On the Systematic variations in colour along the surface of spindle nebulae

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Özet: Spindle nebulaların yüzeyi boyunca yüzey parlaklığının ve renk eksesinin komplike bir şekilde değişikliği müşahade edilmiştir. Parlaklık merkez düzleme doğru azalır. Renk evvelâ artar, fakat bu düzlemin civarında tekrar azalır.

Bu makalede, değişik modelde extragalatik sistemler, absorbe edici maddenin ve yıldızların muhtelif şekilde dağılımına göre nazarı itibara alınmıştır. Yıldızların, absorbe edici maddenin içinde bulunması ve absorbe edici maddenin dağılımında yıldızların dağılımından çok farklı olmaması lâzım geldiği görülür. Bu şartlar tahakkuk ettiği zaman, normal yıldız dağılımına haiz olan sistemlerde, parlaklık ve renk değişimi rasat edilen şekilde olması lazım gelir.

1. From a study of the surface brightness of spiral nebulae, which are seen edgewise, Holmberg [1] concludes, that the surface brightness rapidly decreases towards the central absorbing belt. At the same time the colour increases. Within the central belt itself the remaining intensity is small, but near the plane of symmetry the mean colour instead of being redder appears to be bluer than in the adjacent regions. Therefore at some distance from the plane of symmetry a reversal in the sign of dC/dz occurs. The changes of colour as described here must be the combined effect of the change in the mean colour of the stars and the increase of selective absorption towards the plane of symmetry.

Within the normal spectra range the selectivity of the absorption is given by a λ^{-1} law [2]. Therefore the ratio of the total absorption $A(\lambda_0)$ in the wavelength λ_0 to the degree of selectivity in the two adjacent wavelengths λ_0 and λ_1 is equal to:

$$\frac{A(\lambda_0)}{E(\lambda_0, \lambda_1)} = \frac{\lambda_0^{-1}}{\lambda_0^{-1} - \lambda_1^{-1}}$$

or $A(\lambda_1) = F \cdot A(\lambda_0)$ where $\lambda_1 > \lambda_0$ and $0 < F < 1$

Let $\mathfrak{J}(\lambda_0, z)$ represent the total amount of light of wavelength λ_0 emitted by all stars within an unit volume of space at a distance z from the plane of symmetry (fig. 1) With a view to the large distances of the spiral nebulae z may be considered as an angular measure. The total amount of light emitted through the surface O into outer space will be :

$$\mathfrak{J} = O \cdot \mathfrak{J}(\lambda_0 \cdot z) \int_0^\infty \exp\left(\int_0^s - K\lambda_0^{-1}\, ds\right)$$

$$= O \cdot \mathfrak{J}(\lambda_0 z) \int_0^\infty \exp\{-A(\lambda_0)s\} \cdot ds = O \cdot \frac{\mathfrak{J}(\lambda_0, z)}{A(\lambda_0)}$$

or expressed in magnitudes

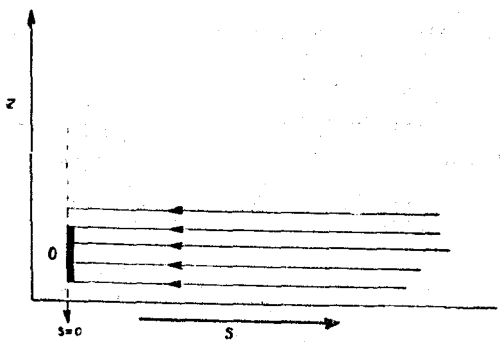

Fig. 1

$$\mathfrak{S} = -2.5 \log \mathfrak{J} = \text{const} - 2.5 \log \mathfrak{J}(\lambda_0 z) + 2.5 \log A(\lambda_0) \quad \cdots\cdot \quad (1.1)$$

For λ_1 a similar expression is obtained. Therefore the mean colour is

$$C = -2.5 \log \frac{\mathfrak{J}(\lambda_0 z)}{\mathfrak{J}(\lambda_1 z)} + 2.5 \log \frac{A(\lambda_0)}{A(\lambda_1)}$$

or

$$C = -2.5 \log \frac{\mathfrak{J}(\lambda_0 z)}{\mathfrak{J}(\lambda_1 z)} + 2.5 \log \frac{1}{F} \quad \cdots\cdot \quad (2.1)$$

If λ_0 and λ_1 are taken to coïncide with the mean isophotic wavelengths of the international photographic and photovisual scale we have

$$C = -2.5 \log \frac{\Im_f(z)}{\Im_r(z)} + 2.5 \log \frac{1}{F} \qquad \ldots . (3.1)$$

2. In our local galaxy the blue stars are more highly concentrated towards the galactic plane than the red stars. Therefore the ratio $\Im_f(z)/\Im_r(z)$ must systematically depend on z. We first consider a simplified case. It is supposed that the mixture of stars is composed of two groups only. Of these two groups the first is supposed to have colour index $C = 0$ and an absolute magnitude M_0, while for the second the corresponding values are $C = 1.0$ and M_1. For $z = 0$ the numbers of stars per unit volume of space in each group are taken to be N_0 and N_1 respectively.

