Smooth Threshold Autoregressive models and Markov process: An application to the Lebanese GDP growth rate®

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ABSTRACT

This paper analyzes the evolution of the Lebanese GDP growth rate over the period 1970-2019 by estimating two kinds of switching models: The Smooth Transition Autoregressive (STAR) model and the model of the Markov process. These models show, on the one hand, asymmetries in the evolution of GDP growth with an abrupt transition from a regime to another and, on the other hand, a high probability that the economy remains in the recession regime. Even though the duration of the expansion phase is longer than the duration of the recession phase, the Lebanese economy experiencing the greatest difficulties in moving from a recession regime to an expansion regime. In addition, such an evolution is explosive and volatile during the lower regime (recession phase) but stationary and damped in the upper regime (expansion phase). Finally, the paper shows that the STAR model, taking a logistic form, better fits the Lebanese GDP growth than the Markov model.

Key words: *GDP growth rate, Business cycle, Asymmetry, Markovian*

JEL Codes : C13, C22, E23, E32

1. INTRODUCTION

Several studies about the GDP fluctuations have been conducted in developed countries (Cette, 1997; OCDE, 2002; Ferrara, 2008 and 2009) as well as in developing countries (Fathi, 2009; Ndongo and Francis, 2007; Hoffmaister and Roldos, 2001). For example, regarding Lebanon, GDP fluctuations are considerably noticeable over the 1970-2019 period and particularly during the civil war. Indeed, before 1975 (i.e., the beginning of the civil war), the GDP exhibited a growth rate of 11% in 1972. Then, this rate fell drastically to – 84% in 1976 and increased to 61% the next year (United Nations, 2020). This instability continued until the end of the civil war in 1990. In fact, during the civil war, the GDP growth rate displayed abnormal values and such values may distort the analysis of its evolution.

The graph for the annual difference of the GDP (in logarithm) is presented below.

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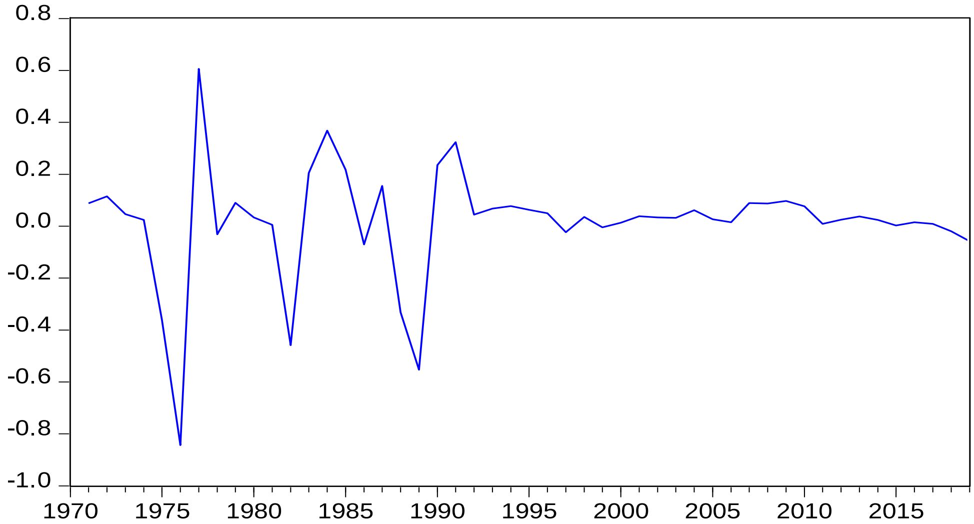
Figure 1: Lebanese GDP growth 1971-2019



Source: Author

This volatility in the evolution of the GDP cannot be found in other developed countries as well as in most of the MENA countries (Appendix 1). Indeed, Lebanon is the country showing the biggest GDP fluctuations and, literature regarding the analysis of the Lebanese GDP growth evolution is relatively rare (Verne, 2011). Therefore, it is important to use nonlinear time series models to characterize the asymmetric dynamics in the macroeconomic data such as GDP growth. As a matter of fact, these models were not estimated in Lebanon where the evolution of the GDP growth appears erratic, especially during the civil war. So, these models seem particularly well adapted for this country to take into account the nonlinearity and possible asymmetries in the cyclical evolution of its GDP growth.

Thus, the purpose of this paper is to estimate nonlinearity and asymmetries in the Lebanese GDP growth that are captured by the analysis of the passage from a regime (expansion) to another (recession). For apprehending these asymmetric evolutions of the Lebanese economy, we shall estimate and compare two kinds of switching models: the Smooth Transition Autoregressive model (called, thereafter, STAR model, which is well known and often compared with other linear models in the literature) to examine the dynamic behavior of the GDP growth, and the model of discrete-state Markov process developed by Hamilton (1989) to see the characteristics of these changes of regime. We also don't forget that the Lebanese economy has been largely perturbed by civil war and other conflicts after 1990.

Thus, to estimate these evolutions of the Lebanese GDP growth, in a context of political instability, the present paper contains the following sections. After a brief literature review, comparing nonlinear models with linear models developed in Section 2, Section 3 presents and proposes an estimation of a STAR model to investigate the threshold from which the Lebanese economy undergoes a regime change. Section 4 presents an estimation of a Markovian model developed by Hamilton (1989) to see the likelihood of change from a regime to another. Moreover, it makes a comparison to determine which model better fits the observed data. Section 5 concludes and comments.

2. LITERATURE REVIEW OF THE STAR MODEL COMPARED WITH OTHER LINEAR MODELS

Nonlinear time series models are relevant to characterize the asymmetric evolution of the macroeconomic data. The most famous nonlinear model that analyzes regime-switching is the Smooth Transition Autoregressive (STAR) model. The STAR model was developed by Chang and Tong (1986) and later popularized by Granger and Teräsvirta (1993), Teräsvirta (1994) and Van Dijk et.al. (2002). It is particularly well adapted to capture nonlinear characteristics of business cycle indicators. For example, at the macroeconomic level, Skalin and Teräsvirta (1999) use such a model for examining the nonlinearity in the Swedish business cycle. The general specification of this model nets the linear autoregressive model as a special case. It is also more general in comparison to other nonlinear models like the threshold autoregressive (TAR) models, initially developed and discussed by Tong (1983).

