Avrupa Bilim ve Teknoloji Dergisi
Özel Sayı, S. 381-388, Ağustos 2020
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Araştırma Makalesi

www.ejosat.com ISSN:2148-2683

European Journal of Science and Technology
Special Issue, pp. 381-388, August 2020
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Research Article

ARM Based Hybrid Measurement System Design for Liquid Level Measurement with High Accuracy under Vibrating Case

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(Bu yayın 26-27 Haziran 2020 tarihinde HORA-2020 kongresinde sözlü olarak sunulmuştur.)

(**DOI:** 10.31590/ejosat.7802632)

ATIF/REFERENCE: Artan, M., Soydemir, M. U. & Şahin, S. (2020). ARM Based Hybrid Measurement System Design for Liquid Level Measurement with High Accuracy under Vibrating Case. *Avrupa Bilim ve Teknoloji Dergisi*, (Special Issue), 381-388.

Abstract

In this study, the design and implementation of a 32-bit ARM microcontroller based measurement system which can measure the liquid level with an accuracy and resolution that have maximum 1% error level in liquid tanks under the mechanical vibrations produced by the actuators during their work is realized. It is aimed to prevent losses of sensor's measurement accuracy over time and so preventing the erroneous results of the measurements by measuring the liquid level without contact and without adding any measuring material or device to the liquid in the tank. The measurement system is created by using hybrid method of ultrasonic, temperature and humidity sensors, and unlike conventional tactile liquid level measurement methods, it can measure the level of non-contact liquids such as fuel, chemical solutions, and it might be easily adapted to liquid tanks of different sizes and shapes. Since the of the sound waves sent from the ultrasonic sensor can change with the temperature and humidity, for calculation of the actual speed of the sound waves, taking the measurement data obtained from temperature, humidity sensors into consideration. Thus, the accuracy of liquid level measurement is increased by using this hybrid sensor consisting of ultrasonic, temperature and humidity sensors. Liquid levels are measured in 3 different vibration intensity as low, medium and high, respectively and the measurement error that might be caused by mechanical vibrations is reduced by using different iterations of the median filter, and the remaining measurement error is eliminated by using data such as liquid level measurement and liquid tank vibration intensity in a regression analysis. The final liquid level measurement result after regression model is transferred from 32-bit ARM based microcontroller to an android application via a Wi-Fi module wirelessly.

Keywords: Median Filter, Regression Analysis, Hybrid Sensor, Liquid Level Measurement, ARM Microcontroller.

Titreşim Altında Yüksek Doğruluklu Sıvı Seviyesi Ölçümü için ARM Tabanlı Melez Ölçüm Sistemi Tasarımı

Öz

Bu çalışmada aktüatörlerin çalışmaları sırasında ürettiği mekanik titreşimler altındaki sıvı tanklarında, sıvı seviyesini hata oranı en fazla %1 olacak şekilde bir doğruluk ve çözünürlükle ölçebilen 32-bit ARM mikrodenetleyici tabanlı ölçüm sisteminin tasarım ve gerçeklemesi yapılmaktadır. Tank içinde bulunan sıvıya herhangi bir ölçüm maddesi veya cihazı eklemeden sıvı seviyesi temassız ölçülerek kullanılan algılayıcının zamanla ölçüm hassasiyetini kaybetmesi ve bundan dolayı ölçümlerin hatalı sonuçlar vermesinin önlenilebilmesi amaçlanmaktadır. Ölçüm sistemi, ultrasonik, sıcaklık ve nem sensörlerinin melez biçimde kullanılmasıyla oluşturulup geleneksel temaslı sıvı seviye ölçüm yöntemlerinin aksine tank içindeki yakıt, kimyasal çözeltiler gibi temassız sıvıların seviyesini ölçebilmekte, farklı boyut ve şekillerdeki sıvı tanklarına da kolayca uyarlanabilmektedir. Ultrasonik algılayıcıdan gönderilen ses dalgalarının tank içerisindeki hızı, sıcaklık ve nem ile değişebileceğinden ses dalgalarının gerçek hızının ölçülebilmesi için sıcaklık, nem algılayıcılarından alınan ölçüm değerleri de göz önüne alınarak sıvı seviyesi hesaplanmaktadır. Böylece, ultrasonik, sıcaklık ve nem algılayıcılarından oluşan bu melez algılayıcı kullanılarak sıvı seviyesi ölçümünün doğruluğu arttırılmaktadır. Düşük, orta ve yüksek olarak 3 farklı titreşim şiddetinde sıvı seviyesi ölçülmektedir ve bu mekanik titreşimlerden kaynaklanabilecek ölçüm hatası ise medyan filtrenin farklı iterasyonları kullanılarak azaltılıp geriye kalan ölçüm hatası ise sıvı seviyesi ölçümü, tankın titreşim şiddeti verilerinin bir regresyon analizinde kullanılmasıyla giderilmektedir. Nihai elde edilen sıvı seviye ölçüm sonucu 32-bit ARM mikrodenetleyiciden bir Wi-Fi modül aracılığıyla kablosuz şekilde bir Android uygulamasına aktarılarak takibi yapılabilmektedir.

