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Research Article

Mode-Matching Analysis of Acoustic Wave Propagation along a Rigid Coaxial Pipe with an Internal Impedance Loading

Hülya Öztürk[1*]

[1*] Gebze Technical University, Department of Mathematics, Kocaeli, Turkey, (ORCID: 0000-0002-7814-718X), h.ozturk@gtu.edu.tr

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Abstract

Mathematical analysis of the acoustic wave propagation along the coaxial waveguide, whose right part of the outer wall is impedance-coated, is considered. By determining the boundary conditions corresponding to the relevant geometry, the boundary value problem is solved by applying the Mode-Matching technique. At the end of the analysis, numerical computations are carried out to illustrate the affect of some parameters. The results demonstrate the significance of the absorbing lining in reducing the unwanted noise.

Keywords: Mode-Matching, Coaxial Waveguide, Acoustics, Absorbent Lining.

Dış Duvarının Yarısı Akustik Yutucu Malzeme ile Kaplı Rijit Koaksiyel Bir Boruda Akustik Dalgaların Yayılımının Mod-Uydurma ile Analizi

Öz

Dış duvarının yarısı akustik yutucu malzeme ile kaplı koaksiyel bir boruda akustik dalgaların yayılımının analizi yapılmıştır. İlgili geometriye karşılık gelen sınır koşulları belirlenmiş ve bu koşullardan faydalanılarak mod uydurma tekniği ile sınır değer problemi çözülmüştür. Analizin sonunda ise, bazı parametrelerin ses yayılımına etkisi nümerik sonuçlar ile sunulmuştur. Akustik yutucu malzeme sayesinde istenmeyen gürültünün azaltılabileceği görülmüştür.

Anahtar Kelimeler: Mod-Uydurma, Koaksiyel Dalga Kılavuzu, Akustik, Yutucu Malzeme.

* Corresponding Author: h.ozturk@gtu.edu.tr

1. Introduction

The propagation of sound waves along ducts has been the focus of attention by researchers for many years due to its importance in noise reduction. There are a lot of analytical and numerical methods have been studied by many scientists [1-8]. One of the most effective methods is the Mode-Matching or eigenfunction expansion technique, which is based on representing the unknown fields in terms of an infinite sum of orthogonal functions in the individual regions and then matching them across the boundaries between these regions. This method has been applied in many papers [9-13] and give accurate results.

The present work deals with the mode-matching analysis of the coaxial waveguide problem. The critical situation here is that part of the outer wall is loaded with impedance. The aim is to show the effect of partial absorbing surface on sound propagation. For this purpose, the geometry is divided into two region and then the potentials satisfying boundary conditions in each region are established in terms of their normal modes by using seperation of variables. The solution leads to infinite set of linear equations which are solved numerically. Finally, graphs are displayed for different parameters. It is observed that, the absorbing surface. has a significant effect on reducing harmful and undesired noise.

2. Formulation of the Problem

Assume an acoustic wave propagating in a coaxial duct shown Figure 1. The material properties of the right part of the outer wall is simulated by a specific admittance of $\eta = \rho_0 c / Z$ where c is the velocity of sound and Z is the liner impedance.

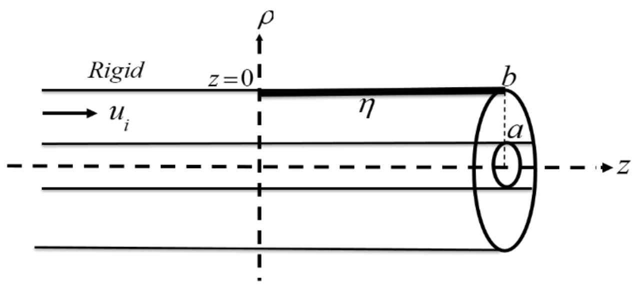

Fig. 1 Geometry of the problem.

Let an incident acoustic wave with time dependence $\exp(-i\omega t)$ where ω is the angular frequency is given by

$$u_i(z) = e^{ikz} \tag{1}$$

Here $k = \omega / c$ is the wave number. As it well known, the total field

$$u_T(\rho, z) = \begin{cases} u_1(\rho,z) + u_i(z) & ; \ a<\rho<b \ ; \ z\in(-\infty,0) \\ u_2(\rho,z) & ; \ a<\rho<b \ ; \ z\in(0,\infty) \end{cases} \tag{2}$$

satisfies the Helmholtz equation

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial z^2} + k^2\right] u_j(\rho,z) = 0\ , \quad j=1,2 \tag{3}$$

and the following boundary and continuity conditions:

$$\frac{\partial}{\partial\rho} u_T(a,z) = 0 \quad , \quad -\infty < z < \infty \tag{4}$$

$$\frac{\partial}{\partial\rho} u_1(\mathrm{b},z) = 0 \quad , \quad -\infty < z < 0 \tag{5}$$

$$\left(ik\eta - \frac{\partial}{\partial\rho}\right) u_2(b,z) = 0 \quad , \quad 0 < z < \infty \tag{6}$$

$$\frac{\partial}{\partial z} u_1(\rho,0) + \frac{\partial}{\partial z} u_i(0) = \frac{\partial}{\partial z} u_2(\rho,0) \tag{7}$$

$$u_1(\rho,0) + u_i(0) = u_2(\rho,0) \tag{8}$$

The functions $u_j(\rho,z),\ j=1,2$ appearing in (3) are the unknown fields.