So, in the literature, this STAR model was compared with other linear and autoregressive (AR) models. Sarantis (1999) investigates the real change rate of G-10 countries and compares forecasting performance between the STAR model and linear models. He concludes that there is not a big difference between both models regarding the accuracy of the prediction. On the contrary, Teräsvirta (1994) analyzes the performance of the STAR model using 47 macroeconomic variables in the G7 countries and shows that the forecasting accuracy of the STAR model outperforms linear autoregressive models. A similar result is found by Bradley and Jensen (2004), regarding the forecasting of industrial production. However, Claveria and Torra (2014), analyzing the evolution of the tourism demand, find that the forecasting of the ARIMA model outperforms the nonlinear self-exciting threshold autoregressive (SETAR) model and artificial neural network (ANN) model as well. In the same activity sector, by allowing structural breaks in the data, Saayman and Botha (2017) show the superior performance of the nonlinear model relative to the SARIMA model. Umer et al. (2018), by employing the travel monthly and leisure index in Turkey from April 1997 to August 2016, show that the nonlinear Logistic STAR model does not improve the forecasting accuracy compared to the linear AR model.

This brief literature indicates several contradictory results about the efficiency of forecasting of the nonlinear models compared with other linear models. However, in a country like Lebanon, where the evolution of the GDP growth is somehow erratic, it seems that the estimation of the nonlinear model as the Logistic STAR model is needed for apprehending nonlinearities and asymmetries. Such a model can be compared with linear models as is the case in the aforementioned literature, but our purpose is to evaluate it with another asymmetric model such as the Markov process that estimates the probability of the switching regime.

3. SMOOTH TRANSITION AUTOREGRESSIVE MODELS AND DYNAMIC BEHAVIOR OF THE LEBANESE GDP GROWTH

The STAR model is estimated on the annual GDP growth in Lebanon over the period 1970-2019. This series has been unloaded from the United Nations Statistic (2020) (at constant US dollars price). We write it in logarithm and the first difference and obtain the GDP growth rate, Y_t.

The STAR model (Dias, 2003) for a univariate series Y_t (rate of GDP growth), can be expressed as:

$$Y_t = a_0 + \sum_{j=1}^{m} b_j Y_{t-j} + \left(a'_0 + \sum_{j=1}^{m} b'_j Y_{t-j} \right) G(Y_{t-d};\ \gamma, c) + \varepsilon_t \qquad (1)$$

Where $\varepsilon_t \rightarrow iid(0, \sigma_\varepsilon^2)$, m represents the lags of Y_t and G, called the transition function. This is a continuous bounded function between 0 and 1and at least twice differentiable in the sample space. It takes two extreme values: $G = 0$ and $G = 1$ with c representing the threshold e.g., the rate of GDP growth around which the economy runs from a regime to another. The transition between these two extreme regimes is allowed to be smooth and is governed by the transition variable Y_{t-d}. When $G = 0$, the economy enters the recession phase, e.g., the lower regime. When $G = 1$, the economy reaches the expansion phase, e.g., the upper regime where equation 1 can be re-written as:

$$Y_t = (a_0 + a'_0) + \sum_{j=1}^{m} (b_j + b'_j) Y_{t-j} + \varepsilon_t \qquad (2)$$

In this model, the GDP growth follows a stationary process and moves more or less smoothly between two extreme regimes characterized by possibly completely different specifications and dynamics instead of switching abruptly as is assumed in a threshold autoregressive (TAR) model. For our purpose, these extreme regimes represent the expansion (positive GDP growth) and the recession (negative GDP growth). The regime that characterizes the dynamic at each moment is linear and depends on the lagged value of the transition variable y_{t-d}.

The transition function can take two different forms: The logistic form and the exponential form.

The logistic form is as follows:

$$G(Y_{t-d};\ \gamma, c) = (1 + exp[-\gamma(Y_{t-d} - c)])^{-1}, \qquad \gamma > 0 \qquad (3)$$

This function is monotonically in Y_{t-d} so that the two regimes correspond to high and low values of the transition variable. The threshold value c determines the point at which the regimes are equally weighted and γ controls the speed and smoothness of the transition. The γ coefficient is the smoothness parameter. It determines the slopes of the logistic function and consequently governs the speed with which the transition takes place between the lower regime ($G = 0$) and the upper regime ($G = 1$). If γ is very large, the change from 0 to 1 is abrupt and, if $\gamma \rightarrow \infty$, the logistic STAR (LSTAR) model nets a two regimes thresholds autoregressive (TAR) model as a special case. When $\gamma \rightarrow 0$, the LSTAR model nets the AR model.

Regarding the exponential form, we have:

$$G(Y_{t-d};\ \gamma, c) = (1 - \exp[-\gamma(Y_{t-d} - c)^2]), \qquad \gamma > 0 \qquad (4)$$

This function is increasing in the absolute deviation of Y_{t-d} from the threshold c. $G(Y_{t-d}; \gamma, c) = 0$ when $Y_{t-d} = c$ and G approaches 1 as $Y_{t-d} \rightarrow \infty$ and $Y_{t-d} \rightarrow -\infty$. Such a function does not nest the TAR model because when $\gamma \rightarrow 0$ or $\gamma \rightarrow \infty$, the specification becomes linear since G approaches a constant function returning 0 or 1.

To capture nonlinear characteristics of the GDP growth and select between both functions, we run the Teräsvirta test (1994, p. 211) which leads to follow the sequence of steps:

- Specifying a linear autoregressive model of order *p* for the GDP growth using the Akaike and Schwarz information criteria.
- Testing linearity against the nonlinear specified alternative STAR model.
- Determining the transition variable most suitable if the null hypothesis of linearity is rejected.
- Choosing between the Logistic STAR (LSTAR) model and the Exponential STAR (ESTAR) model by testing a sequence of hypotheses.

