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Do We Need More Chaos Examples?

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ABSTRACT A half century after the discovery of the Lorenz attractor, people are still finding and publishing new chaotic systems with special properties. Surely we are nearing the time when additional examples are unnecessary... or maybe not.

Like many others, I first learned about chaos in the 1980s after working several decades in a different field. I thought chaos must be rare, or I would have encountered it earlier. Indeed, most people were then studying a few standard chaotic systems such as those proposed by Ed Lorenz (1963) and Otto Rössler (1976), and any new chaotic system that was discovered merited a publication.

That was also a time when personal computers made their appearance. I set mine to the task of finding just how rare chaos is, and in the process I found hundreds of new chaotic systems (Sprott 1993). I thought that was the beginning of the end of finding and publishing new examples of chaos.

However, people kept asking interesting questions such as "What is the simplest jerk function that gives chaos?" (Gottlieb 1996) and publishing answers to those questions, myself included (Sprott 1997). Two decades later, hundreds of such papers had been published, and I proposed a standard for the publication of new chaotic systems (Sprott 2011), but it did little to stem the tide.

Now another decade has passed, and the latest frenzy of new examples has been inspired by the discovery of "hidden attractors" (Leonov and Kuznetsov 2013) whose basins do not contain the neighborhood of any equilibrium points. As the name implies, such attractors are rare and hard to find. Except that they are neither. It's just that no one had bothered to look.

With age comes wisdom (sometimes), and so I will not proclaim that all the important questions have been asked and answered and that no new examples of chaos are needed. Rather I will conclude by announcing yet another new chaotic system with unusual properties, designed just to show that more surprises likely await us. Here it is:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x - \operatorname{sgn}(z)y \\ \dot{z} &= y^2 - \exp(-x^2) \end{aligned} \tag{1}$$

Why is this system interesting and worth publishing? It's because it is time-reversible and dissipative with a strange multifractal attractor that is hidden but whose basin includes the whole of the three-dimensional space so that every initial condition goes to the attractor. Even more remarkably, every initial condition apparently lies on the attractor which fills all of space with a highly nonuniform measure and a capacity dimension of 3.0, and it is just one member in a large family of systems with those properties. It violates every expectation that most of us have developed over the years about respectable chaotic attractors.

I will not spoil the fun of anyone who wants to explore this system on their own except to say that the Lyapunov exponents are (0.2280, 0, −0.2480) and the Kaplan–Yorke dimension is 2.9194, dramatically different from the vast majority of three-dimensional dissipative chaotic systems whose Kaplan–Yorke dimension is only slightly greater than 2.0. Furthermore, a cross section of the attractor in the $z = 0$ plane shown in Fig. 1, with colors representing the value of the local largest Lyapunov exponent, shows intricate structure that belies the simplicity of the equations that produced it and invites further study.

The long-term future of chaos research is an open question, but in the near-term, I look forward to additional, truly novel examples of chaotic systems in this and other similar journals.

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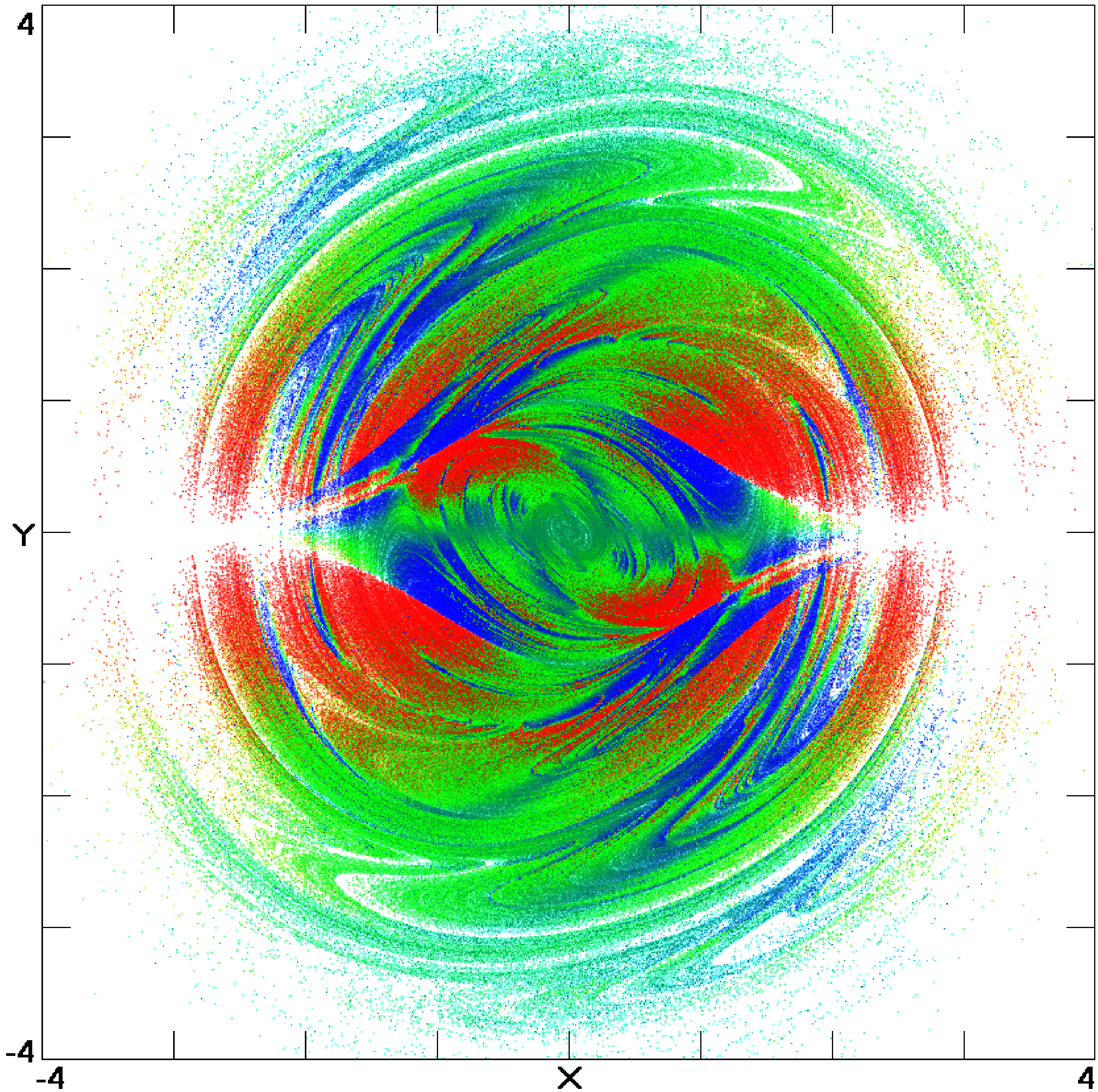

Figure 1 Cross section of the strange attractor for Eq. (1) in the $z = 0$ plane. The attractor fills the whole of space, and the colors indicate the value of the local largest Lyapunov exponent with red positive and blue negative.

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