Anahtar Kelimeler: Medyan Filtre, Regresyon Analizi, Hibrit Sensör, Sıvı Seviye Ölçümü, ARM Mikrodenetleyici.

1. Introduction

Accurate information about liquid level in the tanks are important in automotive, oil, water, pressure, and gas industries (Xu et al. 2017; Priya et al., 2014; Morris and Langari 2012). It is a good example that a pumping oil into a storage tank requires liquid level measurement to prevent spillage borrowed from (TR Ministry of Education 2009).

The costly failures that may occur as a result of fuel depletion of the generators can be prevented by measuring the fuel level of the generators used in the industry. There are two main ways for liquid level measurement, direct and indirect (Gillum 2009; Fraden 2010). Direct level measurement method is a visible measurement method such as level glass or vertical moving float connected parallel to the liquid tanks (Dogan 2015). Indirect level measurement method, on the other hand, is applied in cases where the liquid level is invisible and it is generally used in completely closed liquid tanks containing dangerous substances (flammable, caustic liquids) (Antunes et al., 2015; Shin and Wu 2010). Liquids such as fuel and chemical solution in the liquid tanks, break the liquid level measurement sensors when they contact them for a long time (Dogan 2015). Therefore, continuous level gauges are preferred in industry. Today, the most common indirect and continuous level gauges are made with pressure sensors, capacitive sensors and ultrasonic sensors (Toghyani et al., 2017; Singh et al., 2011). Pressure sensors are based on the principle of determining the liquid level according to the ratio of the reference pressure with the pressure that changes depending on the liquid level in the tank. The liquid level of the tank can be determined by converting the pressure value into a voltage value with the pressure transducer (Dogan 2015). On the other hand, capacitive sensors are created by insulating material between two conductive plates. The conductive plates are capacitive sensor and liquid tank. Between the conductive plates there is insulating materials such as liquid and air which have different dielectric coefficients. The capacitance value of the capacitor changes according to the liquid and air quantities in the tank (Dogan 2015; Toghyani et al., 2017; Terzic et al., 2010). Many of the liquid tanks used in the industry are closed systems and their shape and size vary (Akyıldız et al., 2012; TR Ministry of Education 2009). Neither of these two measurement methods has the flexibility to be used in liquid tanks of different sizes and shapes. Also, measurement is taken by immersing them in the liquid. They give different results according to the dielectric coefficient and density of the liquid in the tank (Canbolat 2009). In addition to accurately measuring the liquid level, measurements should be effectively displayed to the user and an alert must be sent in unexpected situations. In the literature, the user is informed by transferring the obtained measurement results to a human machine interface wirelessly via Bluetooth and GSM modules (Altın and Bulut 2016; Kumar et al., 2018).

In this study, a 32-bit ARM microcontroller based measurement system is implemented in order to measure the liquid level in the tank with utmost 1% error percentage under mechanical vibrations caused by the actuator. Ultrasonic and heat and humidity sensors are used in hybrid form to accurate to ensure liquid level measurement. The system is designed to measure the liquid level with contactless liquid such as the fuel, chemical solutions and to be easily adapted to available danger solutions tanks. By using a median filter, the precision of the measurement is increased so that most of the measurement error caused by mechanical vibrations is reduced and the remaining measurement error was eliminated by applying a regression analysis to the data from the measurement system. Thus the accuracy of the measurement is improved. The obtained measurement results are observed to an Android application with ESP8266-01 wireless module for a user-friendly human machine interface for data acquisition.