The first step requires the specification of a linear autoregressive model throughout 1970-2019 and is to carry out the Lagrange Multiplier test (LM test) to test linearity against STAR models alternative. We choose the rate of GDP growth as a transition variable and estimate the delay parameter. We also include the war dummy variable (*W*) to correct abnormal values. Such a variable represents the civil war and equals 1 from 1975 to 1990 and 2006 (second war with Israel) and 0 otherwise.

To run this test, we follow the Luukkonen et.al. (1988) procedure that consists in substituting the transition function *F* by its fourth-order Taylor expansion that then yields an auxiliary regression as:

$$Y_t = b_0 + \sum_{j=1}^{m} b_{1,j} Y_{t-j} + \sum_{j=1}^{m} b_{2,j} Y_{t-j} Y_{t-d} + \sum_{j=1}^{m} b_{3,j} Y_{t-j} Y_{t-d}^2 + \sum_{j=1}^{m} b_{4,j} Y_{t-j} Y_{t-d}^3 + \sum_{j=1}^{m} b_{5,j} Y_{t-j} Y_{t-d}^4 + wW_t + \varepsilon_t \quad (5)$$

If the null hypotheses $H0_i$ of the sequential test are rejected, then the model is non-linear. Thus, in Appendix 2 (Table A1), the Fisher test $F_{(k,\ n\ -\ k\ -\ 1)}$ and the associated *p-value* regarding the null hypotheses $H0_i$ of the sequential test indicates that we can reject the null hypotheses of linearity ($\gamma \neq 0$).

At the same time, we identify the delay of the transition variable Y_{t-d}. To determine such a delay, we have chosen the value for which the above-mentioned test rejects the null hypothesis more strongly. So, the delay is $d = 2$.

Also, for choosing between the ESTAR model and the LSTAR model, we estimate the relation (5) based on the third-order Taylor expansion ($b_{5,j} = 0$). By using the OLS method based on Akaike and Schwarz criterion (that give us $m = 3$) and on the delay of the transition variable ($d = 2$) we obtain:

$$Y_t = -0.018 + 0.18Y_{t-1} - 0.07Y_{t-2} - 0.16Y_{t-3} + 6.77Y_{t-1}Y_{t-2} + 4.11Y_{t-2}Y_{t-2} + 4.67Y_{t-3}Y_{t-2}$$

(0.37) (0.60) (0.08) (0.54) (2.33)** (1.32) (1.94)*

$$+ 2.98Y_{t-1}Y^2_{t-2} - 5.83Y_{t-2}Y^2_{t-2} - 3.94Y_{t-3}Y^2_{t-2} - 31.98Y_{t-1}Y^3_{t-2} - 25.27Y_{t-2}Y^3_{t-2}$$

(0.51) (0.90) (1.04) (1.17) (1.40)

$– 25.32\, Y_{t-3} Y^3_{t-2} – 0.16W + e_t$

(2.50)** (2.12)** (6)

$R^2 = 0.60$, $N = 46$

$Q(13, 14, 15)$= [18.94(0.13) ; 19.03(0.16) ; 19.5(0.19)]

(.) = t-ratio and ***, ** significance at the one-percent level and five percent level respectively.

R^2 = coefficient of determination and *N*, the number of observations.

Q is the Ljung Box Statistic with 13, 14, 15 lags and the *p*-value are also presented in parenthesis.

The *Q* statistics and the associated *p-value*, larger than the five-percent level, show that residuals are following a white noise process. The war dummy variable shows a significant coefficient (at the five-percent threshold).

Knowing that the model is nonlinear with the transition variable $Y_{t-d} = Y_{t-2}$, we follow the Teräsvirta's procedure (1994, p. 211) for choosing between the LSTAR model and the ESTAR model, by carrying out the three tests in the following sequence:

H01: $b_{4,j} = 0$ against H11 ($b_{4,j} \neq 0$), with an *F*-test (F_4) $j = 1,\ldots,m$
H02: $b_{3,j} = 0 \mid b_{4,j} = 0$ against H12 ($b_{3,j} \neq 0 \mid b_{4,j} = 0$,), with an *F*-test ($F_3$) $j = 1,\ldots,m$
H03: $b_{2,j} = 0 \mid b_{3,j} = b_{4,j} = 0$ against H13 ($b_{2,j} \neq 0 \mid b_{3,j} = b_{4,j} = 0$,), with an *F*-test ($F_2$) $j = 1,\ldots,m$
This test gives us the following results (with the *p*-value in parenthesis):

$F_4 = 2.18(0.09)$; $F_3 = 0.91(0.68)$; $F_2 = 7.14(0.00)$.

Because we reject H03 after failing to reject H02, we may choose the LSTAR model. Such a result is confirmed by the Escribano and Jorda test (2001, p. 14) which recommends substituting the transition function *F* by its fourth-order Taylor expansion to consider that the threshold parameter *c* is different from zero (Appendix 3).

Therefore, we choose the LSTAR model that takes the following form:

$Y_t = (a_0 + a_1Y_{t-1} + a_2Y_{t-2} + a_3Y_{t-3} + w_1W) + (b_0 + b_1Y_{t-1} + b_2Y_{t-3} + b_3Y_{t-3} + w_2W)G(Y_{t-d}; \gamma; c)$

With $G\,(Y_{t-d}; \gamma; c) = \{1 + \exp\,[-\gamma(Y_{t-2} - c\,)]\}^{-1} + \varepsilon_t$ (7)

$\{1 + \exp[-\gamma(Y_{t-2} - c)]\}^{-1}$ represents the transition function (described above) and W is the dummy variable representing the war.