If the relative density at a distance z from the plane of symmetry is given by Oort's empirical relation [3)]

$$\rho = \rho_0 \exp\left(-2 \frac{|z|}{T}\right)$$

the ratio $\Im_f(z) ; \Im_r(z)$ is equal to

$$\frac{\Im_f(z)}{\Im_r(z)} = \text{const.} \frac{\left\{ \frac{N_0}{N_1} \exp\left[-2z\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] \times 10^{-0.4\, M_0 - M_1)} + 1 \right\}}{\left\{ \frac{N_0}{N_1} \exp\left[-2z\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] \times 10^{-0.4(M_0 - M_1)} + 2.5 \right\}} \quad (1.2)$$

$$\left.\begin{array}{l} \text{We put } T_1 = a\, T_0 ; \frac{N_0}{N_1} \cdot 10^{-0.4(M_0 - M_1)} = B ; \frac{2}{T_0}\left(1 - \frac{1}{a}\right) = b \\ \text{where } a > 1 ; B > 0 \text{ and } b > 0 \end{array}\right\} \quad (2.2)$$

Then (1.2) reduces to

$$\frac{\Im_f(z)}{\Im_r(z)} = \text{const} \frac{(Be^{-bz} + 1)}{(Be^{-bz} + 2.5} = \text{const} \frac{\left(\frac{1}{B} e^{bz} + 1\right)}{\left(\frac{2.5}{B} e^{bz} + 1\right)} \quad (3.2)$$

Next from (3.2) and (3.1) we have

$$C = \text{const} - 5\,76\, ln \frac{\left(\frac{1}{B} e^{bz} + 1\right)}{\left(\frac{2.5}{B} e^{bz} + 1\right)} + 2.5 \log \frac{1}{F} \quad (4.2)$$

For large values z the term $ln\left(\frac{1}{B} e^{bz} + 1\right) - ln\left(\frac{2.5}{B} e^{bz} + 1\right)$

rapidly converges to the limit ln 0.4 So we need only concern ourselves with small values of z.

For sufficiently small values of z instead of (3.2) we have

$$C = \text{const} + 8.64 \frac{b}{B} z + 2.5 \log \frac{1}{F} \qquad \cdots\cdot(5.2)$$

For the intermediate and large values of z the second and higher powers of z can not be neglected, but we know that with these larger values the function by which the term $8.64 \frac{b}{B} z$ is to be replaced becomes a constant.

For small values of z from (5.2) we have

$$\frac{dC}{dz} = + 8.64 \frac{b}{B} \qquad \cdots\cdot (6.2)$$

while for large values of z we have

$$\frac{dC}{dz} = 0 \qquad \cdots\cdot (7.2)$$

It is to be remarked that the relations (1.2) .... (4.2) were derived for two groups of stars only. For other groups of stars different values of N_0, N_1, M_0, M_1 etc. would have to be used and different values for b and B would be obtained. Apart from this, identical results would be obtained. Our relations therefore are general ones which hold for any mixture of stars, though it would be difficult to determine the actual values of the coefficients b and B.

3. With the case considered in the sections 1 and 2, we find that near the plane of symmetry ($z = 0$) there first will be a steady increase of colour with increasing values of z (5.2). With the intermediate values of z the increase dC/dz levels off and finally becomes zero with the larger values of z. From (4.2) we see that there the only influence of the absorption is that for all values of z the colour is increased by a constant amount. There will be a steady increase in colour and no reversal in the sign of dC/dz can occur.

Until now the numerical value of F was adopted to be constant. As exposed in section (1) this assumption seems to be correct as it is based on the physical nature of the absorption. However, dark clouds which consist of relatively large particles have a neutral absorption. They diminish the light that passes through them, but light of all wavelengths will be weakened in the same proportion. So if large neutral clouds are intermingled with the gaseous substratum which causes the selective absorption the coefficient F might vary from one place to the other.

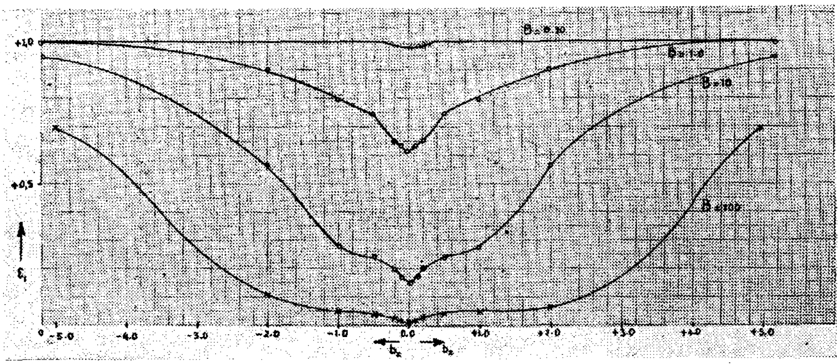

Fig. 2

From the investigations by Stebbins and Whitford 2) it would appear that in the mean the influence of neutral absorption is low in our local galaxy. Still in the lower galactic latitudes some neutral clouds have been observed by B. J. Bok [4] and Müller [5] (r Car with $\beta = 0^\circ$ and ζ Oph with $\beta = +4^\circ$).

If with small values of z fairly extensive neutral clouds are present, the total absorption will increase and F will increase. In the limiting case F would be $F = 1$ and $2.5 \log 1/F = 0$ while $\frac{d}{dz} (\log 1/F) > 0$.

Consequently near the plane of symmetry the increase of C with increasing values of z would be given by a curve which is slightly steeper than that obtained for the stars alone. In this case also a reversal in the sign of dC/dz is not possible.

However, the case which we have considered is a very special one. In the plane of symmetry the density of the stars was considered to be the same with all values of S so that the ratio $\mathfrak{J}_f(z) : \mathfrak{J}_r(z)$ also was taken to be constant for all values of S.

4. Near the surface O (fig. 1) the density ρ of the stars will be $\rho = 0$, while ρ increases with increasing value of S until at $S = S_0$ a maximum is reached.