Solutions of Helmholtz equation (3) obtained by means of separation of variables method, with boundary conditions (4-6) are given by

$$u_1(\rho,z) = \sum_{n=1}^{\infty} a_n e^{-i\alpha_n z} \varphi_n(\rho) \tag{9}$$

$$u_2(\rho,z) = \sum_{n=1}^{\infty} b_n e^{i\chi_n z} \psi_n(\rho) \tag{10}$$

where a_n and b_n are the magnitudes of the reflected and transmitted duct mode, respectively. The eigenvalues α_n and χ_n are the solutions of

$$K_n\left[J_1(K_n a) - \frac{J_1(K_n b)}{Y_1(K_n b)} Y_1(K_n a)\right] = 0,\ n=1,2,... \tag{11}$$

$$\xi_n\left[J_1(\xi_n a) - \frac{\left[ik\eta J_0(\xi_n b) + \xi_n J_1(\xi_n b)\right]}{\left[ik\eta Y_0(\xi_n b) + \xi_n Y_1(\xi_n b)\right]} Y_1(\xi_n a)\right] = 0,\ n=1,2,... \tag{12}$$

with

$$\alpha_n = \sqrt{k^2 - K_n{}^2},\ n=1,2,... \tag{13}$$

$$\chi_n = \sqrt{k^2 - \xi_n{}^2},\ n=1,2,... \tag{14}$$

We define eigenfunctions as

$$\varphi_n(\rho) = \left[J_0(K_n\rho) - \frac{J_1(K_n b)}{Y_1(K_n b)} Y_0(K_n\rho)\right] \tag{15}$$

$$\psi_n(\rho) = \left[J_0(\xi_n\rho) - \frac{\left[ik\eta J_0(\xi_n b) + \xi_n J_1(\xi_n b)\right]}{\left[ik\eta Y_0(\xi_n b) + \xi_n Y_1(\xi_n b)\right]} Y_0(\xi_n\rho)\right] \tag{16}$$

Here J_n and Y_n $(n=0,1)$ stand for Bessel and Neumann functions.

In order to obtain the solution let us apply continuity conditions (7) and (8):

$$-\sum_{m=1}^{\infty} a_m \alpha_m \varphi_m(\rho) + k = \sum_{n=1}^{\infty} \chi_n b_n \psi_n(\rho) \tag{17}$$

$$\sum_{m=1}^{\infty} a_m \varphi_m(\rho) + 1 = \sum_{n=1}^{\infty} b_n \psi_n(\rho) \tag{18}$$

Considering now the integral of (17) and (18) after multiplying the both sides with $\rho\psi_n(\rho)$ and using the following orthogonality relation

$$\int_a^b \psi_m(\rho)\psi_n(\rho)\rho d\rho = \frac{4ik\eta}{\left[ik\eta Y_0(\xi_n b) + \xi_n Y_1(\xi_n b)\right]\pi^2 b K_m Y_1(K_m b)(K_m^2 - \xi_n^2)} \tag{19}$$

give

$$-\frac{2}{\pi b} S(b,k,\eta,\xi_n) \sum_{m=1}^{\infty} \frac{a_m \alpha_m}{K_m Y_1(K_m b)\left(K_m^2 - \xi_n^2\right)} + \frac{k}{\xi_n^2} S(b,k,\eta,\xi_n) = \chi_n P_n b_n \tag{20}$$

$$\frac{2}{\pi b} S(b,k,\eta,\xi_n) \sum_{m=1}^{\infty} \frac{a_m}{K_m Y_1(K_m b)\left(K_m^2 - \xi_n^2\right)} + \frac{1}{\xi_n^2} S(b,k,\eta,\xi_n) = P_n b_n \tag{21}$$

where

$$S(b,k,\eta,\xi_n) = \frac{2ik\eta}{\pi\left[ik\eta Y_0(\xi_n b) + \xi_n Y_1(\xi_n b)\right]}, \quad P_n = \int_a^b \psi_n^2(\rho)\rho d\rho \tag{22}$$

As a result, constructing the system of linear equations properly the coefficients a_m and b_m can be determined numerically

3. Graphical Results

This section contains graphs of the transmission loss, written

$$TL = -20\log_{10}|T_0|$$

for varied values of η, a and b.

Figures 2 and 3 show the modules of the reflection and transmission fields with respect to truncation number for infinite system (N). It can be seen that amplitudes become insensitive for $N > 3$. Hence N is chosen 10 in our computations.