Using the nonlinear least square (NLS) method and dropping the statistically non-significant intercept, we obtain the following results:

$Y_t = (-0.54Y_{t-1} - 1.46Y_{t-2} + 0.90Y_{t-3} - 0.51W) + (1.18Y_{t-1} + 1.50Y_{t-2} - 0.87Y_{t-3} + 0.47W)\ G\ (Y_{t-d};\gamma;\ c)$
(3.89)*** (5.64)*** (3.12)*** (4.77)*** (3.75)*** (3.90)*** (2.65)*** (3.28)***

With $G\ (Y_{t-d};\ \gamma;\ c) = \{1 + \exp[-269.62(Y_{t-2} - 0.048)]\}^{-1} + e_t$ (8)
(0.44) (6.07)***

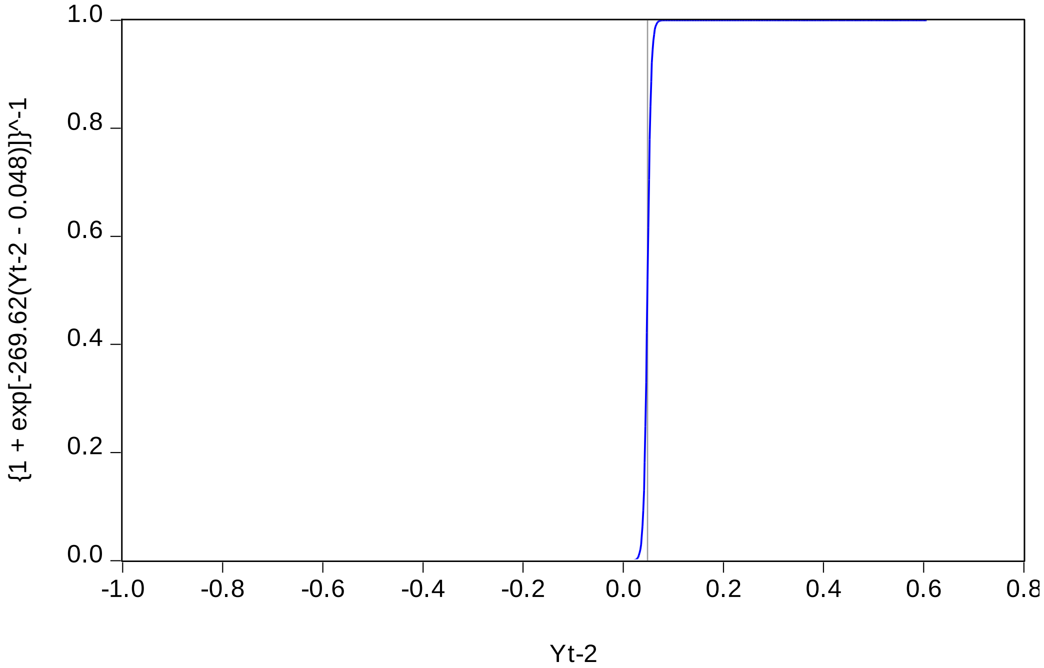
$R^2 = 0.58$, $N = 46$
$Q(13, 14, 15) = [19.16(0.16)\ ;\ 19.31(0.20)\ ;\ 19.41(0.25)]$
ARCH(1) = [0.44(0.66)]; ARCH(2)=[– 0.43(067)]

(.) = t-ratio and ***, ** significance at the one-percent level and five percent level respectively.
R^2 = coefficient of determination and N, the number of observations.
ARCH(p): Heteroskedastic test of order p.

The Q (Ljung-Box) Statistic (with the lags in parenthesis) and the p-value show that residuals follow a white noise process. The LM test of no ARCH effect of order one and order two and the associated p-value (larger than 5%) indicate that they are homoscedastic. All the parameters are highly significant. We also notice that the value of γ coefficient (significantly different from zero, as aforementioned in the relation (6)), is very large.

Figure 2 shows that the rate of GDP growth is switching abruptly from the lower regime (recession) to the upper regime (expansion).

Figure 2: Threshold Weight Function (c = 0.048)



Source: Author

The transition function delivers a specification very close to a regime of self-exciting threshold autoregressive model initially developed by Tong (1983).

The threshold value (c parameter) shows that the Lebanese economy runs from a recession regime to an expansion regime when the rate of GDP growth reaches around 4.8%.

In addition, the war variable significantly corrects the abnormal values taken by the GDP growth, notably during the period of the civil war. Indeed, it entails a reduction in GDP growth of 39% (100*[exp (– 0.50)-1] = 39%) in the lower regime ($G\ (Y_{t-d};\ \gamma;\ c) = 0$), against 4.9% (100*[exp(– 0.50 + 0.45)-1] = 4.9%) in the upper regime (with $G(Y_{t-d};\ \gamma;\ c) = 1$). Thus, the two regimes present different dynamics (Dias, 2003, p. 19). For instance, the lower regime, described by a negative GDP growth rate ($G(Y_{t-d};\ \gamma;\ c) = 0$), has the characteristic polynomial with inverse roots and modulus given by:

$$h(z) = z^3 + 1.44z^2 + 0.53z - 0.88 \qquad (9)$$

with inverse real roots = – 0.546 and inverse complex roots = $0.993 \pm 0.792i$ associated with modulus ρ = 1.27. This shows an explosive dynamic.

Concerning the upper, described by a positive GDP growth rate ($F\ (Y_{t-d};\ \gamma;\ c) = 1$), the characteristic polynomial with inverse roots is given by:

$$h(z) = z^3 - 0.06z^2 - 0.64z - 0.03 \qquad (10)$$

The inverse real roots are: 0.047; 0.75; – 0.845. This means that in the upper regime, where inverse roots are less than one, a stationary dynamic occurs. Such a property implies that once the economy is in the upper regime, it will remain there. The economy will return towards the lower regime after the occurrence of a large negative exogenous shock. However, once the economy is in this regime, the non-stationary dynamic leads the GDP growth out of the lower regime to a normal growth rate as time elapses, even in the absence of positive shocks. In other words, we can estimate the period during which the Lebanese economy stays in a state of recession (lower regime) or a state of expansion (upper regime).

