UJMA

Universal Journal of Mathematics and Applications, 4 (3) (2021) 94-100
Research paper

Universal Journal of Mathematics and Applications

Journal Homepage: www.dergipark.gov.tr/ujma
ISSN 2619-9653
DOI: https://doi.org/10.32323/ujma.977588

On Ideal Convergent Difference Double Sequence Spaces in Intuitionistic Fuzzy Normed Linear Spaces

Esra Kamber[1] and Selma Altundağ[1,2*]

[1]*Science and Technology Institute, Sakarya University, Sakarya, Turkey*
[2]*Department of Mathematics, Sakarya University, Sakarya, Turkey*
[*]*Corresponding author*

Article Info

Keywords: *Ideal filter, Double I-convergence, Difference double sequence spaces, Intuitionistic fuzzy normed space*
2010 AMS: *40D15, 40G99.*
Received: *2 August 2021*
Accepted: *1 October 2021*
Available online: *1 October 2021*

Abstract

In this paper, we introduce difference double sequence spaces $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ and $I_2^{0(\mu,\upsilon)}(M,\Delta)$ in the intuitionistic fuzzy normed linear spaces. We also investigate some topological properties of these spaces.

Introduction

Fuzzy set theory firstly defined by Zadeh [39] has been applied many fields of engineering such as in non-linear dynamic systems [10] , in the population dynamics [5], in the quantum physics [27], but also in various fields of mathematics such as in metric and topological spaces [7,9,12], in the theory of functions [11,38] , in the approximation theory [4]. Fuzzy topology plays an essential role in fuzzy theory. It deals with such conditions where the classical theories break down. The intuitionistic fuzzy normed space and intuitionistic fuzzy n-normed space which were investigated in [32] and [36] are the most important improvements in fuzzy topology. In the last years, the concepts of intuitionistic fuzzy I-convergent difference sequence spaces and intuitionistic fuzzy I-convergent difference double sequence spaces have been studied in [21]- [**?**] and [23]- [24], respectively.

The concept of statistical convergence was given by Steinhaus [34] and Fast [8] using the definition of density of the set of natural numbers. Many years later, statistical convergence was discussed by many researchers in the theory of Fourier analysis, ergodic theory and number theory. Some statistical convergence types were studied in [1]- [3] and [29]. As an extended definition of statistical convergence, definition of I-convergence was introduced by Kostyrko, Salat and Wilczynski [26] by using the idea of I of subsets of the set of natural numbers. I-convergence of double sequences $x=(x_{ij})$ has been studied in [30]- [31]. Recently, I- and I^*- convergence of double sequences have been studied by Das et. al [6]. Also, related studies can be found in [13]- [17].

Some new sequence spaces were introduced by means of various matrix transformations in [18], [19], [28] and [35]. Kızmaz [25] defined the difference sequence spaces with the difference matrix as follows:

$$X(\Delta)=\{x=(x_k)\in\omega:\Delta x\in X\}$$

for $X=l_\infty$, c, c_0, where $\Delta x_k=x_k-x_{k+1}$ and Δ denotes the difference matrix $\Delta=(\Delta_{nk})$ defined by

$$\Delta_{nk}=\begin{cases}(-1)^{n-k}, & \text{if } n\le k\le n+1,\\ 0, & \text{if } 0\le k<n.\end{cases}$$

Email addresses: **e.burdurlu87@gmail.com, (E. Kamber), scaylan@sakarya.edu.tr, (S. Altundağ)**

In this paper, we introduce difference double sequence spaces $I_2^{(\mu,\upsilon)}(M,\Delta)$ and $I_2^{0(\mu,\upsilon)}(M,\Delta)$ in the intuitionistic fuzzy normed linear spaces. We also investigate some topological properties of these new spaces.

Basic definitions

In this section, we give some definitions and notations which will be used for this study.

Definition 2.1. ([33]) A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a*1=a$ for all $a\in[0,1]$,
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for each $a,\ b,\ c,d\in[0,1]$.