For $S > S_0$ the density will decrease again. Although due to the spiral structure irregularities will occur, for the present it is sufficient if the distribution of the densities is represented by a Gaussian error curve

$$\rho_s = \text{const}\exp\{-h^2(S-S_0)^2\} = \text{const}\exp(\alpha S - \beta S^2) \qquad (1.4)$$

where

$$\alpha > 0 \qquad \text{and} \qquad \beta > 0$$

So we have

$$\mathfrak{J}(\lambda_0, z, S) = \mathfrak{J}(\lambda_0 z)\cdot\exp(\alpha S - \beta S^2)$$

and the total amount of light transmitted through the surface O is

$$\mathfrak{J} = O\cdot\mathfrak{J}(\lambda_0 z)\int_0^\infty \exp\{-A(\lambda_0)S + \alpha S - \beta S^2\}\,dS \qquad (2.4)$$

This expression easily reduces to

$$\mathfrak{J} = O\cdot\mathfrak{J}(\lambda_0 z)\frac{\sqrt{\pi}}{\sqrt{\beta}}\exp\left\{\frac{(\alpha - A(\lambda_0))^2}{4\beta}\right\}$$

and so

$$\mathfrak{L} = -2{,}5\log\mathfrak{J} = \text{const} - 2.5\log\mathfrak{J}(\lambda_0 z) - 0\ 271\,\frac{(\alpha - A(\lambda_0))^2}{\beta} \qquad (3.4)$$

while the colour is given by

$$C = -2.5\log\frac{\mathfrak{J}(\lambda_0 z)}{\mathfrak{J}(\lambda_1 z)} - \frac{0{,}271}{\beta}\{(\alpha - A(\lambda_0))^2 - (\alpha - A(\lambda_1))^2\}$$

or

$$C = -2{,}5\log\frac{\mathfrak{J}(\lambda_0 z)}{\mathfrak{J}(\lambda_1 z)} + 0.271\,\frac{(1-F)A(\lambda_0)}{\beta}\{2\alpha - (F+1)A(\lambda_0)\} \qquad (4.4)$$

Consequently instead of (4.2) we have

$$C = \text{const} + 8.64\frac{b}{B}z + 0.271(1-F)A(z)\{2\alpha - (F+1)A(z)\} \qquad (5.4)$$

in which the absorption $A(z)$ is a function of z.

5. Of the numerical values of the coefficients in the relations we know nothing but the fact that they all are positive, while

(1 — F) is also positive. Therefore without any loss in generality instead of (5.4) we may write

$$C = C_0 + C_1 z + C_2 A(z) - C_3 A^2(z) = C_0 + C_1 z + C_3 A(z) \left\{ \frac{C_2}{C_3} - A(z) \right\} \quad \cdots\cdots (1.5)$$

where C_1, C_2 and C_3 are positive constants.

With any value of z the influence of absorption must be such as to increase the colour. Therefore in any case we must have

$$\frac{C_2}{C_3} > A(0) \quad \cdots\cdots (2.5)$$

For the change of colour with increasing values of z we have

$$\frac{dC}{dz} = C_1 - 2\,Kz\,A(0)e^{-Kz^2} \left\{ \frac{C_2}{C_3} - 2\,A(0)\,e^{-Kz^2} \right\} \quad \cdots (3.5)$$

where for the shape of the curve $A(z)$ the form $A(z) = A(0)$. exp $(-Kz^2)$ was adopted.

From (3.5) it is evident that there will be a continuous decrease of the colour with increasing distance from the plane of symmetry as soon as

$$\frac{C_2}{C_3} > 2\,A(0) \quad \cdots\cdots (4.5)$$

Therefore we only need consider those cases for which

$$\frac{C_2}{C_3} > A(0) > \frac{1}{2}\frac{C_2}{C_3} \quad \cdots\cdots (5.5)$$

At first sight it may seem surprising that we must exclude all cases for which $C_2/C_3 < A(0)$

From (1.5) it appears that in such cases the influence of selective absorption would be such as to have the colour decrease instead of increase. Obviously this is impossible. Still it would appear that it should be possible to find an arrangement of stars for which $2\alpha <$ F + 1) A (0) and in such cases $C_2/C_3 - A\,(0)$ would certainly be negative.

The amount of light contributed by the stars a distance S from the surface is const. exp $\{-A\,(\lambda_0)\,S + \alpha S - \beta S^2\}\,dS$ and therefore the maximum amount of light is contributed by the stars

at the distance S_m for which the function $\{-A(\lambda_0)S + \alpha S - \beta S^2\}$ is a minimum and therefore by the stars for which $S_m =$
$= \{\alpha - A(\lambda_0)\} : \beta$. If $\alpha < A(\lambda_0)$ this valus S_m is negative and the maximum amount of light would be contributed by the stars beyond the limit $S = 0$, while for this limit $S = 0$ the density of the stars has sunk to $\rho = 0$.

Evidenty the case $\alpha < (0)$ corresponds to the case where the absorption is so large that the light emitted by the stars within the system is completely obliterated. Therefore we need only consider those cases for which $\alpha > A(0)$ but if $\alpha > A(0)$ we have a fortiori $2\alpha < (F + 1) A(0)$.

6. The curve (1.5) which gives the colour as a function of z results from the superposition of two curves. If we denote $\varepsilon = C - C_0$ as the colour excess we have

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \qquad \dots (1\,6)$$

The curve ε_1 denotes the colour excess resulting from the unequal distribution of the different groups of stars. The curve ε_2 denotes the influence of absorption.

