

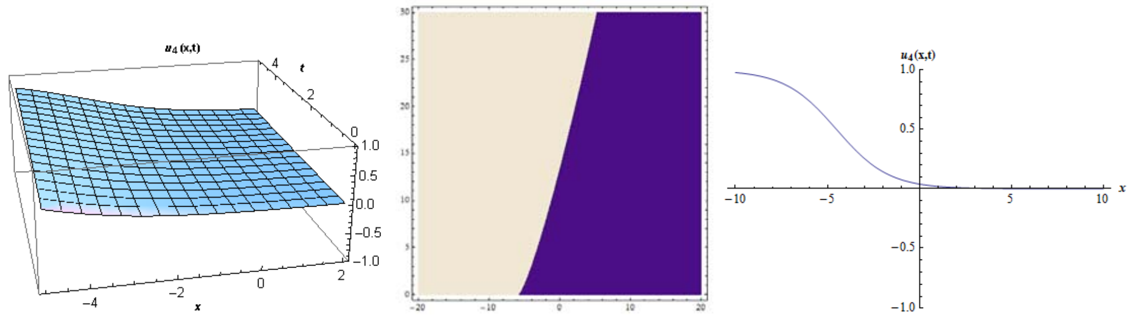
Figure 3. (a) and (b) are the 3D and contour surfaces of Eq.(18) with $\gamma = 0.8$ respectively, and $t = 1$ for the 2D graph (c).

Set 4: For $w \neq d$

$$a_0 = 0, a_2 = 2\sqrt{2}dkb_0, a_1 = 0, a_3 = 2\sqrt{2}dkb_1, w = -\frac{1}{2\sqrt{2}k}, c = \frac{k(1-2\theta)}{\sqrt{2}}, \tag{19}$$

When the Eq.(8) and Eq.(4) are used for Eq.(19), we have

$$u_4(x,t) = \frac{2\sqrt{2}dk}{e^{\frac{x}{\sqrt{2}} + \frac{t^\alpha(-1+2\theta)}{2\alpha}}E + 2\sqrt{2}dk}. \tag{20}$$



(a) (b) (c)

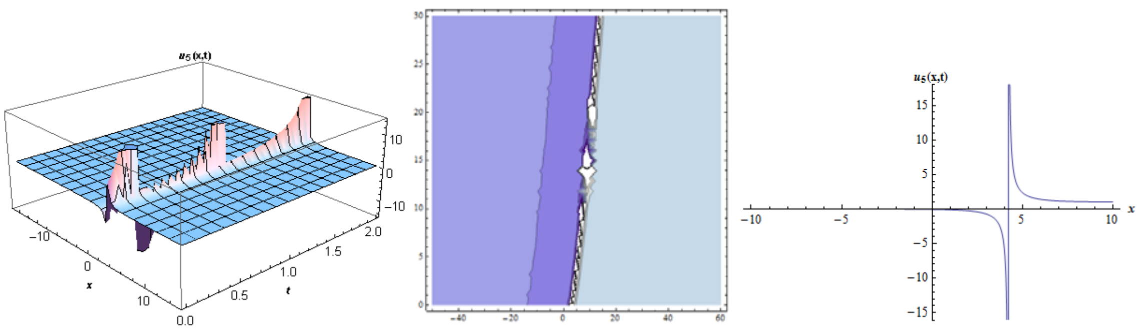
Figure 4. (a) and (b) are the 3D and contour surfaces of Eq.(20) with $\gamma = 0.8$ respectively, and $t = 2$ for the 2D graph (c).

Set 5: For $w \neq d$

$$a_0 = 0, a_1 = 0, a_2 = -\frac{db_0}{w}, a_3 = -\frac{db_1}{w}, c = \frac{-1+2\theta}{4w}, k = \frac{1}{2\sqrt{2}w}, \tag{21}$$

When the Eq.(8) and Eq.(4) are used for Eq.(21), we have

$$u_5(x,t) = \frac{d}{d - e^{-\frac{t^{\alpha}+\sqrt{2}x\alpha-2t^{\alpha}\theta}{2\alpha}} Ew}. \tag{22}$$



(a) (b) (c)

Figure 5. (a) and (b) are the 3D and contour surfaces of Eq.(22) with $\gamma = 0.8$ respectively, and $t = 2$ for the 2D graph (c).