Definition 2.2. ([33]) A binary operation $\circ:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $a\circ 0=a$ for all $a\in[0,1]$,
(iv) $a\circ b\le c\circ d$ whenever $a\le c$ and $b\le d$ for each $a,\ b,\ c,d\in[0,1]$.

Definition 2.3. ([32]) The five-tuple $(X,\mu,\upsilon,*,\circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) is where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, υ are fuzzy sets on $X\times(0,\infty)$, μ denotes the degree of membership and υ denotes the degree of nonmembership of $(x,t)\in X\times(0,\infty)$ satisfying the following conditions for every $x,y\in X$ and $s,t>0$:

(i) $\mu(x,t)+\upsilon(x,t)\le 1$,
(ii) $\mu(x,t)>0$,
(iii) $\mu(x,t)=1$ if and only if $x=0$,
(iv) $\mu(\alpha x,t)=\mu\left(x,\frac{t}{|\alpha|}\right)$ if $\alpha\neq 0$,
(v) $\mu(x,t)*\mu(y,s)\le\mu(x+y,t+s)$,
(vi) $\mu(x,.):(0,\infty)\to[0,1]$ is continuous,
(vii) $\lim\limits_{t\to\infty}\mu(x,t)=1$ and $\lim\limits_{t\to 0}\mu(x,t)=0$,
(viii) $\upsilon(x,t)<1$,
(ix) $\upsilon(x,t)=0$ if and only if $x=0$,
(x) $\upsilon(\alpha x,t)=\upsilon\left(x,\frac{t}{|\alpha|}\right)$ if $\alpha\neq 0$,
(xi) $\upsilon(x,t)\circ\upsilon(y,s)\ge\upsilon(x+y,s+t)$,
(xii) $\upsilon(x,.):(0,\infty)\to[0,1]$ is continuous,
(xiii) $\lim\limits_{t\to\infty}\upsilon(x,t)=0$ and $\lim\limits_{t\to 0}\upsilon(x,t)=1$.

In this case (μ,υ) is called intuitionistic fuzzy norm.

Example 2.1. ([32]) Let(X, $\|.\|$) be a normed space, and let $a*b=ab$ and $a\circ b=\min\{a+b,1\}$ for all $a,b\in[0,1]$. For all $x\in X$ and every $t>0$, consider

$\mu(x,t):=\frac{t}{t+\|x\|}$ and $\upsilon(x,t):=\frac{\|x\|}{t+\|x\|}$.

Then $(X,\mu,\upsilon,*,\circ)$ is an IFNLS.

Definition 2.4. ([32]) Let $(X,\mu,\upsilon,*,\circ)$ be an IFNLS. For $t>0$, the open ball $B_x(r,t)$ with center $x\in X$ and radius $r\in(0,1)$ is defined as

$$B_x(r,t)=\{y\in X:\mu(x-y,t)>1-r \text{ and } \upsilon(x-y,t)<r\}.$$

Definition 2.5.([26]) If X is a non-empty set, then a family of sets $I\subset P(X)$ is called an ideal in X if and only if

(i) $\emptyset\in I$,
(ii) $A,B\in I$ implies that $A\cup B\in I$, and
(iii) for each $A\in I$ and $B\subset A$ we have $B\in I$,

where $P(X)$ is the power set of X.

Definition 2.6.([26]) If X is a non-empty set, then a non-empty family of sets $F\subset P(X)$ is called a filter on X if and only if

(i) $\emptyset\notin F$,

(ii) $A, B \in F$ implies that $A \cap B \in F$, and
(iii) for each $A \in F$ and $A \subset B$, we have $B \in F$.

An ideal I is called non-trivial if $I \neq \emptyset$ and $X \notin I$. A non-trivial ideal $I \subset P(X)$ is called an admissible ideal in X if and only if it contains all singletons, i.e., if it contains $\{\{x\} : x \in X\}$.
A relation between the concepts of an ideal and a filter is given by the following proposition.

