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THE EVOLUTION OF THE IMPORTANCE OF MULTI-SENSORY TEACHING TECHNIQUES IN ELEMENTARY MATHEMATICS: THEORY AND PRACTICE[1]

İLKOKUL MATEMATİĞİNDE ÇOKLU-DUYUMA DAYALI ÖĞRETME TEKNİKLERİNİN ÖNEMİNİN EVRİMİ: KURAM VE UYGULAMA

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ABSTRACT
In recent years, partially because of federal legislation, there have been increases in demand for accountability in all educational venues. Performance in elementary mathematics is no exception. In this paper we review the relevant parts of the learning theories of Piaget, Bruner, and Vygotsky and address the difficulties teachers may face when introducing mathematical concepts. The review of theories, along with a review of previously published empirical studies, supports the use of multi-sensory teaching techniques in the elementary, specifically kindergarten through third grade, classrooms. Since students (both regular and special needs) develop and learn at different rates, it is unlikely that all will be developmentally prepared to assimilate new mathematical concepts at the same time. Multi-sensory techniques allow many students, by assimilation, to grasp elusive concepts and keep up with their peers.

Keywords: Mathematics teaching, multi-sensory teaching techniques.

ÖZ
Kısmen yasal gelişmeler nedeniyle, bütün eğitim alanlarında performansla ilgili sorumluluk alma eğilimi giderek yaygınlaşmaktadır. İlkokul düzeyinde matematik performansı da bu konuda bir istisna değildir. Bu çalışmada Piaget, Bruner ve Vygotsky'nin kuramlarının ilgili bölümlerini ve öğrencileri matematik kavramlarıyla tanıştırırken öğretmenlerin karşılaşabilecekleri zorlukları gözden geçirdik. Yayınlanmış olan kuramsal ve deneysel çalışmalar, özellikle anaokulu seviyesinden üçüncü sınıf sonuna kadarki dönemde, çoklu-duyuma (multi-sensory) dayalı öğretim tekniklerinin kullanımını desteklemektedir. Normal ve engelli çocukların gelişim ve öğrenme hızları farklılık gösterdiğinden, çocukların tümünün yeni matematik kavramlarını idrak etmeye aynı anda hazır olma olasılığı düşüktür. Çoklu-duyum teknikleri, birçok çocuğun anlaşılması zor matematik kavramlarını asimile etme yoluyla öğrenmesini sağlayarak akranlarından geri kalmamasını sağlamaktadır.

Anahtar Kelimeler: Matematik öğretimi, çok duyuma dayalı öğretim teknikleri

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INTRODUCTION

In recent years, there has been an increase in the accountability of student performance across all educational venues. This has been manifested in the appearance of performance indicators (i.e., student achievement tests) of student learning and growth and has forced educators to reexamine didactic methods to ensure that all students have the opportunity to learn. The learning area of elementary mathematics is no exception. In this paper, we review the relevant aspects of learning theories that impact mathematics learning among young children. We argue that the theories (Piaget, 1958; Bruner, 1973; Vygotsky, 1978) indicate the necessity of employing multiple methods of presenting mathematical concepts because children of the same chronological age are not necessarily at the same stage of mental readiness (Van de Walle, 2007; Kamii and Rummelsburg, 2008). We further suggest that these methods should include multi-sensory teaching techniques across a variety of classrooms (Clements, 1999).

When introducing new mathematical concepts to elementary students it is important that teachers help all students understand the material. Since different students may have different learning styles (see below), multiple methods of presentation can facilitate their comprehension of new concepts. Teachers today are not only driven by their teaching values to reach all children, but by legislation as well. The No Child Left Behind Act (2002) has greatly increased districts', principals', and teachers' accountability by requiring schools to report their students' achievement test results to the public. If a school does not meet federal standards, parents have the right to relocate their children to another school (United States Department of Education, n.d.). Because of this increase in accountability, it is more important than ever for teachers to find ways to reach every child and help him master the material to be learned in the classroom.