When deriving numerical results it is better to replace the term $\varepsilon_1 = C_1 z$ of (1.5) and (1.6) by the original function which appears in (4.2). Therefore (1.6) can be written in the form

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -2.5 \log \frac{\left(\frac{1}{B} e^{bz} + 1\right)}{\left(\frac{2.5}{B} e^{bz} + 1\right)} +$$

$$C e^{-p^2 z^2} A(0) \left\{\frac{C_2}{C_3} - A(0) e^{-p^2 z^2}\right\} \qquad \dots (2.6)$$

where $p = \sqrt{K}$

A further simplification is obtained if we put $C_2/C_3 = D.A(0)$ where $D > 1$. Then instead of (2.6) we have

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -2.5 \log \frac{\left(\frac{1}{B} e^{bz} + 1\right)}{\left(\frac{2.5}{B} e^{bz} + 1\right)}$$

$$+ \text{const}\, e^{-p^2 z^2} (D - e^{-p^2 z^2}) \qquad \dots (3.6)$$

We first consider the curve ε_1. In a recent paper [7] for the ratio T_v/T_f the author has obtained values between 2.0 and 1.0 Similar results have been obtained by other authors. Therefore the value of T_v/T_f will be around 2.0 so that the numerical value of the constant b in the relation (2.2) becomes $b = 1/T_f$. At a distance z of the galactic plane the density of the stars is equal to $\rho = \rho_0 \exp\left(-2\frac{|z|}{T}\right)$ and therefore at a distance for which $z = 2T_f$ the density has decreased to 1/55 of its original value. The distance $z = 2T_f$ may be evaluated as being around 1000 parsecs. For $bz = 5.0$ the remaining density is entirely negligible. Therefore with the curve ε_1 we only need consider the values $bz \leq 5.0$

It is hardly possible to evaluate the value of the term B. Therefore the shape of the curve ε_1 has been computed for different values of B and the results are given in table 1.

TABLE 1.

The curve ε_1 for different values of B

bz \ B	100	10	1	1/10
0.00	+ .01	+ .14	+ .61	+ .95
0.10	+ .02	+ .17	+ .64	+ .95
0.20	+ .03	+ .20	+ .65	+ .96
0.50	+ .04	+ .24	+ .75	+ .98
1.00	+ .05	+ .28	+ .80	+ .99
2.00	+ .11	+ .57	+ .90	+ .1.00
5.00	+ .70	+ .95	+ .1.00	+ .1.00

The curves in table 1. are graphically represented in fig. 2. It is apparent that a $bz = 0$ the function $\varepsilon_1(z)$ has a minimum, but this minimum becomes increasingly shallow as B decreases. From (3.2) it is apparent that B crudely denotes the ratio of (the total amount of light emitted by the blue stars in the galactic plane) over (the total amount emitted by the red stars in that plane). The curves therefore indicate that if there is a large predominance of red stars, only a very shallow minimum can be expected.

On the other hand with large values of B a deep minimum is found. This indicates that the curve $\varepsilon_1(z)$ will have a deep minimum if in the plane of symmetry there is a preponderance of blue stars At the same time if B is too large (f. i. $B = 100$) a large colour excess only occurs with large values of bz for which the space density of the stars has become negligible.

Therefore for the phenomenon to occur which has been observed by Holmberg [1)] circumstances seem to be most favourable if the numerical values of B are somewhere between 1 and 10, that is to say if in the plane of symmetry there is a limited preponderance of the blue stars.

7. We next consider the curve $\varepsilon_2 = \text{cont}\, e^{-p^2 z^2} (D - e^{-p\, z^2})$. We already know that D must be larger than 1, while for $D > 2$ there will be a continuous decrease of colour with increasing dis tance from the plane or symmetry

Therefore the shape of the curve $\varepsilon_2(z)$ is mainly interesting for values of D in the interval $1 < D < 2$. For different values of D the function const. $\varepsilon_2(z) = e^{-p^2 z^2} (D - e^{-p^2 z^2})$ has been tabulated in table 2. In the first part of the table the values const. $E_2(z)$ as directly computed are given.

In the second part of this table for each given value of D the computed numbers are multiplied by a constant factor so as to make const $E_2(0) = 1.00$. These latter curves have been plotted in fig. 3.

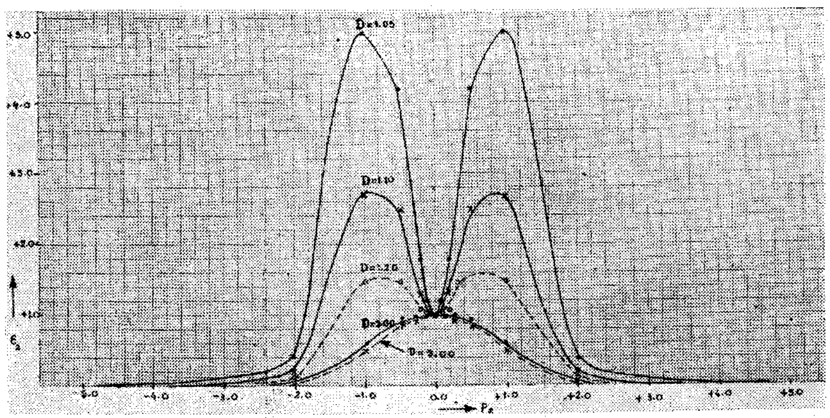

Fig. 3

8. There can hardly be any doubt, but that the complicated variations of the colour observed along the surface of the spindle nebulae can be fully explained by the effect of selective absorption such as indicated in the figure 3 for $D = 1.05$ and

TABLE 2.

Curves $E_2(z)$ Directly computed values

pz / v	1.05	1.10	1.20	1.30	1.50	2.00	3.00
0.00	+ 0.05	+ 0.10	+ 0.20	+ .30	+ 0.50	+ 1.00	+ 2.00
0.10	0.06	0.11	0.21	0.30	0.50	1.00	2.00
0.20	0.09	0.13	0.23	0.33	0.52	1.00	1.96
0.50	0.21	0.25	0.33	0.41	0.56	0.95	1.73
1.00	0.25	0.27	0.31	0.34	0.42	0.60	0.97
2.00	+ 0.02	0.02	+ 0.02	+ 0.02	0.03	0.04	0.06
5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Reduced values

pz / v	1.05	1.10	1.20	1.30	1.50	2.00	3.00
0.00	+ 1.00	+ 1.00	+ 1.00	+ 1.00	+ 1.00	+ 1.00	+ 1.00
0.10	1.20	1.10	1.05	1.00	1.00	1.00	1.00
0.20	1.80	1.30	1.15	1.10	1.04	1.00	0.98
0.50	4.20	1.50	1.55	1.34	1.12	0.95	0.86
1.00	5.00	2.70	1.50	1.10	0.84	0.60	0.48
2.00	+ 0.40	+ 0.20	+ 0.10	+ 0.06	+ 0.06	+ 0.04	+ 0.03
5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

D = 1,10. The variation of the total colour excess $E = E_1 + E_2$ with distance z is found by the addition of the curve in fig. 2 to one of the curves in fig. 3. However, it is obvious that the resulting relation is mainly determined by the shape of the curve E_2. It would hardly be possible to say which of the curves in fig. 2 should be added to which of the curves in fig. 3, except that in the latter case one of the curves with a small value of D must be chosen. Moreover the correct addition of the two curves would offer an additional difficulty.