Proposition 2.1. ([26]) Let $I \subset P(X)$ be a non-trivial ideal. Then the class $F = F(I) = \{M \subset N : M = X - A, \ for\ some\ A \in I\}$ is a filter on X. $F = F(I)$ is called the filter associated with the ideal I.

Definition 2.7 ([30]) Let I_2 be a non-trivial ideal of $N \times N$ and $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A double sequence $x = (x_{ij})$ of elements of X is said to be I_2-convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, the set

$$\left\{(i,j) \in N \times N : \mu\left(x_{ij} - L, t\right) \leq 1 - \varepsilon \ \text{ or } \ \upsilon\left(x_{ij} - L, t\right) \geq \varepsilon\right\} \in I_2.$$

In this case, we write $I_2^{(\mu,\upsilon)} - \lim x = L$.

Definition 2.9. ([20]) An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If the convexity of Orlicz function M is replaced by $M(x+y) \leq M(x+y) + M(y)$, then this function is called modulus function.

Remark 2.1. ([20]) If M is an Orlicz function, then $M(\lambda x) \leq \lambda M(x)$ for all λ with $0 < \lambda < 1$.

Main results

In this paper, we introduce a variant of ideal convergent difference double sequence spaces in the intuitionistic fuzzy normed linear spaces. We also investigate some topological properties of these new spaces.
Let w_2 be the space of all double sequences in the intuitionistic fuzzy normed linear spaces. We define the following sequence spaces:

$$I_2{}^{(\mu,\upsilon)}(M, \Delta) =$$

$$\{(x_{ij}) \in w_2 : \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - L, t\right)}{\rho}) \leq 1 - \varepsilon \ \text{ or } \ M(\frac{\upsilon\left(\Delta x_{ij} - L, t\right)}{\rho}) \geq \varepsilon\right\} \in I_2\}$$

and

$$I_2^{0(\mu,\upsilon)}(M, \Delta) =$$

$$\{(x_{ij}) \in w_2 : \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij}, t\right)}{\rho}) \leq 1 - \varepsilon \ \text{ or } \ M(\frac{\upsilon\left(\Delta x_{ij}, t\right)}{\rho}) \geq \varepsilon\right\} \in I_2\}.$$

Theorem 3.1. The spaces $I_2{}^{(\mu,\upsilon)}(M, \Delta)$ and $I_2^{0(\mu,\upsilon)}(M, \Delta)$ are linear spaces.

Proof. We prove the result for $I_2{}^{(\mu,\upsilon)}(M, \Delta)$. Similarly, it can be proved for $I_2^{0(\mu,\upsilon)}(M, \Delta)$. Let $(x_{ij}), (y_{ij}) \in I_2{}^{(\mu,\upsilon)}(M, \Delta)$ and α, β be scalars. The proof is trivial for $\alpha = 0$ and $\beta = 0$. Let $\alpha \neq 0$ and $\beta \neq 0$. For a given $\varepsilon > 0$, choose $s > 0$ such that $(1-\varepsilon) * (1-\varepsilon) > 1 - s$ and $\varepsilon \circ \varepsilon < s$. Hence, we have

$$A_1 = \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) \leq 1 - \varepsilon \ \text{ or } \ M(\frac{\upsilon\left(\Delta x_{ij} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) \geq \varepsilon\right\} \in I_2,$$

$$A_2 = \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - L_1, \frac{t}{2|\beta|}\right)}{\rho}) \leq 1 - \varepsilon \ \text{ or } \ M(\frac{\upsilon\left(\Delta x_{ij} - L_1, \frac{t}{2|\beta|}\right)}{\rho}) \geq \varepsilon\right\} \in I_2,$$

$$A_1^c = \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) > 1 - \varepsilon \ \text{ and } \ M(\frac{\upsilon\left(\Delta x_{ij} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) < \varepsilon\right\} \in F(I_2),$$

and

$$A_2^c = \left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - L_1, \frac{t}{2|\beta|}\right)}{\rho}) > 1 - \varepsilon \ \text{ and } \ M(\frac{\upsilon\left(\Delta x_{ij} - L_1, \frac{t}{2|\beta|}\right)}{\rho}) < \varepsilon\right\} \in F(I_2).$$