4. CHARACTERISTICS IN THE CHANGE OF THE RATE OF GDP GROWTH: THE MARKOV PROCESS

In Lebanon, the economy runs abruptly from a regime to another. Moreover, the lower and upper regimes exhibit completely different dynamics. Thus, by using the Hamilton (1989) procedure, we show that the dynamics of the GDP growth are state-dependent and follow a Markov process. In this model, there is a finite number of unobserved states. For our purpose, we have two states e.g., state 1: Expansion (upper regime) and state 2: Recession (lower regime). We set a random variable called u_t such that $u_t = 1$ or $u_t = 2$ at any time. The u_t variable follows a first-order Markov process because its current value depends only on the immediate past value. Moreover, we do not know which state the process is, but can only estimate the probabilities.

The process can switch between states repeatedly over the sample. So, we estimate the state-dependent parameter and transition probabilities as: $P(u_t = j/u_{t-1} = i) = p_{ij}$. This means the probability of transitioning from state i to state j. We may write these probabilities in a transition matrix as follows:

$$p_{ij} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

This matrix represents the probability of transitioning from regime i in period t-1 to regime j in period t with $p_{12} = 1 – p_{11}$ and $p_{22} = 1 – p_{21}$.

In the Markov model, the probabilities are parameterized in terms of a multinomial logit:

$$p_{ij} = \frac{1}{1 + \exp\left(-p_{ij}\right)}$$

In this model, the values of the GDP growth rate (Y_t), have to be corrected by the dummy variable (W). The number of states is imposed a priori. Because of the heteroscedasticity exhibited by the GDP growth from 1970 to 2018 and especially during the civil war (1975-1990 period), we estimate the model with specific error variance. For our purpose, the two states model can be expressed as:

$$Y_t = \begin{bmatrix} \alpha_1 + \varphi_1 W + \varepsilon_{t,1} \;\; if\, u_t = 1 \\ \sigma^2_{\varepsilon_t} = \exp(\beta) + v_{t,1} \\ \alpha_2 + \varphi_1 W + \varepsilon_{t,2} \;\; if\, u_t 2\, if\, u_t = 2 \\ \sigma^2_{\varepsilon_t} = \exp(\gamma) + v_{t,2} \end{bmatrix} \qquad (11)$$

$\varepsilon_{t,i}$ and $v_{t,i}$, are the white noises such as $\varepsilon_{t,i} \rightarrow N(0, \sigma_\varepsilon^2)$ and $v_{t,i} \rightarrow N(0, \sigma_v^2)$ and $\sigma_{\varepsilon t}^2$ is the error variance (called sigma) that measures volatility into the regime. W is a common dummy variable and the residuals $e_{t,}$ are following an autoregressive process of order 1.

The results of equation (11) are the following:

$Y_t = 0.03 - 0.02W + e_{t,1}$ *if* $u_t = 1$ (upper regime) (12)
(3.5)*** (1.05)

$\sigma_{\varepsilon t}^2 = 0.026 + v_{t,1}$ (12')
(24.23)***

$Y_t = -0.12 - 0.02W + e_{t,2}$ *if* $u_t = 2$ (lower regime) (12'')
(1.99)** (1.05)

$\sigma_{\varepsilon t}^2 = 0.42 + v_{t,2}$ (12''')
(4.28)***

$e_{t,i} = 0.38e_{t-1,i} + v_t$ (12'''')
(3.65)***

$N = 48$
(.) = z-ratio and ***, significance at the one percent-level.

All parameters are statistically significant at the five-percent threshold, except the coefficient of the war variable (W) because the model corrects for heteroscedasticity of the GDP growth. We see that in the lower regime, the value of the sigma parameter is higher than in the upper regime. This shows larger heteroscedasticity of the GDP growth in the recession regime than in the expansion regime.

Regarding the coefficients of the transition matrix parameter, p_{11}, denotes the probability that the economy remains in an expansion regime in the next period, and it equals 92%:

$$p_{11} = \frac{1}{1 + \exp(-2.46)} = 0.92 \qquad (13)$$

p_{21} denotes the probability of transitioning from an expansion regime to a recession regime in the next period. It equals 16%:

$$p_{21} = \frac{1}{1 + \exp(1.61)} = 0.16 \qquad (14)$$

With $p_{12} = 1 - p_{11} = 0.08$, the probability of transitioning from a recession regime to an expansion regime and $p_{22} = 1 - p_{12}, = 0.84$, the probability that the economy remains in a recession regime we have:

(row = i / column = j)

	1	2
1	0.92	0.08
2	0.16	0.84

Table 4.1. Constant transition probabilities

From these results, we can estimate the expected duration of the expansion regime E[D_{exp}] and the recession regime E[D_{rec}] as follows:

$$E[D_{exp}] = \frac{1}{1-0.92} = 12.5 \qquad (15)$$

And

$$E[D_{rec}] = \frac{1}{1-0.89} = 6.25 \qquad (16)$$

We see a high probability of remaining in the regime origin: 92% regarding the upper regime and 84% for the lower regime. The corresponding expected durations in the upper regime (positive GDP growth) and, in the lower regime (negative GDP growth) are approximately 12.5 years and 6.25 years, respectively.