In fig 2 the values E_1 are plotted against the argument bz, while in figure 3 the values E_2 are plotted against pz while we do not know the exact relation between the coefficients b and p. From the scanty data, which we have at our disposal, it is not possible to determine the exact value of this ratio.

On the other hand it is possible to show that the coefficients b and p should be of the same order of magnitude. Therefore the various relations between ε and z which are possible are obtained by simple addition of the curves in figures 2 and 3. We will refrain from doing so, because this might give an exaggerated impression of the accuracy which can be obtained. We can only state that the variations of the total colour are represented by one of the curves in fig. 3 (small value D), while to this curve small corrections must be applied, which are due to the unequal distribution of the different groups of stars and which are represented by one of the curves in fig. 2.

9. It is of interest to study the relation between p and b in order to be sure that all observational conditions have been fulfilled. Up till now attention has been paid only to the variations of the colour. It is given however that the surface brightness of the spindle nebulae decreases towards the plane of symmetry. This should be interpreted as follows. From $z = 0$ the magnitude first increases and reaches a minimum for $z = z_m$ Beyond the limit z_m the magnitude increases again until finally the surface brightness becomes zero. To all practical purposes the surface brightness should be zero when $bz = n$ where n is a small number say 5, 6, 7 or another number of the same order of magnitude. Obviously the tentative solution for the variations of colour which was given above can not be accepted unless the conditions for the variation of the total surface brightness are simultaneously fulfilled. As follows from the equation (2.1) *a.o.* the total surface brightness at distance z from the plane of symmetry is given by a relation of the form:

$$\mathfrak{J} = \text{const.} \exp(-b'z). \exp \frac{\{\alpha - \text{A}(z)\}^2}{4\beta}$$

and therefore the surface brightness expressed in magnitudes is equal to

$$\mathfrak{L} = \text{const} + 2.5\, b^1 z \log e - 2.5 \frac{\{\alpha - \text{A}(z)\}^2}{4} \log e \quad \cdots (1.9)$$

With sufficient accuracy we can put $(\text{F}+1) = 2$ (see (5.4)) and next in the same way as before for A(z) we write $\text{A}(z) = \text{A}(0).$ $\exp(-p^2 z^2)$ and for $\alpha = \text{D} \cdot \text{A}(0)$ Then instead of (1.9) we have

$$\mathfrak{L} = \text{const} + 1.08\, b^1 z - \text{K}(\text{D} - e^{-p^2 z^2})^2 \quad \cdots\cdots (2.9)$$

Due to the term $1.08\, b'z$ the magnitude will increase with increasing distance from the plane of symmetry. Due to the influence of the second term the magnitude will first decrease and later on remain a constant. Obviously the variations due to the second term largely depend on the numerical values of the constants K and D.

The numerical value of D should be small, because with large values of D the whole term becomes a constant. This however is a condition which we have previously considered and we concluded that only such values of D are acceptable for which $1 < D < 2.0$. Therefore we can limit ourselves to consider these values of D only. The constant K must be related to the total effect of the absorption expressed in magnitudes. For $z = 0$ the effect of absorption is $K(D-1)^2$ while for $z = \infty$ the effect is equal to KD^2. Now it is supposed that at $z = 0$ the total effect of absorption is to produce an increase of the surface brightness equal to 10 magnitudes. Obviously this value 10 is purely hypothetical.

It may not be far from the true value, but we cannot be sure about that. On the other hand the shape of the curve $K(D - e^{-p^2z^2})^2$ is not materially altered if for the influence of the total absorption a value different from 10 is accepted. The only effect is that the scale of the curve will be different. With a value of the total absorption of 10 magnitudes the value of K is found from the relation $K(D-1)^2 - KD^2 = 10$. So we have $K = \frac{-10}{2D+1}$. Inserting this value in (2.9) for the magnitude at a distance z from the plane of symmetry we find

$$\Omega = \text{cont} + 1{,}08\, b'z + \frac{10}{(?D-1)} \cdot (D - e^{-p^2z^2})^2$$

while for the difference in magnitude in surface brightness between two points at distance z from the plane of symmetry and at $z = 0$ we have

$$\Delta M = \Delta M_1 + \Delta M_2 = 1.08\, b'z + \frac{10}{(2D-1)}(D - e^{-p^2z^2})^2 - \frac{10}{(2D-1)}(D-1)^2 \quad \cdots\cdots (3.9)$$

For different values of D the numerical values of the function

$$\Delta M_2 = \frac{10}{(2D-1)}(D - e^{-z^2p^2})^2 - \frac{10}{(2D-1)}(D-1)^2$$

are given in Table 3.

TABLE 3.