The rest of the paper is presented in the following sections, In Section 2, hardware and software design of the liquid level measurement system is described briefly. In Section 3, the results from the prototype of the system are given, and conclusions and recommendations are presented in Section 4.

2. Material and Method

The proposed hybrid measurement system that measures the level of liquid in vibrating tanks consist of hybrid sensor, arm based microcontroller and android application, respectively. Suitable components for these parts are ultrasonic sensor, temperature sensor, vibration sensor, humidity sensor, ESP8266 Wi-Fi module, electronic circuit elements, 32-bit ARM based microcontroller, respectively. A block diagram of the system and circuit diagram of the measurement system is given in Figure 1 and in Figure 2, respectively.

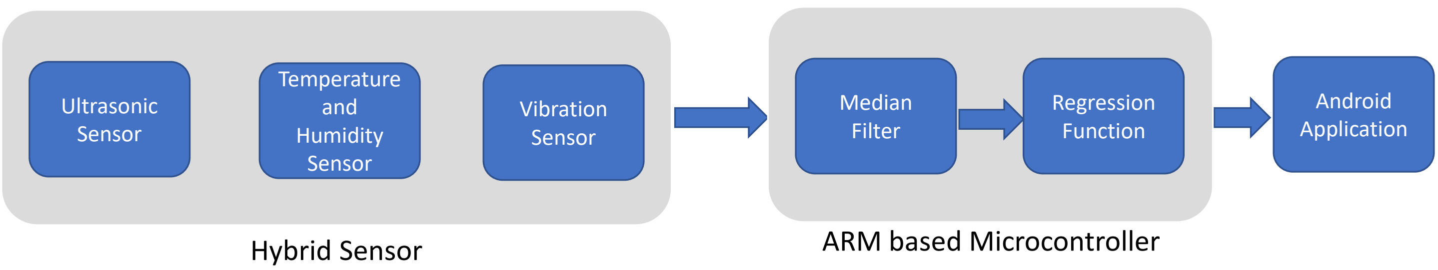

Fig. 1. Block diagram of the hybrid liquid measurement module.

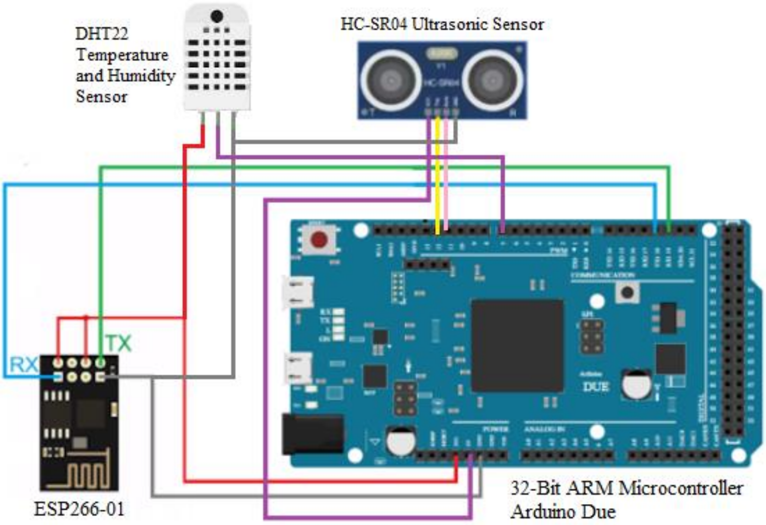

Fig. 2. Circuit diagram of the module.

2.1. The Hybrid Sensor

Ultrasonic sensors are one of the most accurate liquid level measurement sensors and they have advantages such as easy installation and maintenance, adaptability to tanks of any shape and size, and level measurements of contactless liquids such as fuel and chemical solutions. These sensors work according to the principle of detecting backward reflected high frequency acoustic signals with the transducer (Jin at al., 2015; Raj et al., 2002) and they might be detected the distance from the liquid in the tank by using ultrasonic sound waves. The speed of sound wave does not change depending on the frequency, however, it might be changed according to the temperature and humidity in the air. In order to increase the accuracy of the liquid level measurement of the ultrasonic sensor, the actual speed of the sound waves is calculated using ultrasonic, temperature and humidity sensors in hybrid form (Terzic et al., 2010; Cheeke 2012) so the actual speed of the sound wave is calculated using the temperature and humidity values as seen Eq.1 (Bies and Hansen 2009).