This may be challenging since, as proposed by Piaget (e.g., 1965), not all students may be capable of grasping certain mathematical concepts at the same time as others (e.g., Kamii and Rummelsburg, 2008). As a result, students at varying developmental levels and with differing learning preferences can be left behind while the teacher continues on to the next lesson. This leads to the next problematic issue. Research by Hiebert (1988) has shown that students must learn and understand the earlier foundations of mathematical concepts before they will be able to comprehend the next level of processes. This introduces the possible need for additional teaching methods and materials to mediate students' learning when addressing certain mathematical concepts. For example, teachers can introduce the concept of addition by demonstrating with beans, have the class repeat the process verbally and manipulate them physically. What one child does

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not comprehend by hearing and saying it, they have a chance of grasping it by visual and tactile-kinesthetic means.

Some students may not have reached the mental maturity yet (addressed in detail later) by which they are ready to learn the mathematical concepts being taught in class. They may require supplemental learning materials, outside the main curriculum, to enhance their understanding. Multi-sensory supplements, such as math manipulatives, support the child's use of visual, tactile, and/or auditory interactions with the material. These types of materials can help to bridge the gaps that most elementary teachers will encounter when trying to teach young children novel and abstract mathematical concepts (Bullock, 2003).

Multi-sensory learning, as the name implies, is the process of learning new subject matter through the use of two or more senses. This may include combining visual, auditory, tactile-kinesthetic, and/or even olfactory and taste (Scott, 1993). The place for multi-sensory teaching techniques in elementary mathematics classrooms can be illustrated through a brief review of relevant aspects of the theories by Piaget, Bruner and Vygotsky. These theories concern stages of learning and readiness and can provide a better insight into students' development. These theories also can anticipate some of the frustrations students may encounter when learning mathematics. Stage theories suggest that as humans develop, they progress through different stages of cognitive development (i.e., a toddler is not simply a small adult, although adults can develop through stages as well, e.g., Perry, 1970). As children develop through different stages, they are prepared to assimilate different types of material at each stage, but not before. Now, while these stages are related to age, they are not age dependent. These theories, outlined below, for several different reasons, support the use of multi-sensory teaching techniques. These techniques can help students at slightly different stages of mental development interact with to-be-learned material and become familiar with those concepts.

In addition to the discussion of these theories with focus on their relation to the current topic, a review of prior empirical studies will be presented. By reviewing the influence of multi-sensory teaching techniques across both different student populations and academic disciplines, including elementary mathematics, its success in reaching students' understanding can be demonstrated.

THEORIES OF LEARNING

In the fields of psychology and education there have been many theories of learning and development advanced over the years. The theoretical positions involving stages of development are seen as most pertinent to the question of readiness-to-learn, and the stage theories seen as most relevant to early classroom learning are those of Piaget, Bruner, and Vygotsky. Relevant aspects of these theories will be briefly summarized.

Piaget

Jean Piaget's (e.g., 1958; 1965) theory of stage development describes children's progress through certain stages of development. He gives a loose time frame, in which the children enter each stage, but it must be understood that the ages he gives are approximate; it is the order of the stages that he believes to be universal. For example, studies with mentally retarded children have revealed that these children progress through Piaget's stages of development in the same order, though not at the same ages, as other children. Piaget's stages described below, in chronological order, are: sensorimotor, pre-operational, concrete operational, and formal operational. Furthermore, there is no reason to believe that all "normal" children will progress through stages at exactly, or even roughly, the same chronological ages. The reader should also note the broad range of ages included in each stage.

Sensorimotor**:** This stage encompasses most infants from birth to about two years of age. As the name suggests, the child in this stage is occupied only with his own motor activity and his mental activities are strictly limited to what his senses detect. Because the child is dependent on solely what his senses detect, he has no concept of object permanence; he has no idea that an object exists once it is no longer being sensed (Bybee, 1982; Thorne and Henley, 1997).

Pre-operational: This stage ranges from approximately two years to seven years of age. In this phase of life a child does not yet have the ability to perform the mental activities Piaget called "operations." One of these operations, most applicable to learning math concepts, is conservation of quantity; the concept that the quantity of something remains the same even though its physical appearance changes (explained further later). Also, children in this phase have an egocentric perspective of thinking. These children view themselves as the center of everything and have difficulty accepting any views but their own (Bybee, 1982; Thorne and Henley, 1997).