The curves ΔM_2 (in magnitudes) for different values of D

pz \ D	1.05	1.10	1.20	1.30	1.50	2.00
0.00	0.0	0.0	0.0	0.0	0.0	0.0
0.10	0.0	0.0	0 0	0.0	0.0	— 0.1
0.20	— 0.1	— 0.1	— 0 1	— 0.2	— 0.2	— 0.3
0.30	— 0.2	— 0.2	— 0.3	— 0.3	— 0.5	— 0.6
0.40	— 0.4	— 0.4	— 0.6	— 0.7	— 0 8	— 1.1
0.50	— 0 7	— 0.8	— 1.0	— 1.1	— 1.5	— 1.6
0.60	— 1.1	— 1 3	— 1.5	— 1.7	— 2.0	— 2.3
0.70	— 1.7	— 1.9	— 2.2	— 2.4	— 2.7	— 3.1
0.80	— 2.5	— 2.6	— 2.9	— 3.1	— 3.5	— 3.8
0.90	— 3.4	— 3.6	— 3.9	— 4.1	— 4.4	— 4.7
1.00	— 4.2	— 4.4	— 4.7	— 4·8	— 5.1	— 5.5
1.50	— 8.0	— 8.1	— 8.3	— 8.3	— 8.4	— 8.5
2.00	— 9.6	— 9.7	— 9.7	— 9.7	— 9.7	— 9.6
5.00	— 10.0	— 10.0	— 10.0	— 10.0	— 10.0	— 10.0

From the curves in Table 3 it appears that between the limits set to the value of D variations of D only have a small influence on the shape of the curve ΔM_2 (pz). In figure 4 only the curve ΔM_2 valid for $D = 1\ 10$ has been plotted (dotted line), but this one curve may be considered to represent the family of curves given in table 3. As a result of the decreasing influence of absorption towards the larger values of z there is a steady decrease in brightness, but with high values of z the increase in brightness levels off and the brightness becomes a constant.

The curve ΔM_2 however represents only a part of the total variation in brightness This total variation is equal to the sum of the two curves ΔM_1 and ΔM_2 (equation 3.9). Just as in the previous case direct addition of these two curves is difficult as the curve ΔM_1 is to the argument $b'z$ while ΔM_2 is to the argument pz.

10. In equation (5.2) and in section 6 and 7 the estimated value of b was

$$\frac{2}{T_0}\left(1 - \frac{1}{2}\right) = \frac{1}{T_0}$$

Consequently the coefficient b' must be roughly equal to $2\,b$ and instead of (3.9) we can write

$$\Delta M = \Delta M_1 + \Delta M_2 = 2.16\,(bz) + \Delta M_2\,(pz)$$

In figure 4 for $-\Delta M_1$ three different curves are plotted which are valid for $b = 2p$; $b = p$ and $b = 0.4p$ respectively. In each case the total variation ΔM is equal to the difference between the curves ΔM_2 and $-\Delta M_1$. In the figure the full drawn curve represents the function $\Delta M = \Delta M_1 + \Delta M_2$ for the case that $b = p$. From this curve it appears that from $z = 0$ the luminosity steeply increases with increasing distance from the center. Around $pz = 1{,}5$ a maximum is reached and next the luminosity falls off again until at $pz = 5$ the luminosity is again equal to that at the center. Beyond $pz = 5$ there is a further decrease of the luminosity.

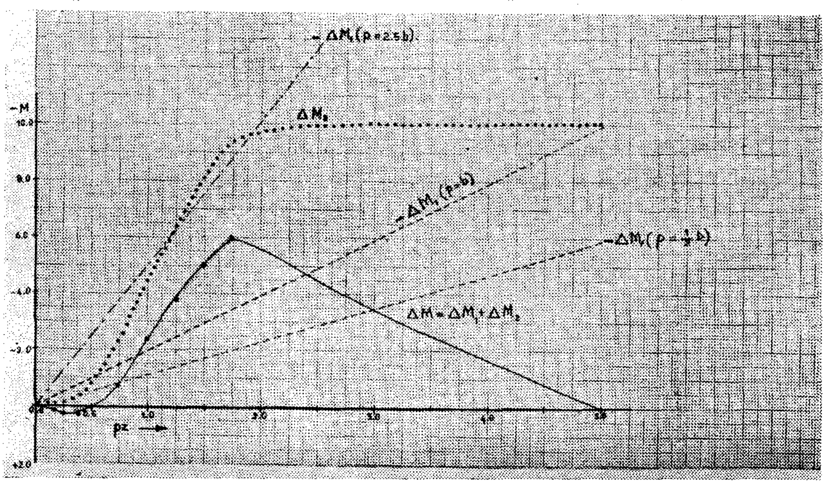

Fig. 4

If b were to be taken $b = 2p$ or conversely $p = 0{.}5\,b$ the resultant curve ΔM would have a similar shape, but the maximum of the luminosity around $pz = 1.5$ would correspond to a smaller value of bz.

Therefore this maximum would occur at a smaller distance from the plane of symmetry. On the other hand if $b \leqq p$ the maximum is shifted towards a larger distance from the center. Evidently the thickness of the absoption belt between the maxima at either side of the central plane is related to the numerical value of the ratio $b : p$

Measurements of the surface brightness are notoriously difficult. Therefore at present an attempt from observational data to establish the correct relation between p and b would be futile. Moreover in our theoretical results too may unknown quantities occur. Still it should be possible to obtain some general information. The importance of obtaining such information is, that the parameter b' describes the distribution of the stars in a direction perpendicular to the plane of symmetry. The parameter p determines the corresponding distribution of the gaseous substratum. So if the relation between p and b could be established, we would at the same time have obtained information about the distribution of the absorbing materials as compared with the distribution of the stars. A large numerical value of the ratio $p : b$ would indicate that the absorbing material is strongly concentrated towards the central plane. Conversely small values of the ratio $p : b$ would indicate that the absorbing materials occur up to large distances from this plane. Further to illustrate this point I have for different values of $p : b$ computed the distance from the central plane at which the density of the absorbing material has decreased to 1/100 of its value in that plane. The result appears in table 4. In this table the unit of distance z_1 has been taken equal to $z_1 = 2/b' = 1/b = T$. The last row of numbers indicates the density (z) of the stars at a distance from the central plane corresponding to the values given in the second row. In the central plane the density $\rho(0)$ was taken to be $\rho\,(0) = 1$.

