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Research Article

Free Vibration Analysis of Isotropic Plates Using Regressive Ensemble Learning

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Abstract

The Finite Element Method (FEM) is a popular technique that is employed to analyze and understand the behavior of a structure. Although it has various advantages, there are some drawbacks such as developing accurate mathematical models, the computational cost for complex systems, and expertise. Thanks to recent advancements in computational science, those drawbacks can be eliminated by integrating artificial intelligence. This study presents an ensemble learning regressor-based technique to evaluate the fundamental natural frequencies of isotropic plate structures. For this purpose, Random Forest Regressor (RFR) has been considered. The isotropic plates have been taken into account as square and rectangular thin and thick plates whose materials have been selected as Structural Steel, Aernet 100, Al 7108, and Al 2024 since they are frequently used in various engineering fields. It has been evaluated that the proposed technique has a 0.9936 correlation score (R^2) and 0.0019 mean square error (MSE). The average prediction accuracy has been obtained by 99.12% for the test set. Those indicated that the proposed approach is not only an appropriate model for such a problem but also predicts the fundamental natural frequency accurately. Considering its success (99.12%) and the execution speed (0.127 seconds), it is concluded that the proposed approach is an advantageous alternative technique to the other mathematical models.

Keywords: Vibration Analysis, Random Forest Regressor, Isotropic Plates, Artificial Intelligence, Natural Frequency.

İzotropik Plakaların Regressif Topluluk Öğrenmesi Kullanarak Serbest Titreşim Analizi

Öz

Sonlu Elemanlar Yöntemi bir yapının davranışını anlamak ve analiz etmek için kullanılan popüler bir tekniktir. Çeşitli avantajları olmasına rağmen doğru matematiksel modelin geliştirilmesi, kompleks sistemler için hesaplama bakımından maliyetli olabilmesi ve uzmanlık gerektirmesi yönünden bazı dezavantajları bulunmaktadır. Bilgisayar biliminde yakın zamanlarda meydana gelen gelişmeler sayesinde bu tip olumsuzluklar yapay zeka kullanılarak giderilebilmektedir. Bu çalışma, izotropik plakaların temel frekanslarını elde etmek için topluluk öğrenmeli regresör tabanlı bir yöntem sunmaktadır. Bunun için Rastegele Orman Regresörü ele alınmıştır. Ele alınan ince ve kalın izotropik plakalar kare ve dikdörgen geometride olup çeşitli mühendislik uygulamalarında kullanılan Yapı Çeliği, Aernet 100, Al 7108 ve Al 2024 malzemeleri dikkate alınarak tasarlanmıştır. Sonuç olarak önerilen yöntemin 0.9936 korelasyon değeri (R^2) ve 0.0019 ortalama karesel hata oranına sahip olduğu görülmüştür. Test seti için ortalama tahmin oranı ise %99.12 olarak elde edilmiştir. Bu sonuçlar göstermektedir ki önerilen yaklaşım sadece bu tip bir problem için uygun olmakla kalmayıp aynı zamanda temel doğal frekansı yüksek doğrulukla tespit edebilmiştir. Önerilen modelin başarısı (%99.12) ve çalışma süresi (0.127 saniye) dikkate alındığında gerçek zamanlı tahmin sistemleri için matematiksel modellere kıyasla avantajlara sahip bir alternatif olduğu sonucuna varılmıştır.

Anahtar Kelimeler: Titreşim Analizi, Rastgele Orman Regresörü, İzotropik Plakalar, Yapay Zeka, Doğal Frekans.

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1. Introduction

Evaluation of dynamic properties of engineering structures is an essential task at the very beginning of the designing phase to understand the structural behavior and to prevent a possible catastrophic event during operation. The most common technique to obtain the dynamic characteristics is Finite Element Method (FEM) based analyses, which provide numerical outcomes that are accurate as those of experimental analyses. Employing FEM reduces the cost that is required to conduct experimental analyses and also enables the understanding of various features of a structure or a system. However, FEM also has some drawbacks such as developing accurate mathematical models, the computational cost for complex systems, and expertise to conduct FEM-based analyses and understand the results. Thanks to recent improvements in computer science, the concept of artificial intelligence is employed for various engineering purposes. In addition to the advancements in artificial intelligence, novel and/or improved FEM techniques are also emerged. Accordingly, researchers are utilized different artificial intelligence and FEM techniques to evaluate structural behavior under healthy and various damaged conditions. Some of those key studies have been briefly presented as follows.

