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PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' CONCEPTION OF AUXILIARY ELEMENTS OF TRIANGLES

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Abstract: In the literature, there have been research examining different grade levels of students' understanding of geometric shapes such as triangles and their main elements as well as their auxiliary elements. The purpose of the current study is to investigate preservice middle school mathematics teachers' (PMSMT) conception of auxiliary elements of triangles. In order to achieve this, the activity sheets about definitions, constructions, and properties of auxiliary elements of triangles were designed and conducted to 23 junior PMSMT. The PMSMT engaged in these activity sheets. The data were collected through their written works and it was analyzed based on the content analysis which is a type of qualitative data analysis technique. It was found that, the PMSMT could effectively define auxiliary elements of triangles. However, they had difficulty in the properties and related theorems about auxiliary elements.

***Keywords*:** Auxiliary elements, conception, triangles.

Introduction

Preservice mathematics teachers need knowledge and skills in order to perform their responsibility of teaching geometrical concepts. They can acquire them in a learning environment designed by effective and useful geometrical tasks related to these knowledge and skills. In this respect, it is necessary that the preservice mathematics teachers should be in supported with a learning environment supported by rich opportunities to obtain experiences in order to understand the geometrical concepts. In this wave, they can improve their knowledge and understanding of geometry (Han, 2007; Henningsen & Stein, 1997).

In geometry, different grade level of learners are expected to attain knowledge about related concepts. Triangle is one of the most important concepts among them. Triangles are essential in teaching geometry, but different grade level of students unfortunately face with obstacles in understanding triangles (Damarin, 1981; Vinner & Hershkowitz, 1980). In the literature, there have been research examining the students' understanding of triangles. Some of these research explain that the triangles should be taught focusing on analyzing and understanding of elements of triangles. For example, in Wang (2011)'s study, it was found that prospective elementary teachers could form accurate logical reasoning and explanations about main elements of shapes such as angles and edges. However, they could not effectively reason using auxiliary elements. Also, Uygun (2016) and Gutierrez and Jaime (1999) focused on preservice teachers' understanding about altitudes of triangles as one of auxiliary elements of triangles. They stated that it was necessary for them to attain deep knowledge about altitudes. The researchers made suggestion to perform studies about other auxiliary elements of triangles for future research. In this respect, the necessity of making investigations about auxiliary elements of triangles is emphasized in the current study. Additionally, while Alatorre and Saiz(2010) investigated preservice and inservice teachers' understanding of altitudes of triangles; Gutierrez and Jaime (1999) examined students' learning of altitudes of triangles as a type of auxiliary element. In their study, they found that student learning was affected by their teachers' understanding, explanations and transferring of the knowledge. The researchers explained this finding by using quotations from a lesson designed by altitudes as a type of auxiliary element. Different from these studies, this study investigates the impacts of teachers' definitions, concept images,

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difficulties and errors about altitudes of triangles on their students. When the effects of teachers on student learning are considered, it is important to educate effective and knowledgeable teachers in preservice years. Moreover, many research in the literature have focused on the altitudes of triangles so it is necessary to pay attention on other types of auxiliary elements of triangles. Hence, preservice teachers should acquire deep knowledge and understanding of auxiliary elements of triangles. In this respect, it is important to examine preservice teachers' understanding of auxiliary elements of triangles. Therefore, in the present study, it was focused on to investigate the understanding of preservice middle school mathematics teachers' understanding of different types of auxiliary elements of triangles.

Method

In this study particularistic case study is used, as the case is determined based on the criteria of the researchers' interest and willingness. In a particularistic case study, the researcher identifies the phenomena that s/he wants to examine and understand the phenomena deeply and document it in detail (Merriam, 2009; Stake, 1995).

The sample of the current study included twenty three preservice middle school mathematics teachers (PMSMT). They were enrolled in the program of elementary mathematics education at a university in the northern part of Turkey. The participants were composed of twelve female and eleven male PMSMT. The participants were selected by using the criterion sampling strategy as a type of purposeful sampling strategy. The selection criteria for the present study were being familiar with the necessary knowledge of geometry related to the concept of triangles and being taken the undergraduate course of Geometry in their teacher education programs. The data were gathered by using the video recordings of whole class discussion, audio recordings of peer group discussions and artifact collection of their written documents of activity sheets.

