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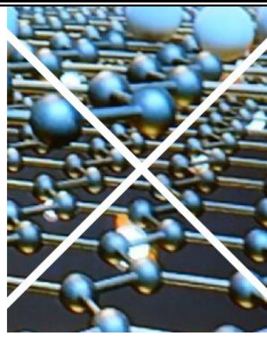

CRITICAL SIZE OF A SLAB REACTOR WITH TRIPLET ANISOTROPIC SCATTERING IN THE NEUTRON TRASNPORT THEORY

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Abstract

Critical thickness for one-group energy neutrons are determined for the triplet anisotropic-scattering in plane geometry by using Legendre polynomials of P_N method, and Chebyshev polynomials of first type, T_N method. Triplet anisotropic scattering is the fourth term of the scattering function. The neutron flux moments in the neutron transport equation comprises the Eigen function of the neutron flux. By solving the Eigen functions, the eigenvalues are obtained from Chebyshev polynomial solution. The resultant neutron flux equation composes of the Eigen function, Chebyshev polynomial term and the number of secondary neutrons "c". The critical size of the system is found by the Mark boundary condition for different scattering types. The resultant critical thickness values are presented in the following tables. It is seen that our results are compatible with the existing literature.

Keyword: Critical Thickness, Slab Reactor, Neutron Transport Theory

1.Introduction

The neutron transport equation is used for describing the behavior of the neutrons in a reactor core. The equation includes three variables for position vector, three variables for the velocity of neutrons and time variable. In totally the seven parameters constituted the general line of the neutron transport equation. Many solutions based on approximations about the geometry of the system, energy of the neutrons in the medium and time dependent or independent case have been suggested. [1-8].

The critical thickness problem is an remarkable problem in the neutron transport theory. The problem can be taken into account for many views. The scattering function types are analyzed in this study.

The scattering function is related with the cosine angle of the scattering and the Legendre polynomials of first type. So the changing with the expansion of the Legendre is directly related with the scattering function and also the critical thickness. In this study the scattering function is expanded up to third term that is called as triplet anisotropic scattering (n=3). We aimed to represent the solubility of the neutron transport equation for triplet anisotropic scattering with the T_N and P_N methods. The calculations are done for a wide range of the number of secondary neutrons c. So the effect of the scattering function with different type of scatterings on the criticality problem can be examined by making this calculations.

If one deals with the steady-state, one-speed, plane geometrical approximation, the equation takes the form of only two variables, one of them is the position which is represented by x, the other variable μ is the cosine direction. Thus, the steady-state neutron transport equation for one-group energetic neutrons in plane geometry can be written as [8],

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \psi(x,\mu) = \frac{c}{2}\int_{-1}^{1} f(\mu,\mu')\psi(x,\mu')\,d\mu' \tag{1}$$

where $f(\mu,\mu')$ is the scattering function and defines the scattering probability of neutrons, μ' is the scattering direction, , $\psi(x,\mu)$ is the neutron flux at position x and direction μ, the parameter c defined by the material cross sections as $c\sigma_t = \upsilon\sigma_f + \sigma_s$, is the number of secondary neutrons per collision in which σ_f is the fission cross section and σ_s is the scattering cross section, υ is the number of neutrons per fission. The scattering function in Eq. (1) can be written in terms of the Legendre polynomials [9] as

$$f(\mu,\mu') = \sum_{n=0}^{N} (2n+1) f_n P_n(\mu) P_n(\mu') \tag{2}$$

where f_n is the scattering coefficient, and $P_n(\mu)$ and $P_n(\mu')$ are Legendre polynomials. Since the scattering function is defined as the probability of scattering, the summation of all these scatterings is equal to unity. That means the individual values of f_n is evaluated for every scattering situation.

The first term (n=0) of Eq. (2) is defined as isotropic, the second term (n=1) is called as linear anisotropic scattering, the third term (n=2) is quadratic anisotropic scattering and the fourth term (n=3) is named as triplet anisotropic scattering. The probability of scattering coefficient is proportional to the multiplication of the μ and μ' which are $P_n(\mu)$ and $P_n(\mu')$

The Chebyshev polynomials first type is an attractive method to solve the neutron transport theory. Mika's anisotropic scattering function can be easily applied to T_N method. The effect of the high order anisotropic scattering on the critical thickness problem of the neutron transport theory is examined.