Let define the set $A_3 = A_1 \cup A_2$. Hence $A_3 \in I_2$. It follows that A_3^c is a non-empty set in $F(I_2)$. We will prove that for every $(x_{ij}), (y_{ij}) \in I_2{}^{(\mu,\upsilon)}(M, \Delta)$,

$$A_3^c \subset \left\{(i,j) \in N \times N : M(\tfrac{\mu((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha.L_1 + \beta.L_2), t)}{\rho}) > 1 - s \right.$$
$$\left. \text{and } M(\tfrac{\upsilon((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha.L_1 + \beta.L_2), t)}{\rho}) < s\right\}.$$

Let $(m,n) \in A_3^c$. In this case,

$$M(\frac{\mu\left(\Delta x_{mn} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) > 1-\varepsilon \quad \text{and } M(\frac{\upsilon\left(\Delta x_{mn} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) < \varepsilon,$$

and

$$M(\frac{\mu\left(\Delta y_{mn} - L_2, \frac{t}{2|\beta|}\right)}{\rho}) > 1-\varepsilon \quad \text{and } M(\frac{\upsilon\left(\Delta y_{mn} - L_2, \frac{t}{2|\beta|}\right)}{\rho}) < \varepsilon \quad .$$

Then

$$M(\frac{\mu((\alpha.\Delta x_{mn} + \beta.\Delta y_{mn}) - (\alpha.L_1 + \beta.L_2), t)}{\rho})$$

$$\geq M(\frac{\mu(\alpha.\Delta x_{mn} - \alpha.L_1, t/2)}{\rho}) * M(\frac{\mu(\beta.\Delta y_{mn} - \beta.L_2, t/2)}{\rho})$$

$$= M(\frac{\mu\left(\Delta x_{mn} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) * M(\frac{\mu\left(\Delta y_{mn} - L_2, \frac{t}{2|\beta|}\right)}{\rho}) > (1-\varepsilon)*(1-\varepsilon) > 1-s$$

and

$$M(\frac{\upsilon((\alpha.\Delta x_{mn} + \beta.\Delta y_{mn}) - (\alpha.L_1 + \beta.L_2), t)}{\rho})$$

$$\leq M(\frac{\upsilon(\alpha.\Delta x_{mn} - \alpha.L_1, t/2)}{\rho}) \circ M(\frac{\upsilon(\beta.\Delta y_{mn} - \beta.L_2, t/2)}{\rho})$$

$$= M(\frac{\upsilon\left(\Delta x_{mn} - L_1, \frac{t}{2|\alpha|}\right)}{\rho}) \circ M(\frac{\upsilon\left(\Delta y_{mn} - L_2, \frac{t}{2|\beta|}\right)}{\rho}) < \varepsilon \circ \varepsilon < s.$$

This proves that

$$A_3^c \subset \Big\{(i,j) \in N \times N : M(\tfrac{\mu((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha.L_1 + \beta.L_2), t)}{\rho}) > 1-s$$
$$\text{and } M(\tfrac{\upsilon((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha.L_1 + \beta.L_2), t)}{\rho}) < s\Big\}.$$

Hence $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ is a linear space.

Theorem 3.2. Every closed ball $B_x^c(r,t)(M)$ is an open set in $I_2{}^{(\mu,\upsilon)}(M,\Delta)$.