Concrete operational: Between the ages of seven and eleven the child enters the concrete operational stage. The child becomes less egocentric. And it is here that the child develops the concept of conservation. Although he has not fully developed the cognitive skills needed to handle abstract problems that require mental manipulations, he can now deal with tangible problems (Bybee, 1982; Thorne and Henley, 1997).

Formal operational: From approximately 12 or 13 years of age through adulthood, Piaget considers a person to be in the formal operational stage. A person is now capable of abstract thought, he can think reflectively, and can test hypotheses either systematically or hypothetically (Bybee, 1982; Thorne & Henley, 1997).

Piagetian Theory and Elementary Teaching: The two stages most relevant to K-3 children, and therefore of most concern to their elementary teachers, are the pre-operational and concrete operational stages. It is not the details of these stages that are so important for teachers to understand, but the differences between these two stages in regards to a child's conceptual abilities. In the preoperational stage, the child is not yet able to complete mental operations. Mental operations may include a task such as adding numbers in their head. Once they reach the concrete operational stage they can complete mental operations and therefore no longer need to use overt trial and error methods; trial and error can now be done in their heads (Bruner, 1973). It should be emphasized that "normal" children reach the concrete operational stage *between the ages of seven and eleven*. However, that is a fairly broad target for educators to use. For the child to be considered in the concrete operational stage their mental operations must be reversible (Bruner). Reversibility is the child's ability to think in two opposite directions at the same time. The lack of reversibility in a child in the preoperational stage can be demonstrated with any one of the many Piagetian conservation tasks.

One conservation task that is easily related to problems early elementary teachers face with their students is the conservation of number task described by Kamii (1982). A teacher lines up eight foam sticks in a row in front of her, and the child lines up a row of eight pieces to match the ones that the teacher has placed down. But if a child's, who is in the preoperational stage, set of pieces is spread out, as demonstrated in Figure 1, she now thinks that she has more pieces than the teacher. A child that does not have mental operations that are reversible is not able to understand conservation of number. They cannot grasp the concept that the row can be brought back to its original state and is therefore the same number as before (Bruner, 1973).

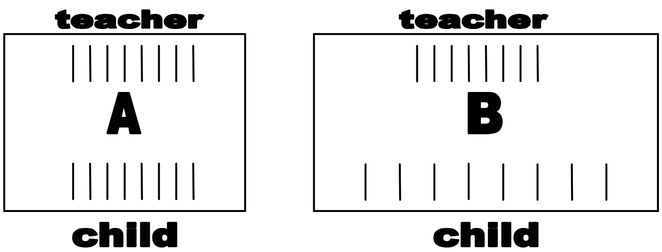

Figure 1. Demonstration of a Conservation Task

According to Figure 1, in situation A the child knows that he has the same number of pieces as his teacher. In situation B, when the child's pieces are spread out in front of him, he believes that he has more pieces than his teacher.

Another example of reversibility in a mathematical task is filling in the missing-addend.

$$3 + ? = 7$$

This task demonstrates the child's ability to look at the 3 as part of the whole, 7. Children that are not yet in the concrete-operational stage may be able to answer this problem due to rote memorization but not due to comprehension (Kamii, Lewis, and Booker, 1998).

As Bruner (1973) states in a discussion regarding Piaget's stages of development, teachers are extremely limited in conveying concepts to children in the preoperational stage. But, the child's development, according to Bruner, is responsive to the learning environment. It is just a matter of finding the right method of delivering the material to help a child progress through the stages of learning. Multi-sensory approaches allow children to receive the information in a variety of ways. These can facilitate development in general and math in specific by providing tools for the students to relate to until the concept is fully embraced.

If any impression is made on a teacher from this theory it should be that if the child has not progressed into at least the beginning of the concrete operational stage of development, then this theory suggests that he may not be capable of understanding the abstract mathematical concepts. Examples of these concepts are linking symbols to the ideas they represent or manipulating items in their heads, and as a result may not be able to advance to where he is expected to be.