2. Solution Methods for Triplet Anisotropic Scattering in Neutron Transport Equation

2.a. P_N Method

The angular flux in terms of the Legendre polynomials [10] are given by

$$\psi\left(x,\mu\right)=\sum_{n=0}^{\infty}\frac{2n+1}{2}\phi_n(x)P_n(\mu) \quad (3)$$

Legendre moments of the flux [3] are defined by

$$\phi_n(x)=\int_{-1}^{1}P_n(\mu')\psi(x,\mu')d\mu' \qquad (4)$$

The scattering function in Eq. (2) for triplet anisotropic scattering is given as,

$$f\left(\mu,\mu'\right)=f_0P_0(\mu)P_0(\mu')+3f_1P_1(\mu)P_1(\mu')+5f_2P_2(\mu)P_2(\mu')+7f_3P_3(\mu)P_3(\mu') \qquad (5)$$

If one replaces Eq. (3) into Eq. (1) with the definition of the Legendre moments, then one gets

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\psi(x,\mu)=\frac{c}{2}\left[P_0(\mu)f_0\phi_0(x)+3P_1(\mu)f_1\phi_1(x)+5P_2(\mu)f_2\phi_2(x)+7P_3(\mu)f_3\phi_3(x)\right] \qquad (6)$$

Using the Eq. (6) with the orthogonality and recurrence relation of the Legendre polynomials of first kind, respectively [11].

$$\int_{-1}^{1}P_m(\mu)P_n(\mu)d\mu=\begin{cases}0 & m\neq n\\ \dfrac{2}{2n+1} & m=n\end{cases} \qquad (7)$$

$$\mu P_n\left(\mu\right)=\frac{1}{2n+1}\left[\left(n+1\right)P_{n+1}(\mu)+nP_{n-1}(\mu)\right] \qquad (8)$$

One obtains the $P_n\left(\mu\right)$ moments $\phi_n(x)$ in general form as

$$\left(n+1\right)\frac{d\phi_{n+1}(x)}{dx}+n\frac{d\phi_{n-1}(x)}{dx}+(2n+1)(1-cf_n\delta_{n0}+cf_n\delta_{n1}+cf_n\delta_{n2}$$
$$+cf_n\delta_{n3})\phi_n(x)=0, \qquad n=0,1,2,....,N \qquad (9)$$

Here, the Kronecker delta is $\delta_{nm}=\begin{cases}1, & n=m\\ 0, & n\neq m\end{cases}$. In order to obtain the eigenvalue spectrum, a well-known solution for the homogeneous Eq. (9), is employed, of the form [12]

$$\phi_n(x)=A_n(v)e^{-x/v} \qquad (10)$$

in which A_n are the Eigen functions corresponding to eigenvalues, v. The Eigen functions of the flux function are taken into account in the equation of criticality depend on the discrete eigenvalues v, c and f_n. After replacing Eq. (10) in Eq. (9), one finds the eigenvalues in the triplet anisotropic scattering. Then, one can write the system of equations for $A_n(v)$ as follows

$$A_1(v) = vA_0(v)(1-cf_0) \tag{11}$$

$$A_2(v) = \frac{3vA_1(v)(1-cf_1) - A_0(v)}{2} \tag{12}$$

$$A_3(v) = \frac{5vA_2(v)(1-cf_2) - 2A_1(v)}{3} \tag{13}$$

$$A_4(v) = \frac{7vA_3(v)(1-cf_3) - 3A_2(v)}{4} \tag{14}$$

An equality corresponding to the forms of $A_n(v)$ through Eq. (11-14) can be written as

$$\begin{aligned}&(n+1)A_{n+1}(v) + nA_{n-1}(v) + (2n+1)(1 - cf_n\delta_{n0} + cf_n\delta_{n1} + cf_n\delta_{n2} \\ &+cf_n\delta_{n3})vA_n(v) = 0, \qquad n = 0,1,2,...,N\end{aligned} \tag{15}$$