Proof. Let $B_x(r,t)(M)$ be an open ball with centre $x \in I_2{}^{(\mu,\upsilon)}(M,\Delta)$ and radius $r \in (0,1)$ with respect to t, i.e.

$$B_x(r,t)(M) = \{y \in I_2{}^{(\mu,\upsilon)}(M,\Delta) :$$
$$\left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij} - \Delta y_{ij}, t\right)}{\rho}) \leq 1-r \text{ or } M(\frac{\mu\left(\Delta x_{ij} - \Delta y_{ij}, t\right)}{\rho}) \geq r\right\} \in I_2\}.$$

Let $y \in B_x^c(r,t)(M)$. So $M(\frac{\mu\left(\Delta x - \Delta y, t\right)}{\rho}) > 1-r$ and $M(\frac{\upsilon\left(\Delta x - \Delta y, t\right)}{\rho}) < r$.

Since $M(\frac{\mu\left(\Delta x - \Delta y, t\right)}{\rho}) > 1-r$, there exists $t_0 \in (0,t)$ such that $M(\frac{\mu\left(\Delta x - \Delta y, t_0\right)}{\rho}) > 1-r$ and $M(\frac{\upsilon\left(\Delta x - \Delta y, t_0\right)}{\rho}) < r$.

Let $r_0 = M(\frac{\mu\left(\Delta x - \Delta y, t_0\right)}{\rho})$. Since $r_0 > 1-r$, there exists $s \in (0,1)$ such that $r_0 > 1-s > 1-r$ and so there exists $r_1, r_2 \in (0,1)$ such that $r_0 * r_1 > 1-s$ and $(1-r_0) \circ (1-r_2) < s$.

Let $r_3 = max\{r_1, r_2\}$. Then $1-s < r_0 * r_1 \leq r_0 * r_3$ and $(1-r_0) \circ (1-r_3) \leq (1-r_0) \circ (1-r_2) < s$.

Consider the closed balls $B_y^c(1-r_3, t-t_0)(M)$ and $B_x^c(r,t)(M)$. We prove that $B_y^c(1-r_3, t-t_0)(M) \subset B_x^c(r,t)(M)$. Let $z \in B_y^c(1-r_3, t-$

$t_0)(M)$. Then $M(\frac{\mu(\Delta y-\Delta z,t-t_0)}{\rho}) > r_3$ and $M(\frac{\upsilon(\Delta y-\Delta z,t-t_0)}{\rho}) < 1-r_3$. Hence

$$M(\frac{\mu(\Delta x-\Delta z,t)}{\rho}) \geq M(\frac{\mu(\Delta x-\Delta y,t_0)}{\rho}) * M(\frac{\mu(\Delta y-\Delta z,t-t_0)}{\rho}) > r_0 * r_3 \geq r_0 * r_1 > 1-s > 1-r,$$

and

$$M(\frac{\upsilon(\Delta x-\Delta z,t)}{\rho}) \leq M(\frac{\upsilon(\Delta x-\Delta y,t_0)}{\rho}) \circ M(\frac{\upsilon(\Delta y-\Delta z,t-t_0)}{\rho})$$
$$< (1-r_0)\circ(1-r_3) < s < r.$$

Thus $z \in B_x^c(r,t)(M)$ and it proves that $B_y^c(1-r_3,t-t_0)(M) \subset B_x^c(r,t)(M)$.

Remark 3.1. It is clear that $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ is an IFNLS. Define

$$\tau_2{}^{(\mu,\upsilon)}(M,\Delta) = \{A \subset I_2{}^{(\mu,\upsilon)}(M,\Delta) :$$
$$for\ each\ x \in A,\ there\ exist\ t>0\ and\ r \in (0,1)\ such\ that\ B_x^c(r,t)(M) \subset A\}.$$

Then $\tau_2{}^{(\mu,\upsilon)}(M,\Delta)$ is a topology on $I_2{}^{(\mu,\upsilon)}(M,\Delta)$.

Theorem 3.3. The topology $\tau_2{}^{(\mu,\upsilon)}(M,\Delta)$ on $I_2^{0(\mu,\upsilon)}(M,\Delta)$ is first countable.