$$V_s = 331.4 + (0.606 * T) + (0.0124 * H) \tag{1}$$

where T is the temperature (degrees Celsius), H is the Humidity and V_s is the actual speed of sound wave. Herein, 331.4 is speed of sound wave (m/s) in an air at a temperature 0 degrees Celsius. Additionally, as the vibration intensity increases, the error in liquid level measurements increases (Panigrahy et al., 2009). Therefore, to decrease the error in the level measurement, the intensity of the vibration is measured with the vibration sensor in order to be used in a regression analysis which is detailed in section 2.2. A simplified diagram hybrid sensor and liquid tank is given in Figure 3.

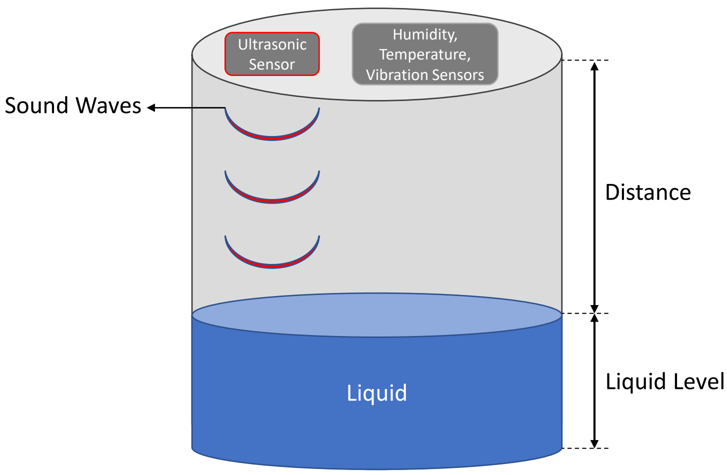

Fig. 3. Simplified diagram of the hybrid sensor design.

2.2. ARM Microcontroller

In the proposed measurement system, a 32-bit ARM based Arduino Due takes measurements from the hybrid sensor and it is evaluated in a C programme in it (Arduino 2020). It calculates actual speed of sound wave (Eq.1) and elapsed time (half the time until sound waves are sent from the top of the tank and back again) data is measured as much as the desired number of iterations. Then, elapsed time data is filtered through the median filter through iteration number and the distance from the top of the tank to the liquid surface is found. This distance is subtracted from the height of the tank and liquid level is found. Found liquid level is evaluated in a regression relationship for final liquid level measurement. Then it is transferred to an android application with the ESP8266 Wi-Fi module (Benchoff 2014).

2.2.1. Median Filter

The median filter is a non-linear digital filtering technique, often used in signal processing to remove noise from a signal. It is a pre-processing step to improve the results of measurements (Mathworks 2020). Mechanical vibrations caused by actuators in liquid tanks cause an error in liquid level measurements similar to the noise signal. The error in liquid level measurements increases as the vibration intensity increases. By applying a median filter, the error is reduced and the precision of the liquid level measurement is increased. The actual sound wave speed and iterations of the median filter are some of the factors that determine the performance and accuracy of the measurement system. Median filter is performed by taking the average of the measurement as much as the number of iterations, as seen in Eq.2.

$$M = \frac{\sum_{i=1}^{n}(y_i)}{n} \tag{2}$$

where y_i is the i^{th} iteration and n is the iteration number. In this study y_i is taken as elapsed time at i^{th} iteration, a median value of elapsed time for n iteration number is found and the liquid level is found by multiplying median value of elapsed time and the speed of sound wave found according to (1) equation and subtracting the found result from tank height.