Reddy et al. (2012), employed Artificial Neural Networks (ANN) to predict the natural frequency of composite plates having different stacking sequences. They found that their proposed approach successfully predicted the natural frequency by a correlation factor (R^2) of 0.998. Avcar and Saplıoğlu (2015) used ANN to predict the first ten natural frequencies of isotropic beams under different dimension parameters and boundary conditions. Besides, they examined different ANN models considering activation function and the number of neurons as differentiating parameters. They concluded that the suitable activation function selection and number of neurons may differ from case to case. Rouzegar and Sharifpoor (2016) employed the Two-Variable Refined Plate Theory to conduct free vibration analysis of isotropic and orthotropic plates using FEM. They found that the presented technique was in good agreement with the true values and the results existed in the literature. Lieu et al. (2018) used isogeometric analysis to conduct bending and free vibration analyses of functionally graded plates (FGM) having a variable thickness. For such a purpose, they employed Generalized Shear Deformation Theory and Non-Uniform Rational B-Spline (NURBS) to represent geometry and to provide a meaningful solution. They found that the bending and vibration characteristics of FGM plates can be obtained by the employed method. Pathirage et al. (2018) examined the effectiveness of autoencoder neural networks and deep learning in identifying damage considering a steel frame structure. Their results indicated that the corresponding proposed approach successfully identified the damage with a correlation factor of 0.996. Nikoo et al. (2018) utilized Heuristic Search and ANN to obtain the natural frequencies of cantilever beams. For this purpose, they employed the genetic algorithm (GA), particle swarm optimization (PSO), and imperialist competitive algorithm (ICA). They found that the GA-ANN technique was superior to the other models with a correlation factor of 0.9291 and 0.010 mean square error value. Aktaş et al. (2019) considered ANN to solve the transcendental equation of longitudinal vibration of a bar, which was fixed from one end and had a mass on the other end. They concluded that the proposed ANN technique was in good agreement with the analytical calculations. Liu and Yang (2019) employed ANN to measure the vibration frequency via the Kinect V2 sensor. They considered frame, beam, and plate structures to validate the success of the proposed approach. They found that ANN can provide accurate frequency measurement when compared with an industrial vibrometer. Le et al. (2019) considered the Adaptive Neuro-Fuzzy Inference System (ANFIS) with GA and PSO to evaluate the buckling damage of steel columns subjected to axially compressive load. They concluded that the ANFIS-PSO technique slightly overperformed ANFIS-GA by a correlation factor of 0.929. Jung et al. (2020) employed Back-Propagated Linear Regression and ANN to evaluate the tensile properties of high strength steel. They claimed that employing a deep learning algorithm resulted in high accuracy in predicting the yield strength, yield ratio, and tensile strength. Cuong-Le et al. (2021) proposed PSO optimized Support Vector Machine (SVM) for damage identification in truss and frame structures. They also compared the proposed method with ANN, Deep Neural Networks (DNN), and ANFIS. They concluded that the proposed approach outperforms the other techniques and successfully identified both damage and the damage level for truss and frame structures. Kallannavar et al. (2021) employed ANN to examine the impact of moisture and temperature on the vibration of skew laminated composite sandwich plates. They found that ANN predicted the outcomes accurately. Hirane et al. (2021) proposed a High-Order Layerwise Finite Element Model to perform static and free vibration analysis of FGM sandwich plates. For this purpose, they considered sandwich plates with FGM face sheets and homogeneous core, isotropic FGM plates, and sandwich plates with homogeneous face sheets and FGM core structures. They concluded that the proposed model gave accurate results for both thin and thick FGM structures. Belarbi et al. (2021) presented an eight-node quadrilateral element to conduct a free vibration analysis of multilayer sandwich plates. They considered different parameters such as aspect ratio, boundary conditions, number of layers, modular ratio, core-to-face thickness ratio, skew angle, geometry, and ply orientations. They claimed that the proposed technique was superior to other traditional methods in terms of both simplicity and accuracy. Zang et al. (2022) employed Isogeometric Scaled Boundary Finite Element Method (IGSBFEM) for free vibration and static analyses of FGM plates. They also measured the impact of the aspect ratio and gradient index on the free vibration and static responses of FGM plates. They concluded that IGSBFEM was in good agreement with other techniques for both free vibration and static analyses. In addition, the proposed method posed an excellent performance on mesh adaptability and it was a shear-locking free approach.