Proof. It is clear that $\{B_x^c(\frac{1}{n},\frac{1}{n})(M) : n \in N\}$ is a local base at $x \in I_2{}^{(\mu,\upsilon)}(M,\Delta)$. Then, the topology $\tau_2{}^{(\mu,\upsilon)}(M,\Delta)$ on $I_2^{0(\mu,\upsilon)}(M,\Delta)$ is first countable.

Theorem 3.4. $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ and $I_2^{0(\mu,\upsilon)}(M,\Delta)$ are Hausdorff spaces.

Proof. Let $x,y \in I_2{}^{(\mu,\upsilon)}(M,\Delta)$ such that $x \neq y$. Then $0 < M(\frac{\mu(\Delta x-\Delta y,t)}{\rho}) < 1$ and $0 < M(\frac{\upsilon(\Delta x-\Delta z,t)}{\rho}) < 1$.

Define r_1, r_2 and r such that $r_1 = M(\frac{\mu(\Delta x-\Delta y,t)}{\rho})$, $r_2 = M(\frac{\upsilon(\Delta x-\Delta y,t)}{\rho})$ and $r = max\{r_1, 1-r_2\}$. Then for each $r_0 \in (r,1)$ there exist r_3 and r_4 such that $r_3 * r_4 \geq r_0$ and $(1-r_3)\circ(1-r_4) \leq (1-r_0)$.

Let $r_5 = max\{r_3,(1-r_4)\}$ and consider the closed balls $B_x^c(1-r_5,\frac{t}{2})(M)$ and $B_y^c(1-r_5,\frac{t}{2})(M)$. Then, clearly $B_x^c(1-r_5,\frac{t}{2})(M) \cap B_y^c(1-r_5,\frac{t}{2})(M) = \emptyset$.

Suppose that $z \in B_x^c(1-r_5,\frac{t}{2})(M) \cap B_y^c(1-r_5,\frac{t}{2})(M)$. So,

$$r_1 = M(\frac{\mu(\Delta x-\Delta y,t)}{\rho}) \geq M(\frac{\mu(\Delta x-\Delta z,t/2)}{\rho}) * M(\frac{\mu(\Delta y-\Delta z,t/2)}{\rho})$$
$$\geq r_5 * r_5 \geq r_3 * r_4 \geq r_0 > r \text{ and}$$
$$r_2 = M(\frac{\upsilon(\Delta x-\Delta y,t)}{\rho}) \leq M(\frac{\upsilon(\Delta x-\Delta z,t/2)}{\rho}) \circ M(\frac{\upsilon(\Delta y-\Delta z,t/2)}{\rho})$$
$$\leq (1-r_5)\circ(1-r_5) \leq (1-r_3)\circ(1-r_4) \leq (1-r_0) < 1-r,$$

which is a contradiction. Hence $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ is a Hausdorff space.

Theorem 3.5. Let $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ be an IFNLS, $\tau_2{}^{(\mu,\upsilon)}(M,\Delta)$ be a topology on $I_2{}^{(\mu,\upsilon)}(M,\Delta)$ and (x_{ij}) be a sequence in $I_2{}^{(\mu,\upsilon)}(M,\Delta)$. Then a sequence (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ) if and only if $M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) \longrightarrow 1$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) \longrightarrow 0$ as $i,j \longrightarrow \infty$.

Proof. Let $B_{x_0}(r,t)(M)$ be an open ball with centre $x_0 \in I_2{}^{(\mu,\upsilon)}(M,\Delta)$ and radius $r \in (0,1)$ with respect to t, i.e.

$$B_{x_0}(r,t)(M) = \{(x_{ij}) \in I_2{}^{(\mu,\upsilon)}(M,\Delta) :$$
$$\left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) \leq 1-r \text{ or } M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) \geq r\right\} \in I_2\}.$$

Suppose (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ). Then for $r \in (0,1)$ and $t > 0$, there exists $k_0 \in N$ such that $(x_{ij}) \in B_{x_0}^c(r,t)(M)$ for all $i,j \geq k_0$. Thus,

$$\left\{(i,j) \in N \times N : M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) > 1-r \text{ and } M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho}) < r\right\} \in F(I_2).$$

So $1-M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})<r$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})<r$, for all $i,j\geq k_0$. Then $M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})\longrightarrow 1$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})\longrightarrow 0$ as $i,j\longrightarrow\infty$.