The activity sheets were prepared in a way that PMSMT were asked to construct the altitude of triangles, prove the formation of altitudes and concurrence of them on triangles as orthocenter, determine the places of this point for different types of triangles and discuss their ideas and explanations about them.

The qualitative data analysis of the present study was performed based on the content analysis technique based on the steps explained by Creswell (2012). The themes were determined as the formation of auxiliary elements, concurrency of them, and naming the concurrency point. The strategies, justifications and explanations were determined as the codes of the data analysis process. In order to provide trustworthiness, the strategies of triangulation by data and investigator were used. The data were gathered from different sources including written documents, audio and video recordings. Also, the transcripts were also analyzed by two researchers independently. Moreover, member checking strategy was used.

Findings

In the process, the PMSMT focused on the formation of auxiliary elements of triangles. They constructed the angle bisector, altitude, perpendicular bisector and median by using compass and straight edge. They studied with their peers and participated in whole class discussions under the guidance of the instructor. The instructor initiated the discussion focusing on the errors of the PMSMT about the construction of auxiliary elements. Hence, their misconceptions and errors were removed and effective understanding was supported. For example, the PMSMT constructed the angle bisector of a triangle in two ways and justified the formation of an angle bisector was performed by two different strategies. In the first strategy, the angle bisector was constructed by the knowledge that the angle bisector of a triangle could be perpendicular bisector at the same time if it was an isosceles triangle. Therefore, they formed an isosceles triangle on the original triangle. Then, by constructing the perpendicular bisector, the angle bisector was formed. In this process, the justification of this construction process was also provided as in Figure 1as follows:

Instructor: Can you construct the angle bisector?
Ali: Firstly, I identify an isosceles triangle by drawing an arc. Then, I form the perpendicular bisector of the new edge…

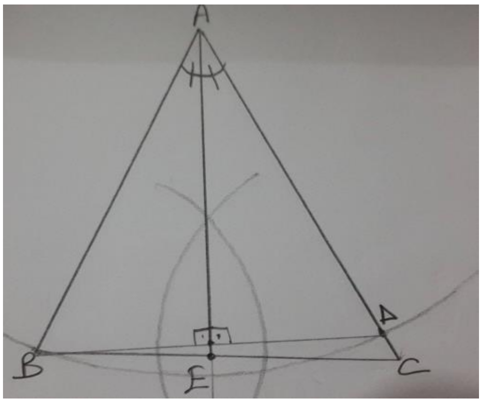

Figure 1. Construction of angle bisector by isosceles triangle

In the other strategy, the angle bisector was constructed using parallelogram. By the knowledge that the diagonal of a parallelogram formed two congruent triangles. They formed a parallelogram and two congruent triangles having a common side. By this way, an angle bisector was constructed in a different way and also it was justified by the knowledge related to parallelograms as in Figure 2 as follows:

Ayşe: I draw a parallelogram by using the sides of AB and BC. Then, by the instructions of finding midpoint of a line, I draw the diagonal of the parallelogram…

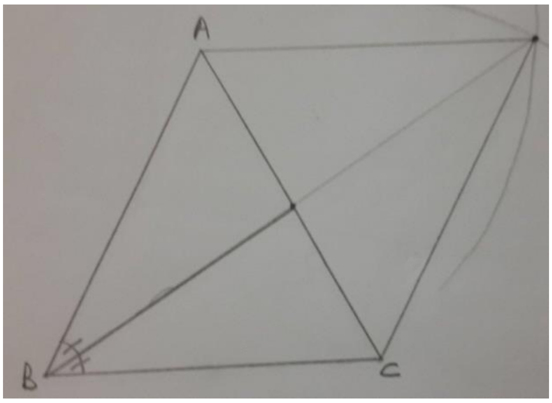

Figure 2. Construction of angle bisector by parallelogram

In the other part of the discussion process, the PMSMT focused on the concurrency of auxiliary elements of triangles. For example, in the activity sheet about angle bisectors, they showed the concurrency of them by the construction of all angle bisectors and justified the process by using the previous whole class discussion as explained above. Moreover, the PMSMT justified the concurrency of angle bisectors by Ceva Theorem. They explained that the angle bisector was a type of ceva and the concurrency of them could be showed by this theorem. Furthermore, they provided another justification using angle bisector theorem as in Figure 3 as follows:

Instructor: What can you say about the intersection of all angle bisectors of a triangle?
Ayşe: They are concurrent.
Instructor: How?
Ayşe: They all intersect each other at a point. I can show this by constructing all of them...
Instructor: What else?
Arzu: An angle bisector is a type of ceva so I can use Ceva Theorem.
Instructor: How?
Arzu: If I can apply the theorem for the related lengths of the sides, I can show…
Instructor: Is there another way?
Elif: Assume that there is a point as the intersection of the angle bisectors. Then, I draw the perpendicular lines from this point to the edges. Hence, congruent triangles such as BGO and KBO are formed… (see Figure 3)

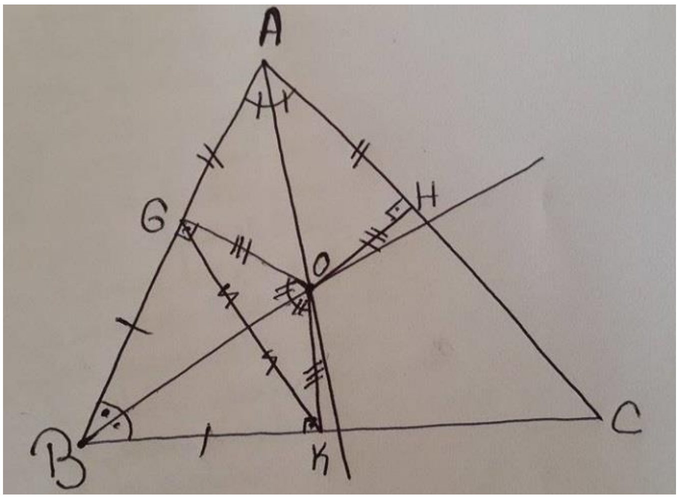

Figure 3. Concurrency of angle bisectors by angle bisector theorem

In the last part of the whole class discussion, the PMSMT talked about the names of the auxiliary elements of triangles. They discussed the incenter as the concurrency point of angle bisectors and the position of this point on the triangle. They provided the justification by using angle bisector theorem as in Figure 3. Also, a different justification was explained using the angles based on the arcs and the corners of the formed triangles by drawing arcs on Figure 3 and focusing on the measures of these arcs and angles of the figure. Then, they examined this situation focusing on the properties of these concurrency points. They also improved their understanding of these points. Afterwards, similar activities and discussion procedures were performed other auxiliary elements of triangles. They examined all auxiliary elements' properties.

Discussion and Conclusion

In the study, it was observed that geometric constructions facilitated the formation of auxiliary elements of triangles, relating with other concepts and justifying the formation of them. Through geometric constructions, the learners make these examinations by broadening their views, thinking and understanding of geometry (Cherowitzo, 2006). In this respect, it can be stated that geometric constructions can be useful in mathematics teacher education in order to help them acquire and develop their geometric thinking and knowledge and understanding about geometry concepts (Erduran & Yeşildere, 2010; Hoffer, 1981; Napitupulu, 2001). The present study was designed and conducted to examine preservice teachers' understanding about a particular geometry concept. This finding is parallel to the findings of some research in the literature since preservice middle school mathematics teachers' understanding of geometrical concepts can be improved by geometric constructions (Cheung, 2011; Napitupulu, 2001). It was also found that whole class discussions improved the PMSMT' understanding of auxiliary elements of triangles since they analyzed and criticized the concepts and their ideas about the solutions, formation of auxiliary elements, and justification of the ideas. This result is parallel to the study of Olkun and Toluk (2004). They stated that class discussions facilitated students' geometric thinking and understanding of geometrical concepts. Therefore, it is important to support PMSMT by providing an environment where they can have class discussions to understand geometric constructions. Also, the tasks and learning environments can be designed in this way in order to develop preservice teachers' knowledge and understanding of mathematical concepts especially the topics in geometry.

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