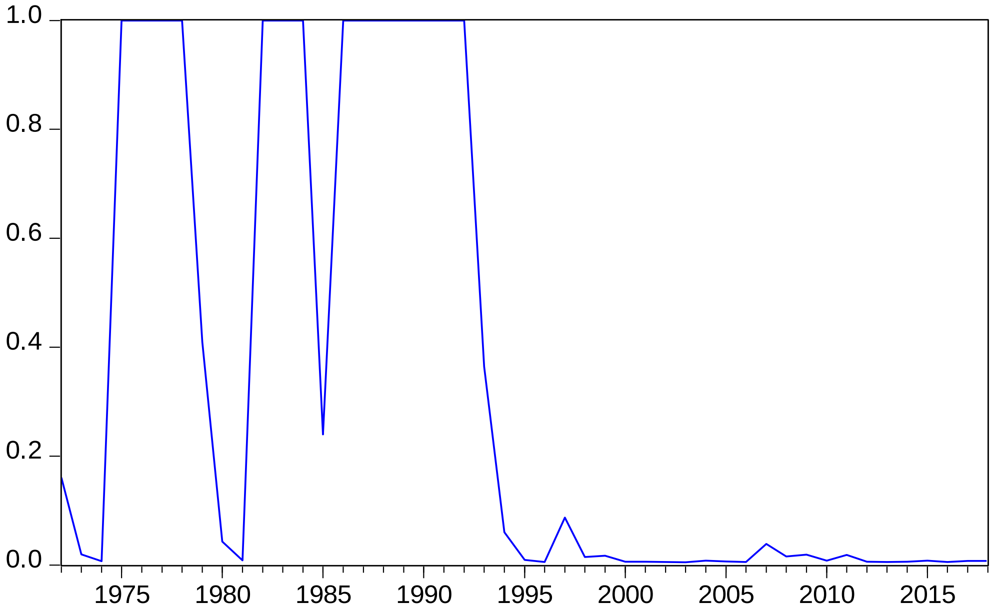
These durations of the regime can be exhibited by the graph of the filtered probabilities of being in the expansion regime and the recession regime (Figure 3).

We can see the regime of each state according to the higher probability:

Thus, $Ut = \underset{K,\,N}{argmax}\ Pr\ (Ut \setminus \Omega t)$

Where U_t represents the state of the economy estimated at the t period, K, the number of regimes, N the size of the sample and, Ωt is the ensemble of information. A recession occurs if the probability of being in the recession regime (state 2) is larger than 50%: $Pr(Ut = 2\setminus \Omega t)$ > 0.5 (Figure 3). An expansion occurs if the probability of being in expansion regime (state 1) is larger than 50%: $Pr(Ut = 1\setminus \Omega t) > 0.5$ (Figure 4).

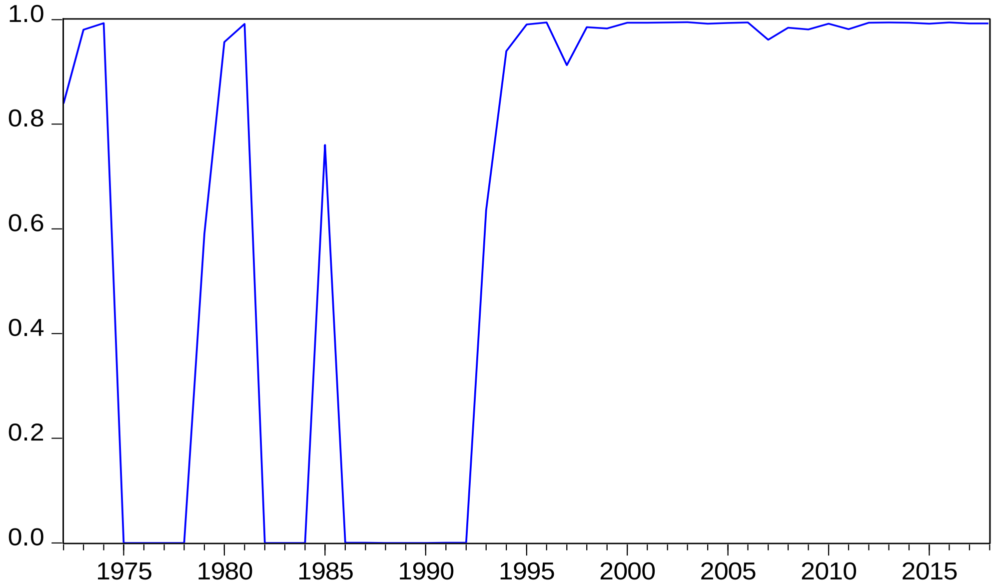
Figure 3: Probability of being in a recession regime



Source: Author

This Figure shows that $Pr(Ut = 2\backslash \Omega t) > 0.5$ between 1975 and 1978 (period of total war); between 1982 and 1984 (period of Israeli invasion) and between 1986 till the end of the war and the beginning of the reconstruction period in 1991.

Figure 4: Probability of being in an expansion regime



Source: Author

This Figure shows that $Pr(Ut = 1\backslash \Omega t) > 0.5$ for 1974; for 1980 and 1981; for 1985; between 1993 and 2018.

In Lebanon, the probability that the economy remains in the upper regime is larger than the probability of staying in the recession regime (92% against 84%) and the duration of the state of expansion is longer than the duration of the state of recession. Such a result indicates that a cyclical asymmetry in the evolution of GDP growth occurs. Indeed, the two main phases of the business cycle, recession and expansion, are not equally long (Luukkonen and Teräsvirta, 1991) and, overall, show different volatility. Furthermore, we notice that the probability of transitioning from an upper regime to a lower regime is twice higher than the probability of transitioning from a lower regime to an upper regime (16% against 8%).

From such results, we can compare both models (Markov process or logistic STAR model) in Figures 5 and 6 to determine which better fits the Lebanese GDP growth data.