The existing studies in the literature mainly focused on employing ANN as the machine learning technique to predict the static or dynamic properties of a structure. Although ANN is a powerful technique that has the capability of solving an engineering problem effectively, it is challenging to select the correct hyper-parameters such as activation function, number of hidden layers, number of neurons per hidden layer, learning rate, and momentum. Besides, it may suffer from computational costs as the number of hidden layers and neurons increases. Therefore, it is needed to develop alternative intelligent techniques to overcome such issues. Based on this motivation, this study utilizes Random Forest Regressor (RFR) technique to predict the fundamental natural frequencies of fully clamped isotropic plates. For this purpose, isotropic plates having different aspect ratios, thickness, and materials have been considered. The data required to build the intelligent model was obtained by modeling all the structures using FEM in ANSYS Workbench 18.2 environment. To obtain the optimal intelligent model, parameter tuning has been performed considering the number of trees of the RFR. The flowchart of the study has been shown in Figure 1.

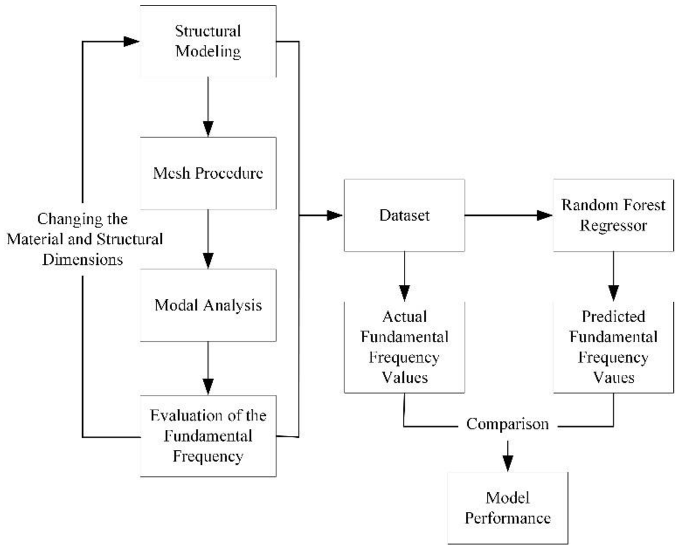

Figure 1. The Flowchart of the Study

The rest of the study has been structured as follows. In section 2, a summary of mathematical expressions of the FEM for free vibration analysis and RFR has been presented. Section 3 gives the convergence results of the developed ANSYS model and presents the performance of the RFR method in predicting the fundamental natural frequency of isotropic plates. The final section draws the key conclusions of the work.

2. Material and Method

2.1. Mathematical Expressions

The free vibration analyses of isotropic plates shown in Figure 2 have been conducted by employing the Finite Element Method via ANSYS Workbench 18.2 environment. As the finite element, the four-node quadrilateral element shown in Figure 3 has been considered.