Conversely, if for each $t>0$,

$M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})\longrightarrow 1$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})\longrightarrow 0$ as $i,j\longrightarrow\infty$. Then for $r\in(0,1)$, there exists $k_0\in N$ such that $1-M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})<r$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})<r$ for all $i,j\geq k_0$. So, $M(\frac{\mu\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})>1-r$ and $M(\frac{\upsilon\left(\Delta x_{ij}-\Delta x_0,t\right)}{\rho})<r$ for all $i,j\geq k_0$. Hence $(x_{ij})\in B^c_{x_0}(r,t)(M)$ for all $i,j\geq k_0$. This proves that a sequence (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ).

Acknowledgements

The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Funding

There is no funding for this work.

Availability of data and materials

Not applicable.

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

References

[1] S. Altundağ, E. Kamber, *Weighted statistical convergence in intuitionistic fuzzy normed linear spaces*, J. Inequal. Spec. Funct., **8**(2017), 113-124.
[2] S. Altundağ, E. Kamber, *Weighted lacunary statistical convergence in intuitionistic fuzzy normed linear spaces*, Gen. Math. Notes, **37**(2016), 1-19.
[3] S. Altundağ, E. Kamber, *Lacunary Δ- statistical convergence in intuitionistic fuzzy n-normed linear spaces*, Journal of inequalities and applications, **40**(2014), 1-12.
[4] G.A. Anastassiou, *Fuzzy approximation by fuzzy convolution type operators*, Comput. Math. Appl., **48**(2004), 1369-1386.
[5] L.C. Barros, R.C. Bassanezi, P.A. Tonelli, *Fuzzy modelling in population dynamics*, Ecol. Model., **128**(2000), 27-33.
[6] P. Das, P. Kostyrko, W. Wilczynski, P. Malik, *I- and I^*- convergence of double sequences*, Math. Slovaca, **58**(2008), 605-620.
[7] M.A. Erceg, *Metric spaces in fuzzy set theory*, J. Math. Anal. Appl., **69**(1979), 205-230.
[8] H. Fast, *Sur la convergence statistique*, Colloq. Math., **2**(1951), 241-244.
[9] A. George, P. Veeramani, *On some result in fuzzy metric space*, Fuzzy Sets Syst., **64**(1994), 395-399.
[10] L. Hong, J.Q. Sun, *Bifurcations of fuzzy nonlinear dynamical systems*, Commun. Nonlinear Sci. Numer. Simul., **1**(2006), 1-12.
[11] G. Jäger, *Fuzzy uniform convergence and equicontinuity*, Fuzzy Sets Syst., **109**(2000), 187-198.
[12] O. Kaleva, S. Seikkala, *On fuzzy metric spaces*, Fuzzy Sets Syst., **12**(1984), 215-229.
[13] V.A. Khan, R.K.A. Rababah, K.M.A.S. Alshlool, S.A.A. Abdullah, A. Ahmad, *On ideal convergence Fibonacci difference sequence spaces*, Adv. Differ. Equ.,**2018**(2018), 1-14.
[14] V.A. Khan, M. Ahmad, H. Fatima, M.F. Khan, *On some results in intuitionistic fuzzy ideal convergence double sequence spaces*, Adv. Differ. Equ., **2019**(2019), 1-10.
[15] V. Kumar, K. Kumar, *On ideal convergence of double sequences in intuitionistic fuzzy normed spaces*, Selçuk J. Appl. Math. **24**(2009), 27-41.
[16] A. KÄ±lÄ±Ã§man, S. Borgohain, *Generalized difference strongly summable sequence spaces of fuzzy real numbers defined by ideal convergence and Orlicz function*, Adv. Differ. Equ., **2013**(2013), 1-10.
[17] S.A. Mohiuddine, A. Alotaibi, S.M. Alsulami,*Ideal convergence of double sequences in random 2-normed spaces*, Adv. Differ. Equ.,**2012**(2012), 1-8.
[18] E.E. Kara,*Some topological and geometrical properties of new Banach sequence spaces*, J. Inequal. Appl.,**2013**(2013), 1-15.
[19] E.E. Kara, M. Ilkhan, *On some Banach sequence spaces derived by a new band matrix*, Br. J. Math. Comput. Sci., **9**(2015), 141â€“159.
[20] E.E. Kara, M. Ilkhan, *Lacunary I-convergent and lacunary I-bounded sequence spaces defined by an Orlicz function*, Electronic Journal of Mathematical Analysis and Applications, **4**(2016), 150-159.
[21] E. Kamber, *Intuitionisic fuzzy I-convergent difference sequence spaces defined by modulus function*, J. Inequal. Spec. Funct., **10**(2019), 93-100.
[22] E. Kamber, *Intuitionisic fuzzy I-convergent difference sequence spaces defined by compact operator*, submitted.
[23] E. Kamber, *Intuitionisic fuzzy I-convergent difference double sequence spaces* , Research and Communications in Mathematics and Mathematical Sciences, **10**(2018), 141-153.
[24] E. Kamber, S. Altundağ, *Intuitionisic fuzzy I-convergent difference double sequence spaces defined by modulus function*, submitted.
[25] H. Kızmaz, *On certain sequence spaces*, Canad. Math. Bull., **24**(1981), 169-176.
[26] P. Kostyrko, T. Salat, W. Wilczynski, *I-convergence* , Real Analysis Exchange, **26**(2000), 669-686.
[27] J. Madore, *Fuzzy physics*, Ann. Phys., **219**(1992), 187-198.
[28] E. Malkowsky, *Recent results in the theory of matrix transformation in sequence spaces*, Math. Vesnik, **49**(1997), 187-196.
[29] M.A. Tok, E.E. Kara, S. Altundağ, *On the $\alpha\beta$-statistical convergence of the modified discrete operators*, Adv. Diff.Equ.,**2018**(2018), 1-6.
[30] M. Mursaleen, S.A. Mohiuddine, H. Osama, H. Edely, *On the ideal convergence of double sequences in intuitionistic fuzzy normed spaces*, Computers and Mathematics with Applications, **59**(2010), 603-611.

