

European Journal of Science and Technology Special Issue 28, pp. 675-683, November 2021 Copyright © 2021 EJOSAT **Research Article** 

# Impact of Fe<sub>3</sub>O<sub>4</sub>/water on Natural Convection in Square Enclosure

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### Abstract

Natural convection characteristics in a 2D square enclosure (10 mm x 10 mm) have been investigated detailly under natural convection conditions ( $10^3 \le Gr \le 10^5$ ). A model has been developed to analyze the dispersed nanoparticle effect on natural convection performance in an enclosure. The left vertical wall is maintained at a high temperature, while the right vertical wall is kept at a low temperature, whereas horizontal walls are assumed to be insulated. Fe<sub>3</sub>O<sub>4</sub>/water ( $0 \le \varphi \le 1.0$ ) nanofluid has been utilized to analyze the convection enhancement in the enclosure. To elucidate flow characteristics and heat transfer performance of Fe<sub>3</sub>O<sub>4</sub>/water nanofluid, temperature, velocity streamline, and vorticity contours have been taken place. It is concluded that nanoparticle dispersion in base fluid enhances the natural convection heat transfer. Also, Grashof number has an important role in heat transfer mechanism.

Keywords: Natural convection, enclosure, Grashof number, Fe3O4/water, nanofluid, CFD

# Kare Kutu İçinde Fe<sub>3</sub>O<sub>4</sub>/Su'yun Doğal Konveksiyona Etkisi

### Öz

Doğal konveksiyon altında  $(10^3 \le Gr \le 10^5)$  2D kutu içindeki (10 mm x 10 mm) Fe<sub>3</sub>O<sub>4</sub>/water karakteristiği detaylı olarak incelenmiştir. Modelimiz bir kutu içinde doğal konveksiyonun nano akışkanın varlığı durumunda analiz etmek için geliştirilmiştir. Sol duvar sıcak duvar, sağ duvar soğuk sıcaklıkta sabit tutulurken, yatay duvarlar yalıtımlı olarak tutulmuştur. Fe<sub>3</sub>O<sub>4</sub>/water ( $0 \le \varphi \le 1.0$ ) nanoakışkanı kutu içindeki konvektif iyileştirmeyi analiz etmek için kullanılmıştır. Fe<sub>3</sub>O<sub>4</sub>/water nanoakışkanının akış karakteristiğimi ve ısı transferi performansı analiz etmek için sıcaklık eş eğrisi, hız akış çizgisi ve girdap akış çizgisi oluşturulmuştur. Sonuç olarak, baz akışkan içine nanoakışkan karıştırılması doğal konveksiyonu iyileştirmektedir. Ayrıca Grashof sayısı ısı transferi mekanizmasında önemli bir role sahiptir.

Anahtar Kelimeler: Doğal konveksiyon, kutu, Grashof sayısı, Fe<sub>3</sub>O<sub>4</sub>/su, nanofluid, HAD