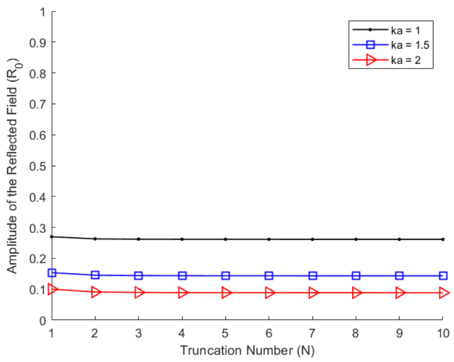

Fig. 2 Reflection field amplitude versus the truncation number N.

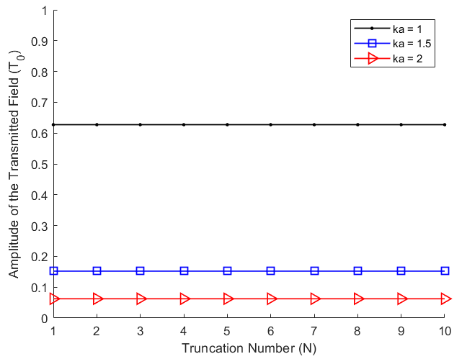

Fig. 3 Transmission field amplitude versus the truncation number N.

Figures 4 and 5 show the variation of transmission loss for different values of surface impedance (η). It can be easily seen that it is possible to decrease the transmission loss for different values η.

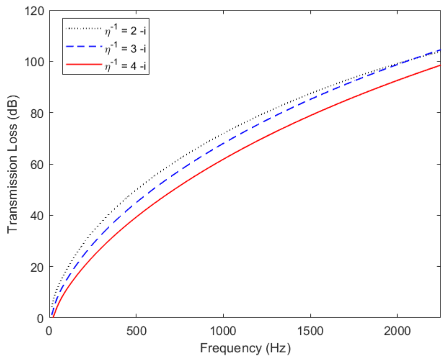

Fig. 4 Transmission loss for different values of real part of η^{-1}.

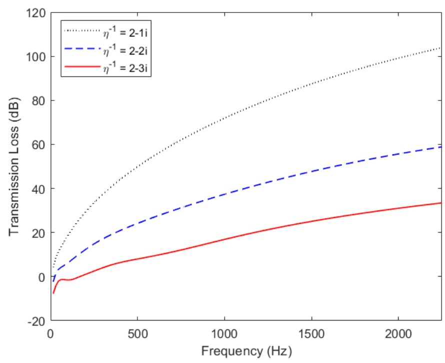

Fig. 5 Transmission loss for different values of imaginary part of η^{-1}.

Figures 6 and 7 depict the transmission loss variation for different values of duct radii a and b. These graphs were obtained when η^{-1}=1-i. One can see that the when a and b increase, the transmission loss decreases.

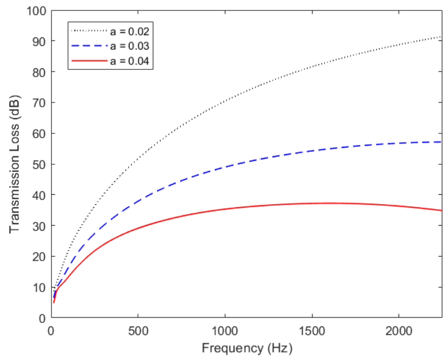

Fig. 6 Transmission loss for different values of duct radius a.

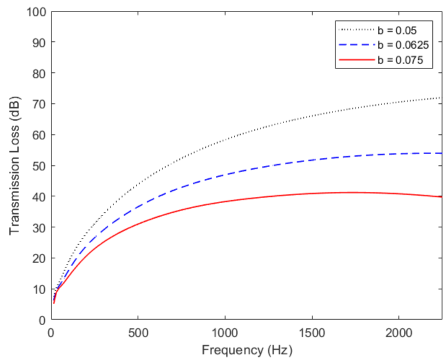

Fig. 7 Transmission loss for different values of duct radius b.

Finally, Figure 8 shows the contribution of the lining on noise reduction. As expected, the transmission field decreases with the effect of surface impedance.

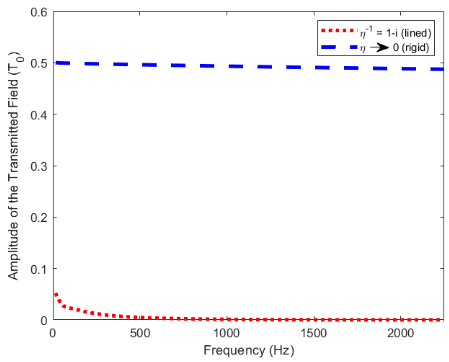

Fig. 8 Transmission field for rigid-lined duct.

5. Conclusion

In this study, the effect of partial lining to the sound transmission is studied through the Mode-Matching technique. Some numerical results are presented for some particular parameters The results show that it is possible to reduce sound by changing the parameters. In addition, the results are compared with the rigid duct. It is observed that the use of lining provides significant sound absorption.

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