2.2.2. The Regression Analysis

Regression analysis is a series of statistical processes used to predict relationships between a dependent variable and one or more independent variables. It allows researchers to find the best fit for the data based on a specific mathematical criterion (David A. Freedman 2009). After increasing the precision of the measurement results with median filter and finding more accurate liquid level, error in the liquid level caused by vibration is might be decreased by a regression analysis. If the measured liquid level and vibration intensity are chosen as inputs; and measurement error percentage is chosen as outputs then the liquid level measurement errors is prevented by constructing a regression model to the dataset. Then, creating mathematical regression model might be embedded to the C program in the microcontroller to eliminate the liquid level measurement error. If the error found with the regression equation is added to the measured liquid level while the measurement system is running, the error in the liquid level will be reduced. For regression analysis, multiple linear regression is applied in study. The multiple linear regression might seen in Eq.3.

$$y = a_0 + b_1x_1 + b_2x_2 + b_3x_1x_2 + b_4{x_1}^2 + b_5{x_2}^2 + \cdots \tag{3}$$

where y is measurement error (actual liquid level-measured liquid level) for liquid level and x_1, x_2 are measured liquid level and vibration intensity measurement, respectively.

2.3. The Android Application

The obtained measurement results are transferred over a network via serial communication between ESP8266 Wi-Fi module and 32-Bit ARM microcontroller Arduino Due. In this way, the measurement system is turned into the Internet of Things (IoT) appliance and it can be controlled over the internet without any distance. The Blynk program is used to create the Android application (Blynk, 2017). It has various widgets to create the interfaces that ensure control the hardware remotely, display and store the data. Thus, the obtained measurement results can be displayed and observed on the Android application for data acquisition as in Fig.4.

Fig.4. The designed Android application.

3. Results and Discussion

The proposed hybrid measurement sensor system is realized with a beaker which have an actual liquid level is 15 cm is used instead of liquid tank. Actual speed of sound wave is obtained Eq.1. Then elapsed time is calculated for desired iteration number and it is passed through the median filter in order to increase the precision. Distance in Fig.3 is found by multiplying speed and elapsed time, and liquid level is found by subtracting distance from beaker height. To determine the effect of the number of iterations under low, medium and high vibration intensities, lots of measurements are taken for times of 10000 ms durations in three different cases where the median filter iterations are 10, 100 and 400 for each of the vibration intensities as in Fig.5, Fig.6 and Fig.7, respectively. In these figures, trace denotes a fixed line for understanding precision. In this way, it is observed that the precision of the measurements increased as the iteration number of the median filter increased. Although the time between the two measurements increased as the number of median filter iterations increased, it might be ignored as the time was less than four seconds in the 400 iteration and highest vibration case. It has been observed that the measurements are more precise when the median filter is 400 iterations and measurement error deviate up to maximum 0.02% under high vibration intensity case. The measurements with median filter at 400 iterations for low, medium and high vibration intensities are observed as 14.6 cm, 14.2 cm and 13.6 cm, respectively.