As seen in Eq. (15), the solution of $A_{-1}(v) = 0$ and $A_0(v) = 1$ One can obtain the discrete and continuum v eigenvalues by setting $A_{n+1}(v) = 0,$ for various values of c and f_n. The roots of the eigenvalue is found from the solving of Eq. (15) for any iteration of P_{N+1} (v)=0 for Legendre polynomials and T_{N+1} (v)=0 for the first type of the Chebyshev polynomials solution. If c=1, then one pair of roots is imaginary (±∞i), other pairs are in the range [-1, +1]. If 0<c<1, then all roots are imaginary but one pair of them is greater than 1. If c>1, then one pair of roots are pure imaginary, and the other pairs are in the range [-1, +1]. After obtaining the discrete eigenvalues, the general solution of the flux moments in Eq. (15) can be written by

$$\phi_n(x) = \sum_{k=1}^{\frac{N+1}{2}} \beta_k A_n(v_k)\left[e^{x/v_k} + (-1)^n e^{-x/v_k}\right] \qquad (N+1)/2 < k \le (N+1) \tag{16}$$

for odd numbers of N. Here, β_k are the coefficients as results of linear combinations of the solutions corresponding to each v_k, and they are determined by the boundary conditions of the system where parity relation is used as $A_n(-v) = (-1)^n A_n(v)$. Thus, the general solution to Eq. (1) is obtained by replacing Eq. (16) into Eq. (6). Finally, one writes

$$\psi(x,\mu) = \sum_{n=0}^{\infty}\sum_{k=1}^{\frac{N+1}{2}} \frac{2n+1}{2}\beta_k A_n(v_k)\left[\left(1+(-1)^n\right)\cosh(\frac{x}{v_k}) + \left(1-(-1)^n\right)\sinh(\frac{x}{v_k})\right]P_n(\mu) \tag{17}$$

2.b.T_N Method

The angular flux defined in terms of first type Chebyshev polynomials [13] is given as

$$\psi(x,\mu) = \frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu) + \frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu) \tag{18}$$

where $T_n(\mu)$ is the first type Chebyshev polynomial and $\phi_n(x)$ is called as the flux moment. Eq. (18) is replaced in Eq.(1) and one gets

$$\mu\frac{\partial}{\partial x}\left(\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu)+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu)\right)+\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu)+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu)=$$
$$\frac{c}{2}\int_{-1}^{1}f(\mu,\mu')\left(\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu')+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu')\right)d\mu' \tag{19}$$

In Eq. (19), we consider the triplet anisotropic scattering case in scattering function as shown in Eq. (3). When the scattering function is substituted into Eq. (19),

$$\mu\frac{\partial}{\partial x}\left(\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu)+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu)\right)+\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu)+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu)$$
$$=\frac{c}{2}\int_{-1}^{1}\begin{pmatrix} f_0P_0(\mu)P_0(\mu')+3f_1P_1(\mu)P_1(\mu') \\ +5f_2P_2(\mu)P_2(\mu')+7f_3P_3(\mu)P_3(\mu')\end{pmatrix}\left(\frac{\phi_0(x)}{\pi\sqrt{1-\mu^2}}T_0(\mu')+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\phi_n(x)T_n(\mu')\right)d\mu' \tag{20}$$

For the first type Chebyshev polynomial, the recursion relation is given as

$$T_{n+1}(\mu)-2\mu T_n(\mu)+T_{n-1}(\mu)=0 \tag{21}$$

and the orthogonally relation is defined as

$$\int_{-1}^{1}T_m(\mu)T_n(\mu)(1-\mu^2)^{-1/2}d\mu=\begin{cases}0, & m\neq n\\ \pi/2, & m=n\neq 0\\ \pi, & m=n=0\end{cases} \tag{22}$$

Using Eq. (21) and Eq. (22) in Eq. (20), one can obtains the T_N moments of the angular flux $\phi_n(x)$ as followings:

$$\begin{aligned}
&m=0, \quad \frac{d\phi_1(x)}{dx}+\phi_0(x)=cf_0\phi_0(x)\\
&m=1, \quad \frac{d\phi_2(x)}{dx}+\frac{d\phi_0(x)}{dx}+2\phi_1(x)=2cf_1\phi_1(x)\\
&m=2, \quad \frac{d\phi_3(x)}{dx}+\frac{d\phi_1(x)}{dx}+2\phi_2(x)=\frac{2}{3}cf_2\phi_0(x)-\frac{2}{3}cf_0\phi_0(x)+2cf_2\phi_2(x)\\
&m=3, \quad \frac{d\phi_4(x)}{dx}+\frac{d\phi_2(x)}{dx}+2\phi_3(x)=-\frac{6}{5}cf_1\phi_1(x)+\frac{6}{5}cf_3\phi_1(x)+2cf_3\phi_3(x)
\end{aligned} \tag{23}$$