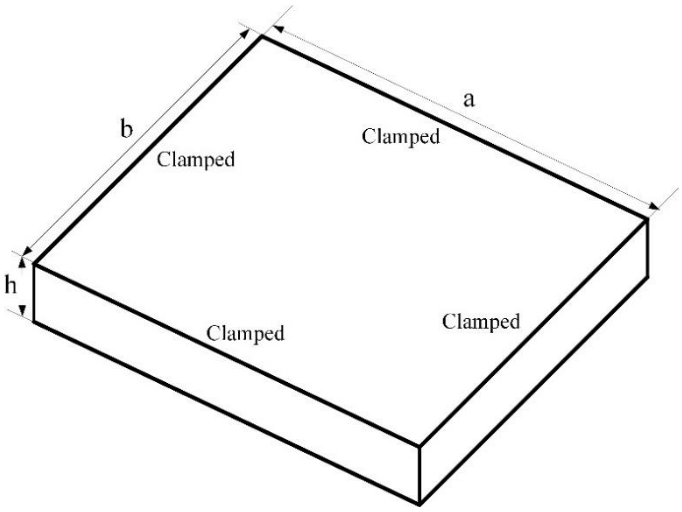

Figure 2. The Illustration of the Isotropic Plates

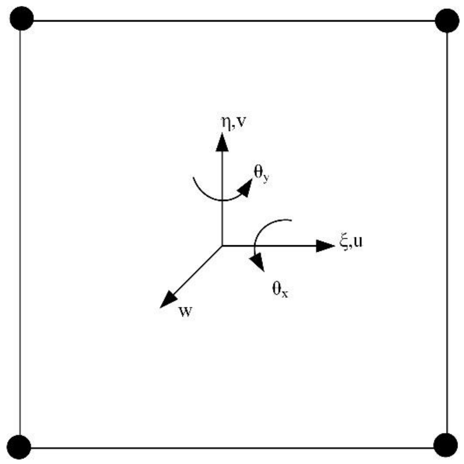

Figure3. The Four-Node Quadrilateral Element

The natural frequencies of those structures have been obtained basically by treating the equation of motion for an undamped conservative system derived by Lagrange equations given in Equation (1) as an eigenvalue problem, which is presented in Equation (2).

$$\boldsymbol{M}\{\ddot{\delta}\} + \boldsymbol{K}\{\delta\} = 0 \tag{1}$$

$$(\boldsymbol{M} - \omega^2 \boldsymbol{K}) = 0 \tag{2}$$

M and ***K*** are the global mass and stiffness matrices, δ is the generalized displacement vector, and ω is the eigenvalues or in other words, natural frequencies of the structure. The global mass and stiffness matrices are evaluated by assembling element mass ($\boldsymbol{m}$) and stiffness ($\boldsymbol{k}$) matrices, which are obtained through kinetic (T) and strain energy (U) relations as follows (Petyt, 2010).

$$T = \{\ddot{\delta}_i\}^{\boldsymbol{T}} \boldsymbol{m} \{\ddot{\delta}_i\} \tag{3}$$

$$U = \{\delta_i\}^{\boldsymbol{T}} \boldsymbol{k} \{\delta_i\} \tag{4}$$

The displacement vector δ_i of the i^{th} node of a finite element considering five degrees of freedom (DOF) per node can be written as (Petyt, 2010)

$$\{\delta_i\} = \{u_i, v_i, w_i, \theta_{x_i}, \theta_{y_i}\} \tag{5}$$

where

$$\begin{aligned} u &= u_0 + z\theta_y \\ v &= v_0 + z\theta_x \\ w &= w_0 \end{aligned} \tag{6}$$

The kinetic and strain relations can also be expressed as (Petyt, 2010)

$$T = \frac{1}{2}\rho \int_A (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dz dA \tag{7}$$

$$U = \frac{1}{2}\left(\int_A \{\epsilon\}^T \boldsymbol{C} \{\epsilon\} dz dA + \int_A \{\tau\}^T \{\gamma\} dz dA\right) \tag{8}$$

where ε and γ are the strain values for normal and shear stress relations, and $\boldsymbol{C}$ is the material matrix that can be evaluated for an isotropic material as (Petyt, 2010)

$$\boldsymbol{C} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ 0 & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G \end{bmatrix} \tag{9}$$

where E is the modulus of elasticity, G is the shear modulus, and ν is the Poisson's ratio of the considered material (Petyt, 2010).