TABLE 4.

Distance z/T at which for different values of p the density of the absorbing material would have decreased to 0.01 of its density in the central plane. The steller densities $p(z)$ corresponding to these distances.

p	2.5 b	2.0 b	1.5 b	1 0 b	0.8 b	0.6 b	0.4b
z/T	0 86	1.07	1.43	2.14	2.67	3.57	5.35
$\rho(z)$	0.18	0.12	0.06	0.014	0.005	0 0008	0.0000

From this table it is abundantly clear that we should only consider those values of p which are $\geq b$.

11. With the curves in table 2 and 3 and in the figures 3 and 4 the argument pz is used. When for different values of the ratio $p:b$ we wish to study the shape of the luminosity curve and the width of the absorbing belt, it is better to use the argument bz. The obvious reason is, that the coëfficient $b = 1/T$ is directly related to the dimensions of the extra galactic system and will remain constant when p is varied.

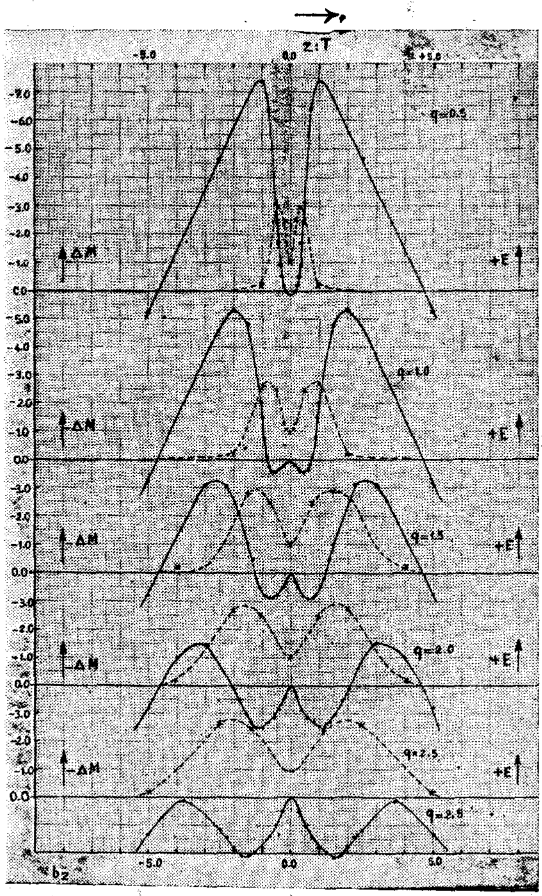

Fig. 5

With the help of table 3 for various values of p the differences ΔM_2 can easily be expressed as a function of bz. It is sufficient if the values pz appearing in the first column are replaced by $q \cdot bz = pz$ where $q \geqq 1.0$. The corresponding numerical value of ΔM_2 is found in the column under D = 1.10. When to this value ΔM_2 the correction $\Delta M_1 = 2{,}16\ lz$ is applied, we obtain the resultant luminosity $\Delta M = \Delta M_1 + \Delta M_2$.

When for q the numerical values $q = 0.5$; $q = 1$; $q = 1$; $q = 2.0$; $q = 2.5$ and $q = 3.0$ are adopted, the resulting luminosity curves are as indicated in fig. 5.

In each separate instance in figure 5 there has also been indicated the curve E_2 for the colour excess valid for D = 1.10. This curve E_2 has been borrowed from figure 3 and from table 2. It has to be observed that both the curves ΔM and E_2 have been expressed on a completely arbitrary scale of magnitudes, while it would be difficult to determine the relation between these two arbitrary scales. Therefore it must be born in mind that in fig. 5 the scales used for plotting the curves ΔM and E_2 are not identical. In figure 5 the values E_2 are plotted against lz instead of against pz. The curves ε_1 can be omitted as the range of the values ε_1 is small as compared with that of ε_2.

It should be remembered that in section 9 in a rather arbitrary way the total influence of absorption was taken to be 10 magnitudes. Therefore all curves ΔM which appear in fig. 5 are based on this assumption.

It seems doubtful whether it is correct to assume that with any degree of concentration of the absorbing material towards the central plane the total influence of absorption remains the same viz. 10 magnitudes. If for this total influence a value > 10 is adopted, the two maxima on either side of the central plane will be less pronounced while the descending branch of the curve intersects the horizontal axis ($\Delta M = 0$) at a distance from the center $pz < 5$. On the other hand from a comparison with fig. 4 it appears that the general shape of the curves remains the same and that there is no substantial shift in the position of the maxima. Consequently from the curves in fig. 5, no quantitative results can be read, but the qualitative results should be correct.

12. As was anticipated from the curves in fig. 5 it appears that when there is a large scattering of the absorbing material ($q = 0.5$) there can only exist a narrow belt of absorption. From its minimum at $z = 0$ the luminosity rises steeply. At $z = T$ a maximum is reached and at larger distances the luminosity decreases again. With increasing concentration (larger values of q)

of the absorbing material towards the central plane, the absorption band broadens and the maxima are shifted in outward direction, while these maxima become less pronounced. At the same time around $z =$ a secondary maximum develops, of which the relative importance steadily increases.

With $q = 2.5$ the absorption band is completely divided into two symmetrical parts. In other words, in the central part of the absorption belt, a narrower belt of greater brightness develops which with $q = 2.5$ reaches the same intensity as the two outer maxima. It is therefore evident, that the relation between p and b can be evaluated from the observed luminosity curve. For reasons exposed in the preceding section $q \geq 1.0$ and consequently the curve obtained for $q = 0.5$ can be rejected. With the spindle nebulae, discussed by Holmberg [1], no secondary maximum at $z = 0$ is observed.