Bruner

Bruner's theory (e.g., 1973) (alluded to earlier) is based on the foundation set by Piagetian Theory. A summary of Bruner's theory indicates that Piaget's stages may not be all or nothing; there may be a "degree" of stage development within each stage. For example, a child is not in sensorimotor, and then out of it; he progresses *through* the stage. This progression through one stage to the next may be facilitated with appropriate teaching techniques. The following is a brief description of Bruner's theory and how it is applied to elementary learning and teaching. The following two sections (how children learn and the teacher's place in learning) are taken from this source.

How children learn: Jerome Bruner is a strong advocate for constructive learning. Constructive learning involves hands-on activities in which the child can create and test his own hypotheses. Bruner believes a child is an active participant in learning and should be encouraged to participate in the learning process. Bruner’s theory is founded on the idea that children construct new ideas based on their previous knowledge. They use their current knowledge to create hypotheses and to help them solve problems and discover relationships. This idea emphasizes the importance of ensuring all children's understanding of a concept before moving on to the next. He also emphasizes that different forms of representation of a single concept may be more appropriate for children at various ages and/or stages of learning than others (discussed more in the next section). From Bruner’s theory, in regards to how children learn, it is most important for a teacher to understand that it will be difficult for a child to grasp new concepts when he has no knowledge on which to base the new information. Also important is that children may require different means of learning; one child’s understanding may not come from the same source as another child.

The teacher’s place in learning: Although Bruner considers the child to be an active participant in the knowledge gathering process, he emphasizes the importance of the instructor as a “translator” of mathematical material to the child. Children may be in different stages of development and as a result have different abilities. Therefore the needs of individual children differ and learning must be made appropriate for each child in a way that he can understand during his particular stage. Bruner's theory supports the idea that there is at least one way in which to reach any child and help her understand a concept. He expresses the availability and the importance of giving children “multiple embodiments of the same general idea” in order to increase their understanding of it. A teacher should address the child’s predisposition towards learning, determine the best way to present the material to be learned, and establish the best sequence in which to present it. With a classroom full of children, probably at different stages of development (or at different levels of sophistication within the same level), it is obvious why a teacher would have difficulty individualizing her method of translation for each student. For this reason, knowledge of supplemental materials that approach the children in a variety of ways in addition to the core curriculum might be helpful for the teacher to have. Materials that teach students via multiple means can address the needs of more students in a classroom.

Vygotsky

The aspect of Lev Vygotsky’s (e.g., 1978) theory of development most applicable to education is his theory of the “zone of proximal development" (ZPD). This theory is similar to the above theories in that it is a stage theory. However, Vygotsky emphasizes how a child transitions from one stage to the next as a function of social interactions rather than on specific stages. The ZPD is defined as the difference between the stage of development at which the child is currently and the potential stage to which he can reach with the proper assistance from adults or more capable peers (Tudge, 1990; Vygotsky, 1978). Vygotsky's ZPD implies a stage development through which children pass with the aid of social interaction. He suggests that cognitive and linguistic skills appear two times to children; first they appear socially, between two people, and then internally within the child (Gallimore and Tharp, 1990). The time for this transition differs for each child. As suggested earlier, it is not all or none.

A teacher's part, according to Vygotsky's (1978) ZPD theory, is to assist the child's performance through the zone into the next phase. Vygotsky's theory suggests that what children can do today with assistance, they will be able to do tomorrow proficiently on their own. According to this theory, with the appropriate guidance, performance can precede competence (Cazdan, 1981). Moll's (1990) summary of Vygotsky's theory suggests that rote drill and practice instruction is not the type of assisting in ZPD that Vygotsky would suggest. He would recommend that teachers should assist in basic activities to learn skills as opposed to teaching the basic skills without activities. According to this perspective, learning materials act as a tool to aid the child in problem solving until the ability to completely comprehend a concept is developed. It is important here to emphasize the distinction between comprehension and performance. The child can "do" the activity and be correct before she grasps to concept completely.

Summary of Theories

The three theorists summarized above offer complementary views on the mental development of children. Piaget (e.g., 1965) introduces the idea of stages of mental readiness in how a child interacts with the world. Bruner (e.g., 1973) builds on this idea but adds the notion that there may be levels within those stages that the teacher can utilize. And Vygotsky closes the circle by introducing the idea that social interaction can facilitate transition from one stage to the next and that a child's performance can be affected independently of the transitions.