Figure 5: Observed data of Lebanese GDP Growth vs Fitted Data with LSTAR Model

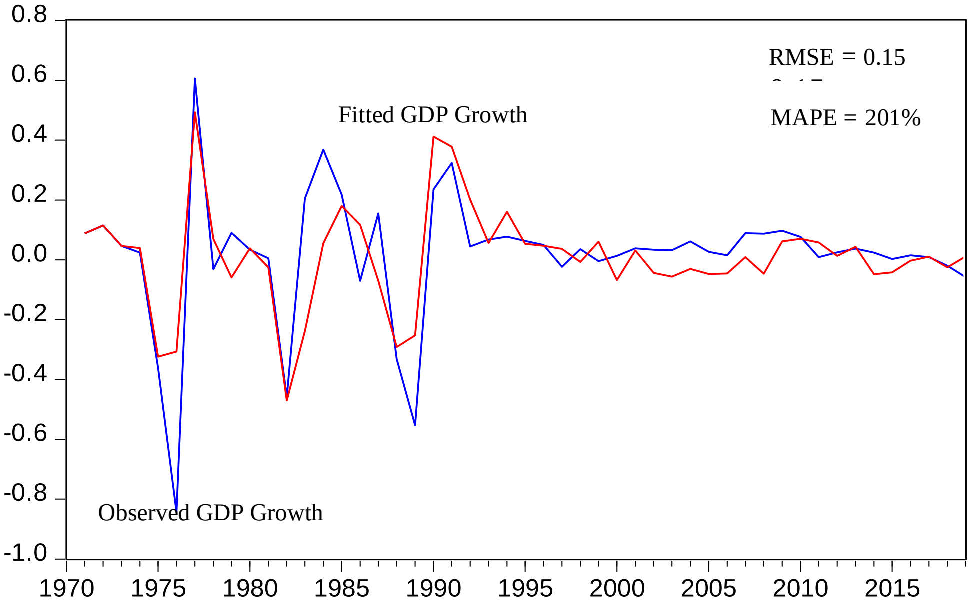
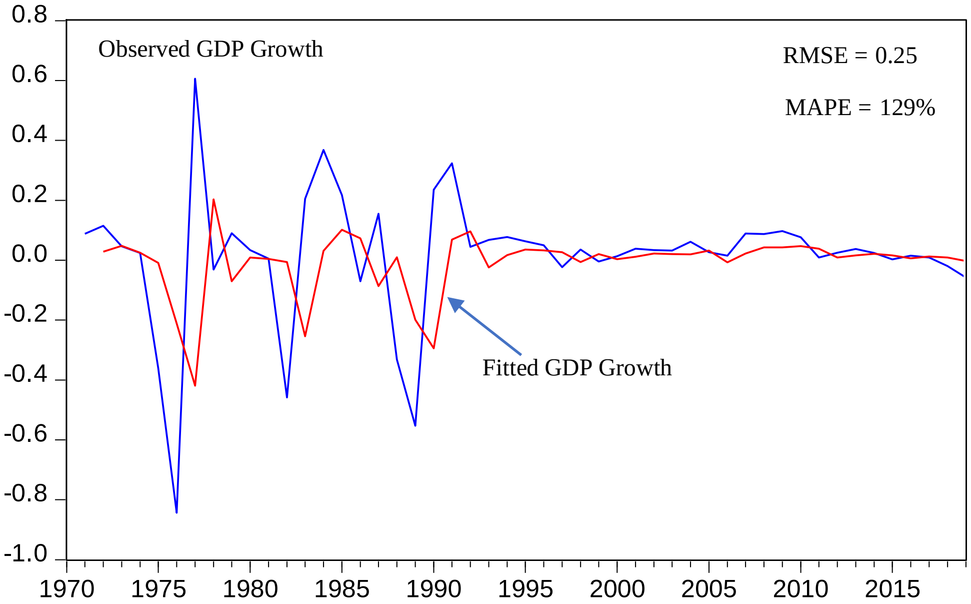


Figure 6: Observed data of Lebanese GDP Growth vs Fitted Data with Markov Switching Model



Source: Author

While the two Figures are showing a relatively good fit of the observed Lebanese GDP growth, the MAPE (Mean Absolute Percentage Error) statistic indicates a very bad fit because it exhibits values larger than 100% regarding both models (Lewis, 1982). However, data are written in logarithm and first differences. So, in this case, MAPE has a disadvantage since it produces indefinite or infinite values when the observed values are zero or close to zero as in the Lebanese GDP growth rate. Indeed, absolute actual values of this macroeconomic variable are very small (often less than one) and this is the reason for which MAPE yields extremely large percentage errors. Such errors might be regarded as outliers. To overcome this problem, Kim and Kim (2016) propose a new measure called the Mean Arctangent Absolute Error (MAAPE) which is less biased than MAPE. This MAAPE is written as follows:

$$\text{MAAPE} = \frac{1}{N}\sum_{t=1}^{N} arctan\left(\left|\frac{Y-\hat{Y}}{Y}\right|\right) \qquad (17)$$

Where Y and $\hat{Y}$ are the values of the observed GDP growth rate and predicted GDP growth rate, respectively.

We obtain MAAPE = 45% regarding the fit of the observed GDP with the logistic STAR model, which is a reasonable prediction, and MAAPE = 53% for the Markov model.

Thus, according to this statistic, it seems that the Logistic STAR model better fits the data, since it indicates a MAAPE value lower than that obtained by the Markov model. Nevertheless, Figure 6 shows the Markov model better fits the observed GDP growth between 1991 and 2019 while Figure 5 indicates that the logistic STAR model better fits the observed GDP growth during the 1970-1990 period.

5. CONCLUSION

From 1975 through 1990, the civil war disturbs the evolution of the Lebanese GDP growth rate and plays an important role in the correction of the abnormal values, notably when estimating the STAR model. By comparing two switching regression models, we saw that the LSTAR model better fits the Lebanese GDP growth than the Markov switching model, especially during the 1970-1990 period. However, the two models are complementary because they enable us to show asymmetries in the evolution of GDP growth.

By estimating a STAR model, we have shown that the Lebanese economy goes brutally from the lower regime (recession) to the upper regime (expansion). Moreover, the evolution of the GDP growth in the lower regime is not only heteroscedastic but also explosive. It becomes less volatile and stationary in the upper regime.