Since the solution of recursion equations gives the flux moments, then a general expression can be given in the solution of the recursion equations

$$\phi_n(x)=G_n(v)\exp(x/v) \tag{24}$$

Using Eq. (24) in Eq. (23) to obtain the values of $G_n(v)$, the eigenfunctions of the flux moments and then the eigenvalues v can be found:

$$
\begin{aligned}
&G_0(v)=1\\
&G_1(v)=cvf_0G_0(v)-vG_0(v)\\
&G_2(v)=2cvf_1G_1(v)-2vG_1(v)-G_0(v)\\
&G_3(v)=\frac{2}{3}cvf_2G_0(v)-\frac{2}{3}cvf_0G_0(v)+2vcf_2G_2(v)-2vG_2(v)-G_1(v)\\
&G_4(v)=-\frac{6}{5}cvf_1G_1(v)+\frac{6}{5}cvf_3G_1(v)+2cvf_3G_3(v)-2vG_3(v)-G_2(v)
\end{aligned}
\qquad (25)
$$

The eigenvalues can be found by $G_{n+1}(v)=0$. In the T1 approximation, the eigenvalues are determined by solving $G_2(v)=0$. The coefficients of the eigenfunctions generate a (N+1)×(N+1) square matrices, and produces a column vector of $\mathbf{G}=\left[G_0,G_1,G_2,,,G_N\right]^T$.

In T_N approximation, there are N+1/2 eigenvalues of v_k, where $k=1,...,N+1$ roots are used to find the flux moments. One obtains the equation of general solution for odd N values as

$$
\phi_n(x)=\sum_{k=1}^{\frac{N+1}{2}}\beta_k G_n(v_k)\left[\exp(x/v_k)+(-1)^n\exp(-x/v_k)\right] \qquad n=1,...,N \qquad (26)
$$

where the parity rule is used as $G_n(-v_k)=(-1)^nG_n(v_k)$, and β_k values are determined by using the boundary condition. Finally, one writes the flux function as

$$
\begin{aligned}
\psi(x,\mu)=&\frac{T_0(\mu)}{\pi\sqrt{1-\mu^2}}\sum_{k=1}^{N+1/2}\beta_k G_0(v_k)\left[(2)\cosh(\frac{x}{v_k})\right]\\
&+\frac{2}{\pi\sqrt{1-\mu^2}}\sum_{n=1}^{N}\sum_{k=1}^{N+1/2}\beta_k G_n(v_k)\left[\left(1+(-1)^n\right)\cosh(\frac{x}{v_k})+\left(1-(-1)^n\right)\sinh(\frac{x}{v_k})\right]T_n(\mu)
\end{aligned}
\qquad (27)
$$

2.c. Criticality Condition

The purpose in the solution of the neutron transport equation is to get a relation between the criticality problem and the number of secondary neutrons "c". This paper focuses on the cases where c>1. Eq. (17) is the result of P_N method. By using the Mark boundary condition, half-slab thickness is obtained. Mark used the concept of the continuity of the angular flux, which implies the continuity of all the angular moments of the neutron flux across the boundaries surrounded by the vacuum, and showed that is condition is equivalent to zero incoming angular flux at the boundaries for the specific values of μ. Here, we use the Mark type boundary condition [14] is

$$
\psi(a,\mu_k)=0, \qquad (N+1)/2<k\le(N+1) \qquad (28)
$$

where μ_k are the roots of the Legendre polynomials found from $P_{N+1}(\mu_k)=0$. The criticality equation can now be obtained for P_N method by using Eq. (17) in the Eq. (28) as following:

$$\psi(a,\mu_k)=\sum_{n=0}^{\infty}\sum_{k=1}^{\frac{N+1}{2}}\frac{2n+1}{2}\beta_k A_n(v_k)\left[\left(1+(-1)^n\right)\cosh(\frac{a}{v_k})+\left(1-(-1)^n\right)\sinh(\frac{a}{v_k})\right]P_n(\mu_k) \quad (29)$$