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### 1. Introduction

Natural convection occurs due to temperature gradient and is mostly encountered as a type of heat transfer mechanism in the industry. Buoyancy-driven natural convection takes place in reactor chambers of nuclear power plants, microelectronics cooling [1], solar air heaters [2], large-scale meteorology [3,4]. Kim et al. [5] investigated the natural convection in a square enclosure numerically using immersed boundary method. The cylinder has been heated in the enclosure and natural convection took place under  $10^3 \le Ra \le 10^7$  conditions. It is seen that vortex generation in the enclosure and the height of the cylinder are related. Bhattacharya and Basak [6] studied the natural convection in a square enclosure with a non-isothermal hot bottom wall, isothermal sidewalls, and insulated top wall. Under the conditions of  $10^2 \le Ra \le 10^6$  steady-state numerical analyses have been accomplished, and flow structures have been visualized. It is understood that increasing Rayleigh number (Ra) has a greater effect on the convective heat transfer rate. Hadidi et al. [7] investigated the natural convection in an inclined square enclosure. The effects of the thickness of a porous layer, Rayleigh number, buoyancy ratio, Lewis number, cavity inclination angle, and the thermal conductivity ratio on the heat transfer performance have been examined in detail. It is concluded that Rayleigh number, cavity inclination angle, and thermal conductivity have a stronger effect on heat transfer rate in comparison with other parameters used in this study. Also, Liang et al. [8] studied the inclined square enclosure between -90° and 90° with 15° intervals under the conditions of  $10^3 \le Ra \le 10^7$ . It is found out that stronger vortexes occurred when heated wall moved from top to bottom wall. Subhani [9] researched the natural convection in a square enclosure with a hot cylinder in the center of the enclosure and cold winglets around it. Winglets increased vorticity in the enclosure and caused higher heat transfer. The best enhancement in heat transfer has been obtained with the winglets at 45° degrees. Sheikholeslami and Shamlooei [10] carried out a study to understand behaviour of Fe<sub>3</sub>O<sub>4</sub>/water nanofluid under natural convection with thermal radiation. Uniform and constant heat flux has been applied to inner wall. The working range of this study is  $10^3 \le Ra \le 10^5$ . Control volume based finite element method has been utilized to perform a study. It is concluded that the usage of thermal radiation has weaker effect in natural convection by using Fe<sub>3</sub>O<sub>4</sub>/water nanofluid. Dogonchi and Hashim [11] studied a wavy circular shape enclosure under natural convection to determine the effect of several parameters such as Rayleigh number  $(10^3 \le Ra \le 10^5)$ , radiation parameter ( $0 \le Rd \le 0.3$ ), the shape factor of nanoparticles (m=3, 4.8, 5.7), the volumetric concentration of Fe<sub>3</sub>O<sub>4</sub>/water (2%, 4%). It is determined that the usage of a higher volumetric concentration with a higher Rayleigh number presents a higher convective heat transfer rate. Sheikholeslami et al. [12] performed natural convection with Coulomb forces to detect the effect of Coulomb force in existence of thermal radiation by using Fe<sub>3</sub>O<sub>4</sub>/water nanofluid. Likewise, other researchers mentioned, in this study determined that the presence of thermal radiation has weaker effect on convective heat transfer rate under natural convection [10,11]. Moraveji and Hejazian [13] carried out an investigation heat transfer effect of Fe<sub>3</sub>O<sub>4</sub>/water nanofluid in rectangular cavity under natural convection. To determine the heat transfer effect, the range of Rayleigh number and volumetric concentration of nanoparticles has been applied as  $10^3 \le Ra \le 10^5$ ,  $0 \le \varphi \le 14\%$ , respectively. While the magnitude of Rayleigh number

presents positive effect in heat transfer rate, addition of nanoparticles to base fluid establishes a negative effect for this study.

Even though natural convection in a square enclosure has been widely investigated in the literature, using nanofluid as working fluid has not been studied detailly and more data presented in this manuscript is going to provide detailed insight on the heat transfer enhancement mechanisms in the enclosure to the literature.

# 2. Material and Method

# 2.1. Problem Description and Mathematical Formulas

In this study, a 2D enclosure has been designed to investigate the buoyancy effect of Fe<sub>3</sub>O<sub>4</sub>/water under natural convection. The dimensions and boundary conditions of the calculation domain can be seen in Fig. 1. As can be seen in Fig. 1, while the left wall  $(T_H)$  is kept at higher temperature condition, which is changing with Grashof (Gr) number, the right wall is also kept at constant lower temperature condition ( $T_L$ =298 K). Also, the top and bottom walls are in an insulated wall condition, which leads to an impermeable situation. The working fluid has been considered Newtonian and incompressible. The nanoparticle has been taken in uniform sizes and well dispersed in the base fluid. So, it is noted that the nanofluid and water have been assumed at the same velocity and in thermal equilibrium. Since the nanoparticle dispersion in base water disperses homogeneously, the computational domain has been modelled using the single-phase approach [14]. Also, the thermophysical properties of both water and Fe<sub>3</sub>O<sub>4</sub>/water nanofluid have been taken as constant, and the thermal expansion coefficient has been modelled with Boussinesq approximation.



Fig. 1. Schematic of study

Regarding these phenomena, the boundary and initial conditions have been adjusted as below: u = 0, v = 0, T = 0 @  $0 \le x \le L, 0 \le y \le H, t = 0$ 

 $u = 0, v = 0, \frac{\partial T}{\partial y} = 0 \quad @ \quad 0 \le x \le L, \quad y = 0 \text{ and } y = H, \quad 0 \le t \le \tau$  $u = 0, v = 0, T = T_H \quad @ \quad x = 0, \quad 0 \le y \le H, \quad 0 \le t \le \tau$  $u = 0, v = 0, T = T_L \quad @ \quad x = L, \quad 0 \le y \le H, \quad 0 \le t \le \tau$ 

By using these boundary conditions above-mentioned, the governing equations have been calculated in dimensional form as following: Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}x\frac{\partial P}{\partial x} + V_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
(2)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}x\frac{\partial P}{\partial v} + v_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \beta_{nf}g\left(T - T_c\right)$$
(3)

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$
(4)

Also, the nanofluid thermo-physical properties can be calculated as below. Also, thermo-physical properties of nanoparticle and base fluid are given in Table 1. Density [15]:

$$\rho_{nf} = \varphi \rho_{np} + (1 - \varphi) \rho_{bf}$$
(5)

Specific heat [16]:

$$C_{p,nf} = \frac{\varphi(\rho C_p)_{np} + (1 - \varphi)(\rho C_p)_{bf}}{\rho_{nf}}$$
(6)