In Equation (8), $\{\tau\}$ represents the shear stress that can be calculated by

$$\{\tau\} = q\boldsymbol{C_s}\{\gamma\} \tag{10}$$

where q is the shear correction factor, $\boldsymbol{C_s}$ is the material matrix for shear stress, which is

$$\boldsymbol{C_s} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} \tag{11}$$

To obtain the global mass and stiffness matrices, it is required to use the relationship between the displacement equations given in Equation (6) and the shape functions of the four-noded quadrilateral element as follows.

$$\begin{gathered} u = \sum_i^4 N_i u\,, \quad v = \sum_i^4 N_i\, v\,, \quad w = \sum_i^4 N_i\, w \\ \theta_x = \sum_i^4 N_i\, \theta_x, \quad \theta_y = \sum_i^4 N_i\, \theta_y \end{gathered} \tag{12}$$

where

$$N_i = \frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta) \tag{13}$$

All of the mathematical expressions indicated above change in accordance with the thickness of the structure. For thin structures, the shear components are negligible, and therefore, the rotations θ_x and θ_y are depended on the slope with respect to the displacement w. In addition, the kinetic and strain energy relations also change due to the assumptions made because of the behavior of the thin structure.

2.1. Random Forest Regressor

Random Forest Regressor (RFR) is an ensemble machine learning technique, which sums decision trees to lower the variance by taking the average of the results (Breiman, 2001). Thanks to this procedure, RFR constitutes a community of low correlated trees. RFR can be briefly explained in two steps. The first one is growing the trees by conducting observations on the sub-data, which is chosen randomly. In the next and final step, the grown trees are going to be assembled to obtain the forest. For regression problems, Random Forest assesses the result by averaging the predictions that are evaluated for each tree. Such an assessment can be expressed mathematically as

$$O(x) = \frac{1}{P}\sum_{i=1}^{P} o_i(x) + e \tag{14}$$

where $o_i(x)$ represents the prediction made for each decision tree, $O(x)$ is the prediction of each observation, P is the number of iterations, and e denotes the error.

3. Results and Discussion

To train and test the RFR model to predict the fundamental frequency of isotropic structures, the dataset comprising isotropic plates having different dimensions and material properties has been obtained. Before proceeding to gather the necessary data, a convergence analysis has been performed to validate the correctness of the FEM model built in ANSYS. For this purpose, a 10 m square plate with 0.05 m thickness has been considered. The same material from an existing literature study (Shojaee, 2012) has been considered for convergence analysis. The structure has been clamped from all edges for all anaylses made within the scope of this study. For the meshing procedure, 500 mm mesh size has been considered for convergence analysis. The meshed structure indicating the element quality has been presented in Figure 4. It is seen from Figure 4 that almost perfect mesh quality (0.999) has been obtained. Table 1 shows the convergence results considering the first three non-dimensional natural frequencies (ω_{nd}), which can be calculated as follows.

$$\begin{gathered} \omega_{nd} = \left(\frac{\omega^2 a^4 \rho h}{\phi}\right)^{\frac{1}{4}} \\ \phi = \frac{Eh^3}{12h(1-\nu^2)} \end{gathered} \tag{15}$$

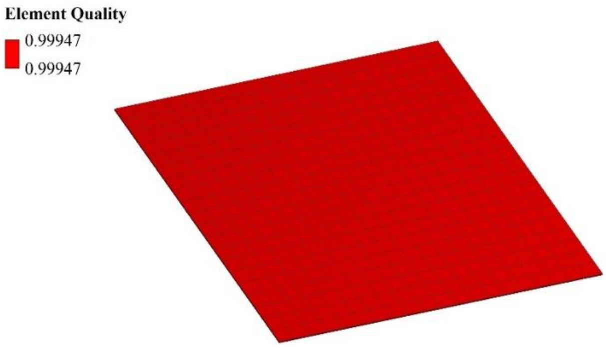

Figure 4. Meshed Structure and Element Quality

Table 1. Convergence Analysis Results

Natural Frequency	Present Study	Shojaee et al., 2012	Bui and Nguyen, 2011	Cheung et al., 1988
ω_{nd1}	2.390	2.207	2.215	2.219
ω_{nd2}	3.410	3.725	3.704	3.725
ω_{nd3}	3.410	3.725	3.706	3.725

As seen in Table 1, the considered FEM model is in good agreement with the results of existing literature. For the rest of the analyses, the isotropic plates have been modeled in the same manner. For data acquisition, the fully clamped isotropic plates have been designed considering the dimension parameters given in Table 2 and material properties given in Table 3. Since the dimensions have varied, it has been considered to select the same mesh size – plate dimension ratio. Therefore, a mesh size of 5 mm has been considered for entire analyses.