[31] E. Dündar, U. Ulusu, N. Pancaroğlu, *Strongly I_2-lacunary convergence and I_2-lacunary Cauchy double sequences of sets*, Aligarh Bull. Math., **35**(2016), 1-15.
[32] R. Saadati, J.H. Park, *On the intuitionistic fuzzy topological spaces*, Chaos Soliton. Fractal., **27**(2006), 331-344.
[33] B. Schweizer, A. Sklar, *Statistical metric spaces*, Pac. J. Math., **10**(1960), 313-334.
[34] H. Steinhaus, *Sur la convergence ordinaire et la convergence asymptotique*, Colloq. Math., **2**(1951), 73-74.
[35] Şengönül, M., *On the Zweier sequence space*, Demonstratio Mathematica, **XL**(2007), 181-196.
[36] S. Vijayabalaji, , N. Thillaigovindan, Y.B. Jun, *Intuitionistic fuzzy n- normed linear space*, Bull. Korean. Math. Soc., **44**(2007), 291-308.
[37] C. S. Wang, *On Nörlund sequence spaces*, Tamkang J. Math., **9**(1978), 269-274.
[38] K. Wu, *Convergences of fuzzy sets based on decomposition theory and fuzzy polynomial function*, Fuzzy Sets Syst., **109**(2000), 173-185.
[39] Zadeh, L.A., *Fuzzy Sets*, Inform. Cont., **8**(1965).