Viscosity [17]:

$$\mu_{nf} = \mu_{bf} \left( 123\varphi^2 + 7.3\varphi + 1 \right) \tag{7}$$

Thermal conductivity [18]:

$$k_{nf} = k_{bf} \frac{\left(k_{np} + 2k_{bf}\right) - 2\varphi\left(k_{bf} - k_{np}\right)}{\left(k_{np} + 2k_{bf}\right) + \varphi\left(k_{bf} - k_{np}\right)}$$
(8)

Thermal expansion coefficient [19]:

$$(\rho\beta)_{nf} = (1 - \varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_{np}$$
<sup>(9)</sup>

Grashof number can be calculated as following [20]:

$$Gr = \frac{g\beta_f \Delta TH^3}{v_f^2} \tag{10}$$

where  $\Delta T$  is the temperature difference between hot wall and cold wall. *H* is the height of the enclosure. *v* is kinematic viscosity.

**Table 1**. Thermophysical properties of medium [21].

Property	Water	Fe <sub>3</sub> O <sub>4</sub>	
$C_p$	4179	670	
ρ	997.1	5200	
k	0.6	6	
β	$2.1x10^{-4}$	$1.3x10^{-5}$	

Then, the local Nusselt and average Nusselt numbers can be calculated with Eq. (11) and Eq. (13).

$$Nu = \frac{Q}{Q_{cond,fluid}} = -\frac{\left(k_{eff}\right)_{stagnant}}{k_f} \frac{\partial \theta}{\partial X}$$
(11)

where,

$$Q = -\left(k_{eff}\right)_{stagnant} A \frac{\partial T}{\partial x}$$
(12)

$$\overline{Nu} = \int_{0}^{L} Nu(y) dy / L$$
(13)

#### 2.2. Mesh study and Validation

The present study has been calculated by using ANSYS Fluent *v2020R2*. The computational study has been applied by using SIMPLE pressure-velocity coupling model with second-order upwind scheme. All convergency criteria have been reached  $10^{-8}$  approximation value. Also, Least-Squares Gradient scheme has been utilized in spatial discretization for the gradient. The finite volume method (*FVM*) has been utilized for all computations. All these procedures have been carried out in both water and nanofluid steps.

Before starting analyses, the mesh study should be performed to obtain better results in the numerical study. Also, all mesh studies have been applied at highest *Gr* number. As can be seen in Table 2, in order to decide the optimum mesh structure, several mesh studies have been done. From the calculations, it is concluded that there is no significant difference between M4 and M5 in terms of average Nusselt number so M4 has been selected as the optimum mesh structure. The typical mesh structure is presented in Fig. 2.



Fig. 2. Mesh structure for the enclosure.

**Table 2**. Mesh study for  $Gr=10^5$ 

Mesh No	Mesh	Avg. Nu (Present study)
M1	11 <i>x</i> 11	6.158
M2	31 <i>x</i> 31	7.121
M3	41 <i>x</i> 41	7.982
M4	61 <i>x</i> 61	8.256
M5	71 <i>x</i> 71	8.467
M6	81 <i>x</i> 81	7.992

For the second step, the numerical domain should be validated with a study published in the literature in the same boundary conditions. For this purpose, numerical results have

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been validated with the data obtained from Khanafer et al. [20]. Comparison of streamlines between the present study and that of Khanafer et al. is given in Fig. 3. Also, numerical values have been compared Table 3. It is concluded from Table 3 that the maximum deviation between the present study and data of Khanafer et al. [20] is 8.06%, so the present computational domain is under acceptable conditions for further studies.

### Present Study ( $\varphi$ =8%) Data of Khanafer ( $\varphi$ =8%)



Fig. 3. Comparison of streamlines between the present study and that of Khanafer et al. [20] ( $\phi$ =8% for Cu/water).

Table 5. Validation of present study with the interature [20].									
	Present Study			Correlation of Khanafer et al. [20]			Deviation %		
φ	$Gr = 10^{3}$	<i>Gr</i> =10 <sup>4</sup>	<i>Gr</i> =10 <sup>5</sup>	<i>Gr</i> =10 <sup>3</sup>	<i>Gr</i> =10 <sup>4</sup>	<i>Gr</i> =10 <sup>5</sup>	Gr=10 <sup>3</sup>	Gr=10 <sup>4</sup>	Gr=10 <sup>5</sup>
%									
0	1.9369	4.1047	8.256	1.9768	4.0574	8.3282	2.0573	1.1514	0.8746
4	2.09576	4.41359	8.8407	2.1141	4.3394	8.9070	0.8765	1.6804	0.7500
8	2.33518	4.87389	9.72715	2.2673	4.6539	9.5526	2.90	4.5129	1.7947
12	2.5779	5.33935	10.6309	2.4272	4.9820	10.226	5.84	6.6923	3.8087
16	2.80433	5.7725	11.4789	2.5915	5.3192	10.918	7.58	7.851	4.8848
20	3.0012	6.14829	12.2193	2.7592	5.6634	11.625	8.06	7.8861	4.8665

## Table 3. Validation of present study with the literature [20]

## 3. Results and Discussion

In this study, natural convection inside a square enclosure with a hot left wall, a cold right wall, and adiabatic top and bottom walls have been examined.