Table 2. Dimension Parameters

Dimension	Initial Value (mm)	Interval (mm)	Final Value (mm)
a	100	100	500
b	100	100	500
h	5, 10, 20, 30, 40, 50 mm		

Table 3. Material Parameters

Material	Modulus of Elasticity (GPa)	Shear Modulus (GPa)	Density (kg/m^3)	Poisson's Ratio
Structural Steel	200	76.92	8000	0.3
Aernet 100	194	74.60	7890	0.3
Al 7108	71	26.70	2770	0.33
Al 2024	73.1	27.50	2780	0.33

Considering the parameters given in Tables 2 and 3, a total of 364 analyses have been conducted. Therefore, the dataset has included 364 instances. To train and test the proposed technique, the two dimension related and one material input has been considered. The dimensional inputs are the aspect ratio (a/b) and the shortest length-thickness ratio (b/h), respectively. As for the input that represents the material properties the value of φ (see Equation (14)) has been taken into account. All analyses have been conducted via Python 3.9. The analyses have been conducted on a computer with an Intel i5-8300H Dual-Core 2.30GHz processor and 16 GB RAM. Before proceeding with the training process, a hyperparameter tuning procedure has been conducted regarding the number of decision trees for the Random Forest Regressor.

Figures 5 and 6 show the convergence diagram of the Random Forest Regressor with respect to the correlation factor (R^2) and mean square error (MSE), respectively.

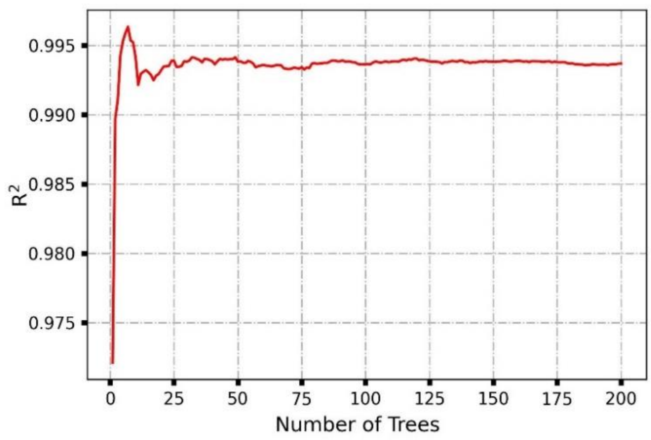

Figure 5. Convergence Diagram of Number of Trees and Correlation Factor

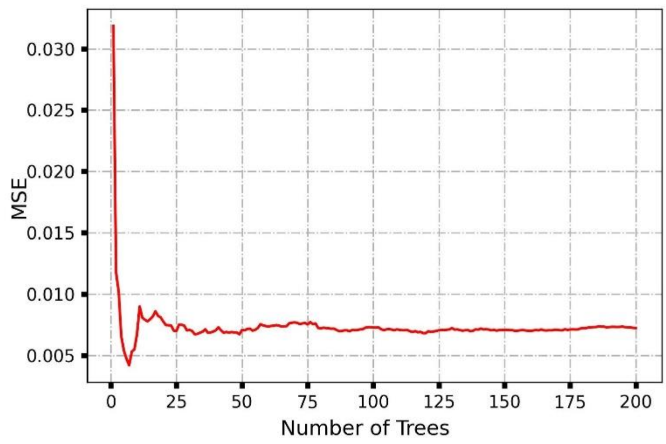

Figure 6. Convergence Diagram of Number of Trees and Mean Square Error

As seen from Figures 5 and 6, the performance of the Random Forest Regressor (RFR) has been optimized after approximately 100 trees. Therefore, the prediction model has been built by using 100 trees. To train and test the proposed approach, the train-test set ratio was chosen as 80-20%, which is the default setting.