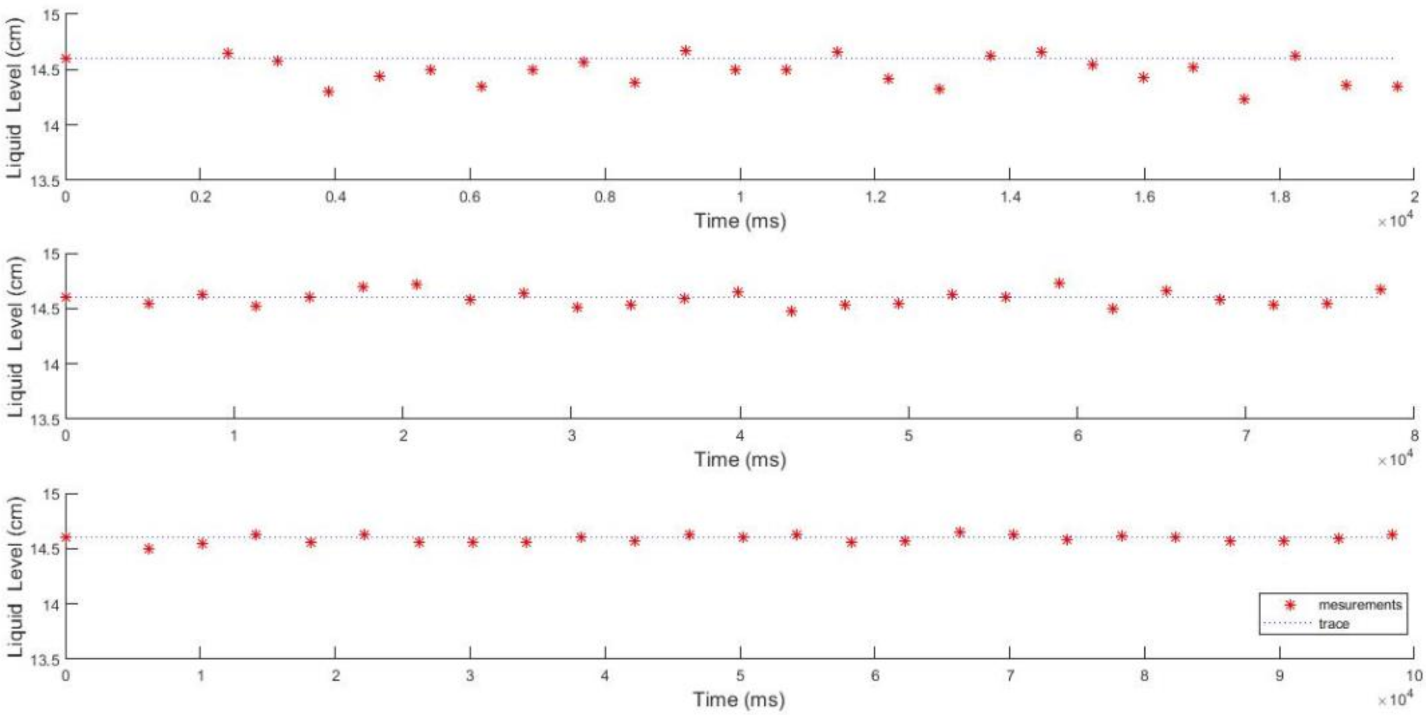

Fig. 5. Under low vibration intensity, performance of 10, 100 and 400 median filter iterations, respectively.