Table 4 depicts the average Nusselt number enhancements using  $Fe_3O_4$ /water nanofluid. Under natural convection condition, the effect of volumetric concentrations of nanofluid on heat transfer performance is weaker. 1.0% volumetric concentrations of nanofluid offers up to 0.74% enhancement, whereas enhancements of lower volumetric concentrations compared to distilled water are indistinctive. Enhancement obtained by using 1.0% volumetric concentrations of nanofluid in average Nusselt number takes place because of increment in thermal conductivity of working fluid. Also, higher *Gr* causes higher average Nusselt number for all cases.

Fig. 4 shows the local Nusselt number across the length of the hot wall using 1.0 vol.% Fe<sub>3</sub>O<sub>4</sub>/water nanofluid. The convection heat transfer rates are higher at lower portions of the hot wall. As the temperature of the fluid increases along the vertical length of the hot wall, the temperature gradient decreases and the heat transfer rate also diminishes. As *Gr* increases, the heat transfer rates increase.

Fig. 5 shows the local temperature across the axial length between the hot and cold walls using distilled water and  $Fe_3O_4$ /water nanofluids as a working fluid.

**Table 4.** Average Nusselt number variations for different Grashof numbers and nanoparticle volume fraction (NPVF) for present study (Fe O (varter))

φ (%)	<i>Gr</i> =10 <sup>3</sup>	<i>Gr</i> =10 <sup>4</sup>	<i>Gr</i> =10 <sup>5</sup>	Enhancement (%) for <i>Gr</i> =10 <sup>3</sup>	Enhancement (%) for <i>Gr</i> =10 <sup>4</sup>	Enhancement (%) for <i>Gr</i> =10 <sup>5</sup>
0	1.9369	4.1047	8.256	-	-	-
0.01	1.93939	4.11302	8.26565	0.1286	0.2027	0.1169
0.1	1.93992	4.11391	8.26735	0.1560	0.2244	0.1375
1	1.95099	4.13506	8.30822	0.7275	0.7396	0.6325



Fig. 4. Local Nusselt number distribution on the hot wall for  $\varphi = 1\%$ .



**Fig. 5.** Local temperature distribution along the axial length for  $Gr=10^5$ 

As expected, nanofluid with higher volumetric concentrations offers higher temperature fluctuations. As the heat transfer occurs from the hot wall, temperatures at the axial lengths where it is closer to the hot wall are higher, whereas temperatures where it is closer to the cold wall are lower. Fig. 6 depicts the vertical velocity distribution along the axial length between the hot and cold walls. As the temperature gradient between the hot and cold walls increases, vertical velocity increases due to enhanced buoyancy-driven forces with higher Gr.



Fig. 6. Local v-Velocity distribution along the axial length for  $\phi=1\%$ .

Fig. 7 shows the temperature contours across the enclosure. Firstly, it is concluded that higher Gr contributes positively to the natural convection in the enclosure and higher volumetric concentrations are seen in a higher temperature gradient. Temperature contours of lower volumetric concentrations are similar as this situation. It shows why researchers study with higher volumetric concentrations of nanofluids in natural convection problems [13]. Fig. 8 represents the velocity contours across the enclosure. As the volumetric concentration of the nanofluid or Gr increase, velocity magnitudes increase because of higher buoyancy-driven forces. This mechanism is also reflected in vorticity contours illustrated in Fig. 9. Higher Gr and volumetric concentration of the nanofluid cause higher vorticity magnitudes.



Fig. 7. Temperature contours for all cases.



Fig. 8. Velocity streamlines for all cases.



Fig. 9. Vorticity streamlines for all cases.

# 4. Conclusions and Recommendations

The natural convection heat transfer characteristics of nanofluid flow in a square enclosure have been examined under the conditions of  $10^3 \le Gr \le 10^5$ .

- Temperature distribution across the enclosure increaes with increasing volumetric concentrations of nanofluid.
- Higher *Gr* number and volumetric concentrations of nanofluid cause higher local Nusselt number, vertical velocity, and vorticity magnitudes.
- Higher heat transfer rates are obtained at the lower portions of the hot wall due to the greater temperature difference between the hot wall and the medium.
- As *Gr* number increases, flow structures develop in enclosure.

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