The critical thickness equation for the first type Chebyshev polynomials T_N method in Eq. (27) can be written by using the Mark boundary condition into Eq. (28) as

$$\begin{aligned}\psi(a,\mu_k)=&\frac{T_0(\mu_k)}{\pi\sqrt{1-\mu_k^2}}\sum_{k=1}^{N+1/2}\beta_k G_0(v_k)\left[(2)\cosh(\frac{a}{v_k})\right]\\&+\frac{2}{\pi\sqrt{1-\mu_k^2}}\sum_{n=1}^{N}\sum_{k=1}^{N+1/2}\beta_k G_n(v_k)\left[\left(1+(-1)^n\right)\cosh(\frac{a}{v_k})+\left(1-(-1)^n\right)\sinh(\frac{a}{v_k})\right]T_n(\mu_k)\end{aligned} \quad (30)$$

One can also write the Eq. (29) and Eq. (30) in a matrix form given by $\left[M_m^k(a)\right]\mathrm{B_k}=[0]$ where Bk is a vector with elements β_k and $\left[M_m^k(a)\right]$ is a square matrix with elements of $\left[(N+1)/2\right]^2$. The matrix $\left[M_m^k(a)\right]$ has a determinant which must be equal to zero for the criticality condition for a nontrivial solution of Eq. (29) and Eq. (30). So the critical thickness can be found by solving the Eq. (29) for P_N method and Eq. (30) for T_N method.

3. Results and Discussion

The neutron transport equation is solved to obtain the critical thickness of a slab reactor for the T_N and P_N method with the triplet anisotropic scattering by using the Mark boundary condition. The resultant criticality equations are showed in Eq. (29) for P_N method and in Eq. (30) for T_N method. It is known that the scattering function is between zero to one and related with the cosine angle $\mu_0\in[-1,1]$. For this reason, the scattering coefficients are determined according to the rule. When we analyzed the scattering function for triplet anisotropic scattering the scattering coefficients are obtained as f_1=0.3 for linear anisotropic; f_2=0.2 for quadratic anisotropic and f_3=0.142 for triplet anisotropic scatterings, respectively, [15], [16].

As seen in Table 1, the critical thickness is calculated with P_N method for triplet anisotropic scattering. It is seen that the critical thickness values decrease as the c value increases. In Tables 2,3, 4 and 5 the secondary neutron number is fixed for 1.01, 1.1, 1.5, and 2.0 respectively. The critical thickness is calculated for different scattering types. As it can be noted that the critical thickness decreases gradually as the c value changes from 1.01 to 2.0 in each scattering, for the 13th order (See the last column in each Table): Legendre polynomial have solutions up to thirteenth order and the convergence is up to three digits in our results.

Table 1. Critical half-thickness for triplet anisotropic scattering in P_N method

c	P_1	P_3	P_5	P_7	P_9	P_{11}	P_{13}
1.01	10.0384	9.82526	9.81031	9.80604	9.80423	9.80329	9.80222
1.05	4.07718	3.83286	3.81531	3.81064	3.80868	3.80767	3.80709
1.10	2.68134	2.43080	2.40904	2.40381	2.40166	2.40057	2.39994
1.20	1.71227	1.47377	1.44256	1.43586	1.43331	1.43203	1.43130
1.40	1.05213	0.85271	0.80956	0.79860	0.79482	0.79310	0.79216
1.60	0.77486	0.60774	0.56201	0.54761	0.54214	0.53972	0.53847
1.80	0.61747	0.47453	0.43025	0.41419	0.40734	0.40410	0.40242
2.00	0.51481	0.39021	0.34864	0.33215	0.32447	0.32055	0.31843

Table 2. Critical half-thickness for different scattering types for c=1.01

Scattering types	P_1	P_3	P_5	P_7	P_9	P_{11}	P_{13}
Isotropic	8.49356	8.34635	8.33616	8.33309	8.33175	8.33104	8.33064
Lin.ans.	10.0384	9.83575	9.82133	9.81695	9.81505	9.81404	9.81297
Pure quadratic	8.49356	8.34750	8.32548	8.32258	8.32133	8.32068	8.32032
Quadratic	10.0384	9.82195	9.80846	9.80378	9.80201	9.80109	9.80012
Pure triplet	8.49356	8.34868	8.33784	8.33466	8.33329	8.33257	8.33214
Triplet	10.0384	9.82526	9.81031	9.80604	9.80423	9.80329	9.80222