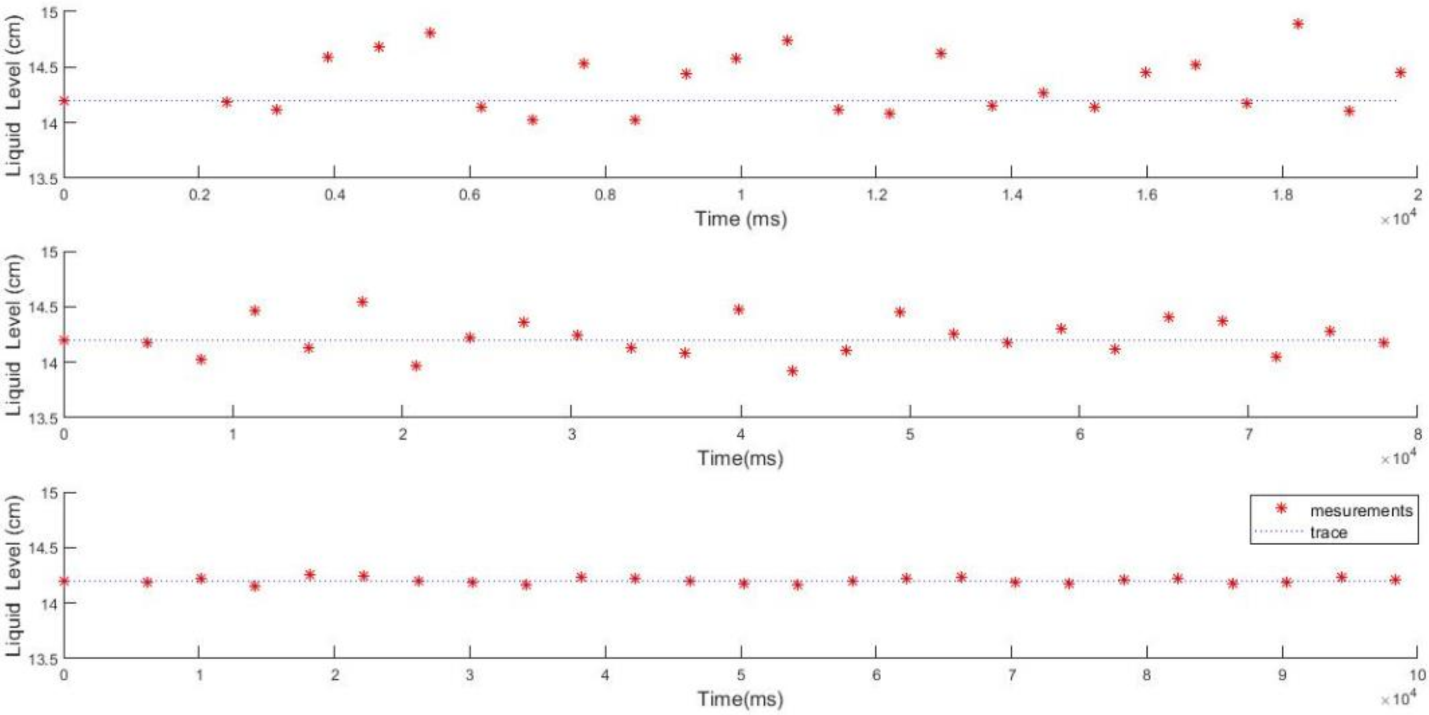

Fig. 6. Under medium vibration intensity, performance of 10, 100 and 400 median filter iterations, respectively.

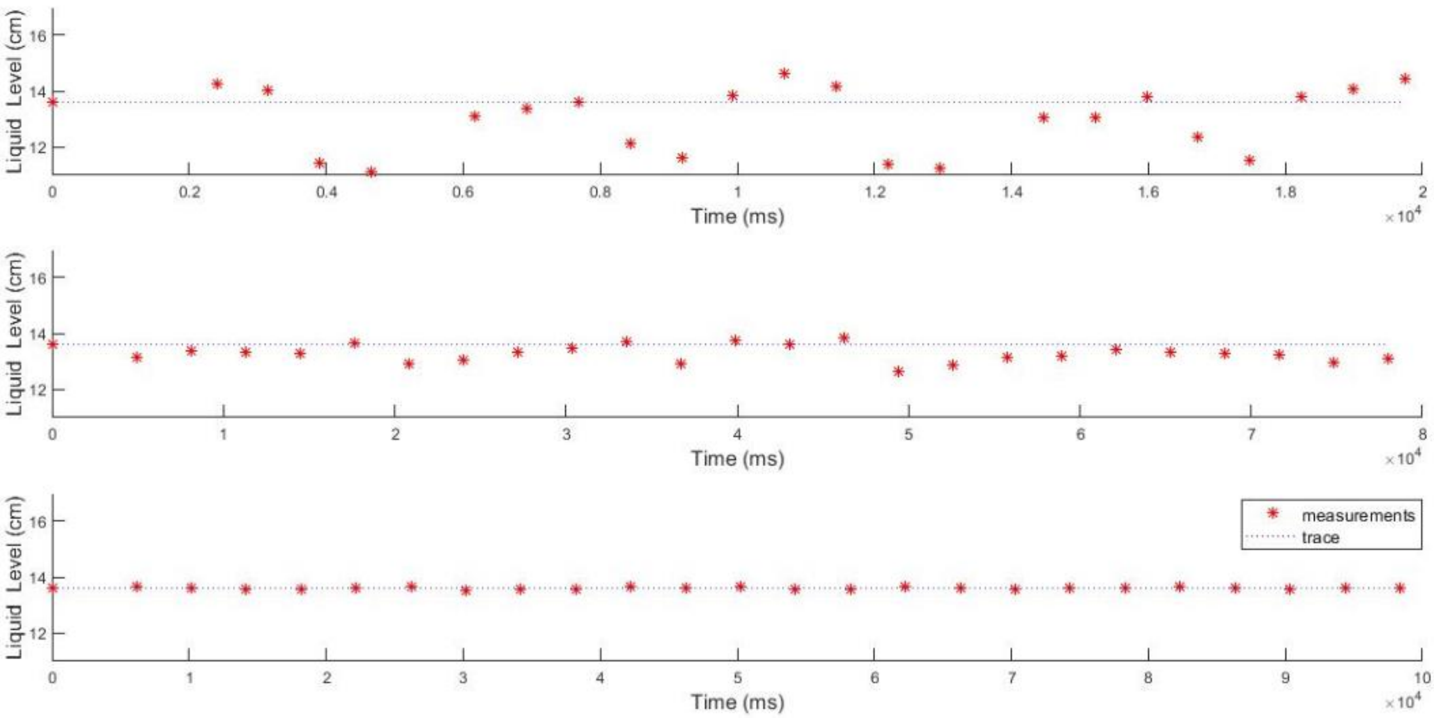

Fig. 7. Under high vibration intensity, performance of 10, 100 and 400 median filter iterations, respectively.