Elementary teachers should take into account the following important ideas that are supported by the above theories. 1) A child's stage in development may be responsible for his inability to understand abstract concepts, which, in turn, may affect his progress in the class's mathematical curriculum. 2) A child builds his knowledge off of his understanding of prior concepts. 3) Teachers play an important role in assisting children via the use of appropriate methods. 4) Teachers may need to approach different children in different ways with the material to be learned. 5) Transition between stages may be different for each child and may be gradual as different "operations" are assimilated.

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If a child needs extra attention to acquire the knowledge needed to move on the next concept or process, and he does not receive it, he may be left behind and will continue to digress as the teacher, without the correct teaching tools, continues on in the curriculum without him. Introducing supplemental multi-sensory materials may facilitate a child to interact with math concepts by two or more means in addition to the core curriculum. As supported by the above theories, these materials may be helpful in teachers' efforts to reach every child's understanding. In addition to the theoretical foundation for multi-sensory teaching techniques, there is some empirical support as well.

EMPIRICAL STUDIES SUPPORT OF MULTI-SENSORY TEACHING

While the theoretical foundations introduced above indicate a need for multi-sensory approaches to early learning, what kind of empirical support is there for such techniques? It is to this small but diverse and provocative body of literature we now turn. There have been only a few studies conducted and they have employed different types of students in different academic subjects.

In a study executed across classrooms in Queensland, Australia, Thorton, Jones, and Toohey (1982) implemented a multi-sensory teaching program, Multisensory Basic Fact Program (MBFP), into remedial classrooms for students' grades two through six. The program incorporates visual learning through pictures, as teachers provide oral prompts. Students are also involved kinesthetically when learning new concepts by tapping or finger-tracing. To test the usefulness of this multi-sensory teaching program, these students were given an addition-facts test before beginning the program and again after the 11-week instruction phase. All of the grade levels except grade two (possibly because they were not yet at the stage in which the material could be absorbed) showed marked improvement from the pretest to posttest. And, although the students had not reviewed the information before the follow-up test, they retained their knowledge of the concepts after a three-week period.

An example of examining the impact of a multi-sensory approach to teaching reading is exemplified in a study by Dev, Doyle, and Valente (2002). They used the Orton-Gillingham technique (Institute for Multi-sensory education, 2000), which involves visual, auditory, and kinesthetic modalities, with first grade children at the special education level. These children improved enough in their reading abilities to advance them out of the special education level. The maintenance of the gains that they achieved with the use of the multi-sensory approach was evaluated after a two year period. None of the children had returned to special education classes (Dev, et al., 2002).

Another empirical study involving the effect of a multi-sensory approach involved testing regular (non-special education) fourth and sixth graders' learning of spelling. Students completed a series of six spelling lessons, which were followed by spelling tests in which they wrote down each word that the teacher pronounced. Each of the students graded their own tests by two different methods. Half of each student's tests were graded by hearing the teacher spell the words aloud. The other half of their tests was graded by comparing their spelling to a correct spelling list followed by the teacher spelling the words aloud. The students were given four post tests after a six week period, and the words graded by the multi-sensory method resulted in significantly higher scores than those graded by the strictly auditory method (Kuhn and Schroeder, 1971).

Multi-sensory learning techniques have also proven to be helpful in the development of a foreign language. Drills that contain visual, auditory, and even tactile involvement by students improve their comprehension of the foreign language (Kalivoda, 1978). Multi-sensory techniques are very helpful when employed in adult ESL classrooms. These students speak various languages other than English and many have no formal schooling background in reading or writing. Teachers must employ activities that involve many, if not all, the senses in order to teach these non-academic students to communicate (Bassano, 1982).