Table 3. Critical half-thickness for different scattering types for c=1.1

Scattering types	P_1	P_3	P_5	P_7	P_9	P_{11}	P_{13}
Isotropic	2.30869	2.13534	2.12100	2.11734	2.11580	2.11501	2.11454
Lin.ans.	2.68134	2.44941	2.43108	2.42613	2.42402	2.42292	2.42227
Pure quadratic	2.30869	2.11492	2.09910	2.09586	2.09383	2.09305	2.09260
Quadratic	2.68134	2.42299	2.40329	2.39831	2.39624	2.39517	2.39455
Pure triplet	2.30869	2.14065	2.12481	2.12097	2.11938	2.11857	2.11809
Triplet	2.68134	2.43080	2.40904	2.40381	2.40166	2.40057	2.39994

Table 4. Critical half-thickness for different scattering types for c=1.5

Scattering types	P_1	P_3	P_5	P_7	P_9	P_{11}	P_{13}
Isotropic	0.78001	0.64949	0.62089	0.61223	0.60904	0.60762	0.60686
Lin.ans.	0.89092	0.71651	0.68261	0.67284	0.66925	0.66759	0.66689
Pure quadratic	0.78001	0.63423	0.59956	0.58880	0.58483	0.58311	0.58222
Quadratic	0.89092	0.69594	0.65485	0.64278	0.63840	0.63646	0.63544
Pure triplet	0.78001	0.65802	0.62654	0.61727	0.61389	0.61240	0.61161
Triplet	0.89092	0.70879	0.66356	0.65065	0.64603	0.64399	0.64292

In literature, the critical thickness of slab is determined for strongly anisotropic scattering by C. Yildiz [16]: There, the solutions are found up to fifteenth order by P_N method, and the convergence is obtained up to three digits.

Table 5. Critical half-thickness for different scattering types for c=2.0

Scattering types	P_1	P_3	P_5	P_7	P_9	P_{11}	P_{13}
Isotropic	0.45343	0.36197	0.33477	0.32350	0.31817	0.31545	0.31396
Lin.ans.	0.51481	0.39320	0.36065	0.34775	0.34184	0.33887	0.33728
Pure quadratic	0.45345	0.35219	0.32014	0.30656	0.29993	0.29647	0.29455
Quadratic	0.51481	0.38020	0.34204	0.32651	0.31919	0.31543	0.31338
Pure triplet	0.45345	0.36889	0.33939	0.32745	0.32186	0.31902	0.31748
Triplet	0.51481	0.39021	0.34864	0.33215	0.32447	0.32055	0.31843

In Table 6, the critical thickness values are tabulated for T_N method. As the iteration order is increased, the critical thickness converges to a certain value (See the last coloumn.)

Table 6. Critical thickness for triplet anisotropic in T_N method

c	T_1	T_3	T_5	T_7	T_9	T_{11}	T_{13}
1.01	12.2945	9.82545	9.81274	9.80669	9.80455	9.80344	9.80074
1.05	4.99350	3.83626	3.81813	3.81133	3.80902	3.80783	3.80682
1.1	3.28396	2.43973	2.41233	2.40456	2.40202	2.40073	2.39960
1.2	2.09710	1.49057	1.44695	1.43674	1.43371	1.43220	1.43088
1.4	1.28859	0.87401	0.81617	0.80005	0.79539	0.79331	0.79151
1.6	0.94900	0.62832	0.56968	0.54973	0.54300	0.54003	0.53764
1.8	0.75624	0.49340	0.43808	0.41676	0.40849	0.40457	0.40166
2.0	0.63051	0.40733	0.35623	0.33494	0.32584	0.32119	0.31799