Liquid level measurement error changes at different vibration intensities and different liquid levels. So, a data set is created for measurements which have low, medium or high vibration intensities effects of the beaker and different liquid levels effects. Data set includes measured liquid level found after elapsed time median filtering, vibration intensity and measurement error percentage for every measurement. Measurement error percentage (e) data is obtained as in Eq. 4.

$$e = (h - \bar{h})\, x\, 100 \qquad (4)$$

where h is the actual liquid level, $\bar{h}$ is the measured liquid level. A regression model is constructed vibration intensity and measured liquid level found after median filtering as inputs and measurement error percentage as output in Wolfram Mathematica (Stephen

Wolfram 1999). A part of the dataset obtained is given in Table 1. Vibration intensity 1, 2, and 3 represent low, medium and high intensities, respectively.

Table 1. A Part of the Obtained Data Set

Measured Liquid Level x_1 (cm)	**Vibration intensity x_2**	**Measurement Error Percentage (%)**
5.1	1	49
4.2	2	94
7.1	3	142
8.8	1	28
12.3	2	86
15.1	3	139
16	1	31
16.9	2	85
20.2	3	138

For regression model, fourth order polynomial model is chosen and data is fitted to model. Final mathematical model is given in Table 1. After the constructing model, R^2 value is calculated to find the accuracy of the constructed model. Correlation Coefficient R^2 which shows the performance of the models can be calculated in Eq.5 (Draper and Smith 1998).

$$R^2 = 1 - \frac{SSE}{SST} \quad (5)$$

where $SST = \sum_{i=1}^{n}(h_i - \bar{h})^2$ (Total Sum of Squares). Herein, h_i is the i^{th} observation measurement error percentage and $\bar{h}$ is the mean of n observation measurement error percentages. And $SSE = \sum_{i=1}^{n}(h_i - \hat{h_i})^2$ (Sum of the Square of Error) where h_i is the i^{th} observation measurement error percentage and $\hat{h_i}$ is the i^{th} predicted measurement error percentage. Training and testing R^2 coefficient are given in Table 1.

Table 2. Regression Analysis Results

Training Data Correlation Coefficient R^2	0.979955
Testing Data Correlation Coefficient R^2	0.943255
The Mathematical Model	$1.355 + 15.53x_1 - 3.214{x_1}^2 + 0.23{x_1}^3 - 0.005{x_1}^4 - 9.87x_2 + 8.24x_1x_2 - 0.836{x_1}^2x_2 + 0.024{x_1}^3x_2 - 2.125{x_2}^2 + 1.45x_1{x_2}^2 - 0.036{x_1}^2{x_2}^2 + 0.372{x_2}^3 - 0.17x_1{x_2}^3 + 0.47{x_2}^4$

While system is running, the result (measured liquid level) of the median filter with 400 iterations and vibration intensity is applied to the mathematical model in Table 2. By applying these results into Eq.4, the actual fluid level of 15.083 cm, 14.997 cm and 14.935 cm are calculated respectively.

By using the median filter with 400 iterations, the measurements taken under high intensity vibration deviated up to a maximum of 0.02% of the liquid level. Also, as a result of the regression analysis, the liquid level measurement caused by vibration is determined as maximum 0.0567%. Thus, the total liquid level measurement error of the designed prototype was calculated as 0.0767% maximum.

4. Conclusions and Recommendations

In this study, design and implementation of the 32-bit ARM microcontroller based measurement system is implemented. Liquid level in the tank with utmost 0.0767% maximum error and resolution under mechanical vibrations caused by the actuator is achieved. The accurate liquid level measurement is obtained by using ultrasonic, temperature and humidity sensors in hybrid form. Precision of liquid level measurements is increased by using a median filter and applying a regression analysis. The obtained measurement results can be observed on an Android application for data acquisition. Although the module will generally be used under a constant vibration intensity in the industry, it is designed to be used under three different vibration intensities, which is more complicated. It has also been shown in the prepared prototype that the liquid level measurement error of the tank under vibration can be determined. In future work directions, pressure variable data is adding liquid level measurement process and robust fitted regression might be used.

5. Acknowledge

This work is supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under 2209A – Research Project Support Programme for Undergraduate Students with project number 1919B011902870.

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