Manipulatives: A Specific Type of Multi-sensory Math Tool

Most of the literature with regard to multi-sensory learning in elementary mathematics involves manipulative materials. These materials, while they have a broad definition, are one way that teachers can incorporate multi-sensory learning into their elementary classrooms. The term "manipulative" has varying, but similar, definitions. Reys (1971) defines manipulative materials as objects that the students can feel, touch, and handle. They are learning tools that appeal to several senses and are distinguished by physical involvement by the student. Manipulatives, according to Chester, Davis, and Reglin (1991), are anything that a student can move either physically or mentally in order to discover the solution to a problem. Sowell (1989) describes two types of manipulatives, concrete and pictorial representations. Concrete manipulatives are items that students can work with directly. And pictorial manipulatives can be audiovisual presentations, observed demonstrations with concrete manipulatives done by others, or even pictures in printed materials. Examples of manipulative devices given by the National Council of Teachers of Mathematics (NCTM) (1973) are colored beads, blocks, and rods, place value devices, games and puzzles, and measurement devices. Specific attention to manipulative use was again reiterated in the *Principles and Standards for School Mathematics* (NCTM, 2000).

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Becoming increasingly available are virtual manipulatives, or electronically-based materials which are interactive, computer-based tools that function much like the more commonly used tangible, concrete manipulatives, but are maneuvered by the computer mouse (Spicer, 2000). Some consider these dynamic, interactive visual representations (manipulatives) the wave of the future (Moyer, Bolyare, and Spikell, 2002) and point out that although there are many computer-generated images being called virtual, they are truly not appropriate for this category unless they are interactive and dynamic (p. 373). In multi-sensory mathematics, interaction with and among materials, both concrete and virtual, is pivotal for deep understanding.

Using concrete materials in class to enhance students understanding is encouraged by the NCTM Standards (2000), which applies to virtual, interactive materials as well. In congruence with Piaget (1965), young children can only sense that which exists in their presence, so concrete objects with which they can explore are important to have available (NCTM, 1973). According to studies done by Suydam and Higgins (1984), lessons with manipulatives are more likely to produce greater achievement by the students than lessons that do not incorporate manipulatives. However, to be effective, manipulatives do not necessarily need to be handled directly by the student for all of the lessons. Hiebert (1989) recommends counting craft sticks as a manipulative device as a means of developing meaning in numeric symbols for young children.

As a final comment regarding any teaching material, it is not adequate for teachers to simply incorporate the materials into the mathematics lessons without specific strategies for their use. In congruence with Vygotsky's (1978) theory of the ZPD, the literature is showered with warnings to teachers about the importance of the teacher's role in the use of these materials, specifically manipulatives (e.g., Baroody, 1989; Hiebert, 1988; Marzola, 1987; NCTM, 1973; Reys, 1971). For example, when teachers use manipulatives in teaching mathematical concepts without reflecting on what these concrete objects represent (one block equals the numeral 1; two tens [base ten] equals the numeral 20), the real mathematics concept may be lost or misconstrued (Clements, 1999). And, when used appropriately, manipulatives can clarify an otherwise unclear concept such as dividing up a pizza to help young children mentally and physically represent the concept of half and quarter of a whole.

CONCLUSION

Based on both theoretical foundation and experimental support, it is important for teachers to be aware that not all students in an elementary mathematics class are at equal levels of mental maturity. For this reason, multiple modalities of presentation need to be incorporated into math lessons. Multi-sensory techniques and materials can help satisfy this requirement. If teachers continually approach new math concepts by only one means of representation, many of their elementary students will not grasp them. This may leave the children unprepared for mathematics lessons yet to come.

We know that using multi-sensory materials such as manipulatives can give a child a tool which he can use until he is truly ready to comprehend difficult mathematical concepts. It may also be true that these same tools can expedite the child's transition through these stages. The previously mentioned study with remedial students in Australia by Thorton, et al (1982) serves to emphasize this idea. Further, if multi-sensory tools can expedite the developmental process for remedial level students, then why should it not be generalized to the regular population as well? While there is only limited support for this from other subject areas, it seems a logical conclusion and should be the topic of future studies.

It has always been important for teachers to attempt to reach the understanding of all of their students; however, now legislation has placed even more urgency upon it. It is encouraging to see through emerging research by Kelly, Durham, and Rains (2004) that teachers are turning towards the use of multi-sensory materials, manipulatives in particular, in the elementary mathematics classroom. By utilizing more avenues of introducing new math concepts to a class, multi-sensory teaching techniques can assist teachers in "translating" novel, abstract mathematics concepts to young learners.

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