Table 4 presents the performance metrics of RFR in predicting the fundamental natural frequency of isotropic plates. As seen from the results, the R^2 score is almost perfect (0.993) and the mean square error (MSE), root mean square error (RMSE), and mean absolute error (MAE) are quite low. These results indicate that the proposed approach is not only suitable for such a problem due to its high correlation factor score, but also it has predicted the fundamental frequency values with high accuracy considering the low error rates.

Table 4. Performance Metrics of the RFR Technique

Phase	R^2	MSE	RMSE	MAE
Training	0.9991	0.0008	0.0292	0.0155
Test	0.9936	0.0073	0.0852	0.0501

Since one of the most common machine learning algorithms used in this field is Artificial Neural Network (ANN), the performance of the Random Forest Regressor (RFR) has been compared with ANN considering a single hidden layer with 100 neurons, and Tanh, Sigmoid, and Rectified Linear Unit (ReLU) activation functions. The number of iterations for ANN is set automatically. Therefore, the algorithm has stopped iterating as convergence has been obtained. Table 5 gives the comparison results considering the test set.

Table 5. Comparison of the RFR Technique with Different ANN Models

Phase	R^2	MSE	RMSE	MAE	Time (s)
RFR	0.9936	0.0073	0.0852	0.0501	0.127
ANN (Tanh)	0.8939	0.1212	0.3481	0.1995	4.935
ANN (ReLU)	0.8918	0.1235	0.3515	0.2182	0.905
ANN (Sigmoid)	0.7842	0.2466	0.4965	0.3022	5.972

It is seen from Table 5 that the RFR technique outperforms ANN in every aspect. RFR is not only the most correlated technique, but also it is the fastest and the most accurate method when compared with ANN.

Table 6 presents some numerical results indicating the prediction performance of the RFR technique considering the non-dimensional fundamental frequency domain.

Table 6. Prediction Performance of the RFR Technique for Randomly Selected Isotropic Plates

Material	a/b	b/h	Actual	Predicted	Difference (%)
Structural Steel	2	4	2.361	2.339	0.90
Structural Steel	3	2	2.264	2.265	0.08
Aernet 100	5	20	4.197	4.181	0.38
Al 7108	1	40	2.384	2.382	0.10
Al 7108	1	80	2.391	2.390	0.04
Al 2024	2	10	2.691	2.654	1.38

As seen in Table 6, the proposed technique successfully predicted the fundamental natural frequency of isotropic plates no matter which type of material and dimensions have been considered. Comparing all the predicted results with the test set indicated that the average prediction accuracy is 99.12%. Therefore, it can be concluded that the Random Forest Regressor is suitable to predict the fundamental natural frequency of isotropic plates as the input values mentioned above are considered. Another key result is the elapsed time during the execution of the proposed approach. It has been calculated that the training and testing procedure has taken only 0.127 seconds. Compared with the traditional methods and ANN-based techniques, the proposed approach is not only accurate but also significantly fast and therefore, reduces the computational cost required to conduct such an analysis.

4. Conclusions

This study proposes an alternative intelligent approach, which employs the Random Forest Regressor to predict the fundamental natural frequency of isotropic plates having different materials and dimensions. According to the numerical analysis, the following conclusions have been drawn.

- The Random Forest Regressor is found to be suitable for predicting the fundamental frequencies of isotropic plates due to its high correlation factor (0.994) and small mean square error, root mean square error, and mean absolute error.
- In addition to its suitability, the Random Forest Regressor predicted the fundamental frequencies of isotropic plates with an average accuracy of 99.88% regardless of the material and structural dimensions. This makes the algorithm not only accurate but also robust and responsive to the possible changes in the properties of the structure.
- Since training and testing the proposed approach has taken only 0.127 seconds, it can be concluded that employing Random Forest Regressor reduces the computational cost that is required to conduct free vibration analyses.
- RFR outperforms ANN in not only in terms of the suitability for predicting the fundamental natural frequency of an isotropic plate but also in terms of execution speed and prediction accuracy.
- Due to the positive aspects of the Random Forest Regressor, it may be utilized for other engineering analyses as well.
- Future works may examine the performance of the proposed method considering a wider range of isotropic and composite materials, structural dimensions, and boundary conditions.

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