Table 7 Critical thickness for different scattering types in T_N method

c	Scattering types	T_1	T_3	T_5	T_7	T_9	T_{11}	T_{13}
1.01	Isotropic	10.4024	8.34422	8.33808	8.33356	8.33203	8.33118	8.33046
	Lin.ans.	12.2945	9.83830	9.82369	9.81770	9.81542	9.81425	9.81213
	Quadratic	12.2945	9.82396	9.81017	9.80448	9.80236	9.80128	9.79875
	Triplet	12.2945	9.82545	9.81274	9.80669	9.80455	9.80344	9.80074
1.1	Isotropic	2.82755	2.13832	2.12361	2.11786	2.11613	2.11516	2.11441
	Lin.ans.	3.28396	2.45749	2.43397	2.42694	2.42443	2.42314	2.42205
	Quadratic	3.28396	2.43288	2.40625	2.39912	2.39664	2.39539	2.39429
	Triplet	3.28396	2.43973	2.41233	2.40456	2.40202	2.40073	2.39960
1.5	Isotropic	0.95531	0.66222	0.62600	0.61348	0.60968	0.60787	0.60653
	Lin.ans.	1.09115	0.73503	0.68794	0.67439	0.66991	0.66792	0.66623
	Quadratic	1.09115	0.71887	0.66123	0.64473	0.63919	0.63684	0.63484
	Triplet	1.09115	0.72998	0.670849	0.65246	0.64674	0.64425	0.64216
2.0	Isotropic	0.55536	0.37316	0.34007	0.32546	0.31926	0.31596	0.31391
	Lin.ans.	0.63051	0.40888	0.36644	0.35011	0.34297	0.33946	0.33697
	Quadratic	0.63051	0.39892	0.34877	0.32944	0.32060	0.31619	0.31310
	Triplet	0.63051	0.40733	0.35623	0.33494	0.32584	0.32120	0.31799

In Table 7, four different scattering types are shown for different values of c changing from 1.01 to 2.0 in T_N method. As seen, the critical thickness converges to a certain value as the coefficient term in the series is increased from f_0 (for isotropic scattering) to f_3 (for triplet scattering).

In literature, there are solutions of the critical thickness in the plane geometry for the triplet anisotropic scattering by F_N method, obtained by R. G. Türeci [17] (as shown in Table 8).

Our results of pure triplet anisotropic scattering are compared with the reference R. G. Türeci [17]. It is found that both methods of our pure triplet anisotropic scattering are consistent with the R. G. Türeci [17]. The number of secondary neutrons c is increased from 1.1 to 2.0 by ~0.2 steps. Corresponding critical thickness values for each c value is compared with the reference. Table 8 shows that values are similar with the results of the reference.

Table 8. Critical thickness (2a) results for pure triplet anisotropic scattering with Ref. [17].

c	**P_N Method**	**T_N Method**	**R. G. Türeci** [17]
1.1	4.23866	4.23925	4.24309
1.3	1.89152	1.89230	1.89248
1.5	1.22764	1.22880	1.22529
1.7	0.90406	0.90567	0.89865
2.0	0.64361	0.64574	0.63220

4.Conclusions

In this study, P_N and T_N methods are compared for different scattering types to calculate the critical (half)-thickness of the slab reactor. As a different approach, we present our results for different scattering types changing from isotropic to triplet anisotropic in a single paper: All calculations have been done simultaneously and presented in independent Tables for different values of c parameter. The neutron scattering function has been enlarged up to f_3 which is called as triplet anisotropic scattering. It is shown that the T_N and P_N methods can be used to solve criticality problem for the triplet anisotropic scattering case. The critical thickness values obtained from the solution of present methods are close to the F_N results given in Table 8.

It is well known that the analytical solution for this type of scattering needs great importance: The equation to be solved becomes more complicated because the scattering function now has more numbers of terms. First, the eigenvalues of the neutron transport equation are found for both methods with the triplet anisotropic scattering. Then the results are placed in the neutron flux equation for finding out the critical thickness of the system. Because the Legendre polynomials provide suitable and rapid results to find the critical half-thickness of the system, we first present the results of the P_N method for the triplet anisotropic scattering. Then, the procedure is repeated for the T_N method. In these calculations for critical half-thickness, many types of scattering are used in the methods separately.

It is clear that the critical half-thickness decreases as the c value increases. The deviation among each scattering coefficient is considerably decreasing by increasing the order of anisotropic scattering. As the number of order is increased, the critical thickness results are converging as expected. It is seen in the Table 1-7 that both methods are found to be consistent. Finally, results for the pure triplet anisotropic scattering are compared with the reference R. G. Türeci [17] and found to be in good agreement. It is thought that the comprehensive and comparative results of all scattering types provided in this study may offer a good source for future studies and/or other researchers.

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