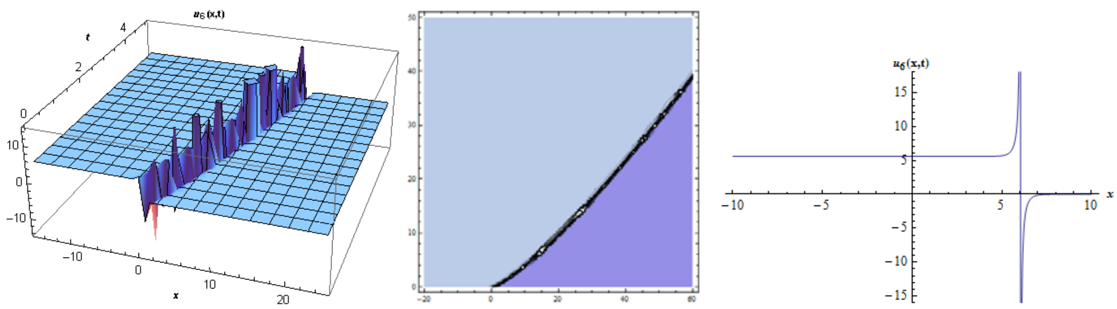
Set 6: For $w \neq d$

$$a_0 = \frac{4wcdb_0 + \sqrt{2}\left(1+8cw-\sqrt{1+4cw}\right)db_0\sqrt{1+2cw+\sqrt{1+4cw}}}{8cdw},$$
$$a_2 = \frac{\sqrt{2\left(1+2cw+\sqrt{1+4cw}\right)}db_0}{w}, k = \frac{\sqrt{1+2cw+\sqrt{1+4cw}}}{2w}, \theta = \frac{a_0}{b_0}, a_3 = a_2 b_1, a_1 = \frac{a_0 b_1}{b_0}, \quad (23)$$

When the Eq.(8) and Eq.(4) are used for Eq.(23), we have

$$u_6(x,t) = \frac{4cdwb_0 h(x,t) - \sqrt{2}\left(e^{\frac{2ct^{\alpha}w}{\alpha}} Ew^2(1+8cw-\sqrt{1+4cw}) + d^2 b_0 e^{\Phi x}(-1+\sqrt{1+4cw})\right.}{8cdwb_0 h(x,t)}$$
$$\frac{\left.\sqrt{1+2cw+\sqrt{1+4cw}}\right)}{8cdwb_0 h(x,t)}. \quad (24)$$

where $\Phi = w\sqrt{\frac{1+2cw+\sqrt{1+4cw}}{w^2}}, h(x,t) = de^{\Phi x} - e^{\frac{2ct^{\alpha}w}{\alpha}} Ew$.



(a) (b) (c)

Figure 6. (a) and (b) are the 3D and contour surfaces of Eq.(24) with $\gamma = 0.8$ respectively, and $t = 2$ for the 2D graph (c).

5. Conclusions

In this study, we have presented some new travelling wave solutions of the conformable time-fractional Fitzhugh–Nagumo model via IBSEFM. Figure 1-6 shows surfaces of the exponential function solutions of fractional-order with $\gamma = 0.8$. The Fitzhugh Nagumo model describes travel waves that can be used for electrical signaling through the lengths of the nerve axons. The fractional cases are more useful than integer order in the diagnosis of psychological and physiological diseases that can be caused by instant electrical conduction between nerve cells. The fractional-order derivative may be using in brain wave oscillations to control their dynamics has been shown in many numerical simulations. Moreover, the conformable fractional derivative definition overcome the mentioned causes therefore the obtained exponential function solutions are more appropriate to test accuracy of numerical simulations. Assuming these facts, it is hoped that the above analytical wave solutions presented for the first time in the literature may be useful in areas such mathematical models such as clinical neurology, neuropsychology and so on.

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