Consequently also the curves for which $q \geq 1.5$ can be rejected. Our conclusion is that $q = p : b$ can vary only between the relatively narrow limits $q = 1.0$ and $q = 1.5$ respectively. With the curves valid for $q = 1.0$ and $q = 1.5$ there is a rapid decrease in the surface brightness from $z = \mathrm{T}$ and $z = 2.5\mathrm{T}$ respectively towards the central plane. At the same time while the brightness decreases, the colour excess increases, but near $z = 0$ a rather sharp decrease of the colour excess sets in. At $z = 0$ itself there is a secondary minimum of the colour excess. Even with $q = 1.5$ a faint secondary maximum of the luminosity curve occurs at $z = 0$. As long as such a secondary maximum has not been observed we must conclude that the numerical value of the coëfficient p is very nearly equal to that of q.

For the cases that $q = 1.0$ and $q = 1.5$ the distribution of the absorbing material in a direction perpendicular to the plane of symmetry as compared with the corresponding mean distribution of the stars is as indicated in table 5.

If conditions in our own galactic system are comparable to those we find for the spindle nebulae, the numbers in table 5 have an additional importance. They would indicate the possible maximum and minimum values of the distribution of the absorbing substratum in our galaxy.

13. It is not necessary to discuss extensively the case where the stars are not embedded in the absorbing clouds.

TABLE 5.

Density of the absorbing material (ρ/ρ_0) a m and density of the stars ρ_0/ρ at different distances from the plane of symmetry.

Distance z/T	ρ/ρ_0 stars	(ρ/ρ_0) a. m $q = 1.0$	(ρ/ρ) a. m $q = 1.5$
0.0	1.00	1.00	1.00
0.2	0.67	0.96	0 91
0.4	0.45	0 85	0.70
0.6	0.30	0.70	0.44
0.8	0.20	0.53	0.24
1.0	0.14	0.37	0 11
1 2	0.09	0 24	0 04
1.4	0.06	0.14	0.01
1.6	0.04	0.08	0.00
1.8	0.03	0.04	0.00
2.0	0.02	0 02	0.00

It is self evident that in this case the colour excess rises to a steep maximum at $z = 0$. If the cloud is sufficiently dense and is sufficiently concentrated towards the central plane, the surface brightnes will decrease toward $z = 0$, but no reversal in the trend of the colour excess is possible. Therefore no detailed discussion is needed.

In other respects our discussion is by no means complete. The ratio $N_1 ; N_1$ (section 2) was tacidly assumed to be constant for all values of S and the same is the case for the ratio $T_v ; T_f$ There is observational evidence, that this may not be the case. Seyfert has observed [8] that with the spiral nebulae the integrated light of the continuous background is of the type G — K while the colour of the spiral arms corresponds to an earlier type. With N.G.C 5164 Carpenter [9] finds from the center to the outer parts a continuous decrease of the colour from yellow to white. Baade [10] also finds a high concentration of the highly luminous blue stars in the outer spiral arms. It therefore seems possible the ratio $N_0 : N_1$ and $T_v : T_f$ will increase towards the limit $S = 0$.

This will not affect our curves ΔM. On the other hand the minima in the curves E_1 and E_2 will be more pronounced than indicated in the figures 2 and 3.

However, for the present anyhow we had to limit ourselves to qualitative results so that even these variations of the ratios $N_0 : N_1$ and $T_v : T_f$ do not effect our results.

14. Our result can be summarised as follows :

a. With spindle nebulae Holmberg has observed that the surface brightness decreases towards the central plane, while at the same time the colour increases. However, very near this plane the colour decreases again.

These effects must be due to the presence of absorbing clouds in the spiral system.

b. No such effect however can be expected if the absorbing clouds are situated in the region of constant stellar density (section 3). On the contrary, these clouds must be situated in a stellar field where a fairly rapid increase of the stellar density occurs (section 4.)

c. The constant denoting interstellar absorption must be smaller than the constant which corresponds to the linear increase in density (section 5).

d. A strong correlation between colour excess and distance from the central plane can only exist if the difference between the two constants mentioned in c is relatively small (section 7).

e. If in a direction perpendicular to the central plane the distribution of the stars is given by the curve $\rho = \rho_0 \exp(-2bz)$ and the density of the absorbing material by $\rho_{a.m} = \rho_{a.m}(0) \exp(-p^2z^2)$ the width and depth of the absorbing belt across the surface of the spindle nebulae is determined by the ratio $p : b$ (section 11).

f. If the absorbing material is widely scattered, only a sharp narrow belt can occur. If the absorbing material is strongly concentrated towards the central plane, the absorbing belt will be wider. With very high concentration in the center of the absorbing belt a narrow belt of relatively higher luminosity must occur. (section 12).

g. From observational evidence it would appear that the ratio

$p : b$ is enclosed between rather narrow limits $p_{max} = 1.5\, b$ and $p_{min} = 1.0\, b$.

h. With values of p within these limits the variations of the curve ΔM giving the surface brightness and of the curve E giving the colour excess qualitatively are identical with the corresponding observed variations of the surface of spindle nebulae. (section 12)

i. Such variations therefore may be expected to occur with spiral nebulae in which both the distribution of the stars and of the absorbing materials are normal but which are observed edgewise.

j. As compared with the rate of the linear increase in density the absoption must be large. Therefore we must expect that only the light from stars in the outer layers of the system is transmitted into empty space.

k. Consequently the effects which have been observed and which we have tried to explain mainly originate near the limit of the system and presumably in the outer spiral arm.

l. If in our galactic system the relative distributions of the stars and of the absorbing material are comparable to those in the spindle nebulae, this would mean that the gaseous substratum extends to almost the same limits as the stars, but at large distance from the galactic plane the absorbing material would be very teneous

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