The model of the discrete-state Markov process has shown that the Lebanese economy exhibits expansion for a relatively longer time than recession. However, the probability that this economy goes from an expansion regime to a recession regime is twice higher than the probability of transitioning from a recession regime to an expansion regime. This means that even though this economy is relatively resilient (until 2019), because of the Lebanese diaspora, which allows Lebanon to benefit from large financial inflows, it is having great difficulties in moving from a recession regime to an expansion regime. A lack of an effective economic policy and a high level of corruption that prevents the sustainable growth of the Lebanese economy can explain such a phenomenon.

APPENDIX

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
ALGERIA	0.03	0.03	0.20	-0.10	0.04	0.77	9.82
EGYPT	0.05	0.05	0.15	0.01	0.03	1.24	4.85
IRAQ	0.04	0.04	0.43	-1.08	0.22	-2.58	14.84
IRAN	0.02	0.03	0.21	-0.24	0.08	-0.60	4.25
ISRAEL	0.04	0.04	0.11	0.00	0.03	0.49	3.15
JORDAN	0.04	0.04	0.19	-0.11	0.05	0.14	4.60
KUWAIT	0.02	0.03	0.60	-0.53	0.16	0.07	8.22
LEBANON	**0.02**	**0.03**	**0.61**	**-0.84**	**0.22**	**-1.41**	**7.90**
LIBYA	-0.01	0.03	0.81	-0.95	0.27	-1.06	7.33
MORROCO	0.04	0.04	0.12	-0.07	0.04	-0.62	3.47
PALESTINE	0.05	0.06	0.21	-0.13	0.08	-0.15	2.39
SYRIA	0.03	0.05	0.22	-0.31	0.10	-1.36	5.90
TUNISIA	0.04	0.04	0.16	-0.02	0.03	0.85	6.36
TURKEY	0.04	0.05	0.11	-0.06	0.04	-1.03	3.54
UAE	0.05	0.05	0.23	-0.21	0.08	-0.40	4.92

Table 1. GDP Growth: Descriptive statistics for MENA region

With Libya and Iraq, Lebanon is the country exhibiting the greatest volatility of the GDP growth with a rate varying from – 84% to 61% and a standard deviation equals 0.22, the second-highest of the sample.

Non-linearity test with Y_{t-2} as transition variable (based on equation (5))

$$Y_t = b_0 + \sum_{j=1}^{m} b_{1,j}\, Y_{t-j} + \sum_{j=1}^{m} b_{2,j}\, Y_{t-j}Y_{t-d} + \sum_{j=1}^{m} b_{3,j}\, Y_{t-j}Y_{t-d}^2 + \sum_{j=1}^{m} b_{4,j}\, Y_{t-j}Y_{t-d}^3 + \sum_{j=1}^{m} b_{5,j}\, Y_{t-j}Y_{t-d}^4\varepsilon_t + wW_t + \varepsilon_t \quad (5)$$

To run this test, we also use the same OLS method to estimate (5), with $m = 3$ and $d = 2$, we obtain:

$$Y_t = -0.03 + 0.04Y_{t-1} + 1.31Y_{t-2} + 0.14Y_{t-3} + 1.84Y_{t-1}Y_{t-2} - 6.57Y_{t-2}\, Y_{t-2} - 3.26Y_{t-3}\, Y_{t-2}$$

(0.51) (0.11) (0.69) (0.43) (0.25) (0.64) (0.36)

$$+ 45.21Y_{t-1}\, Y^2_{t-2} + 7.79Y_{t-2}\, Y^2_{t-2} + 46.69Y_{t-3}\, Y^2_{t-2} - 21.79Y_{t-1}\, Y^3_{t-2} + 29.73Y_{t-2}\, Y^3_{t-2}$$

(1.03) (0.46) (0.81) (0.24) (0.24)

$$- 28.43Y_{t-3}\, Y^3_{t-2} - 307.31Y_{t-1}\, Y^4_{t-2} - 167.03Y_{t-2}\, Y^4_{t-2} - 147.96Y_{t-3}\, Y^4_{t-2} - 0.20W + e_t \quad (18)$$

(1.19) (1.12) (1.0) (0.76) (2.08)**

R^2 = 0.55; N = 46;

Q(13, 14, 15) (the Ljung-Box statistic)= [13.05(0.41) ; 13.74(0.47) ; 14.65(0.48)

(.) = t-ratio and * significance at the ten-percent and one-percent levels respectively.

R^2 = coefficient of determination and N, the number of observations. The Q statistics show that residuals are following a white noise process.

Only the war variable has a significant impact on GDP growth.

Non-linearity test: $Y_{t-d} = Y_{t-2}$

Hypothesis	$b_{1,j} = b_{2,j} = b_{3,j} = b_{4,j} = 0$	$b_{1,j} = b_{2,j} = b_{3,j} = 0$	$b_{1,j} = b_{2,j} = 0$	$b_{1,j} = 0$
$F(k, n-k-1)$=*F-Stat* (*p-value*)	3.47 (0.00)	3.75 (0.00)	3.90 (0.00)	7.33 (0.00)

In all cases, we reject strongly the null hypothesis of non-linearity.

The Escribano-Jorda test

According to Escribano and Jorda (2001, p. 14), we must consider that the threshold parameter c is different from zero. Thus, the authors recommend substituting the transition function F by its fourth-order Taylor (equation (5)).

So, for choosing between LSTAR and ESTAR, they propose to carry out another two tests:

H0L: $b_{2,j} = b_{4,j} = 0$ against $b_{2,j} \neq b_{4,j} \neq 0$ with an F_7 test.
H0E: $b_{3,j} = b_{5j} = 0$ against $b_{3,j} \neq b_{5,j} \neq 0$ with an F_6 test.

If the minimum *p-value*, corresponds to F_7, then we have to select the LSTAR model, and ESTAR model if it corresponds to F_6.

By using the results of relation (17) we have:

F_7 = 2.38(0.06); F_6 = 0.71(0.64)

The results of the F_7-test and the F_6-test with the *p-values* in parenthesis indicate that the *p-value* of the F_7-test is less than the p-value the of F_6-test. Consequently, we can choose the LSTAR model.

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