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NOVEL IDENTITIES INVOLVING GENERALIZED CARLITZ'S TWISTED q-EULER POLYNOMIALS ATTACHED TO χ UNDER S_4

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Abstract. The essential purpose of this paper is to give some novel symmetric identities for generalized Carlitz's twisted q-Euler polynomials attached to χ based on the fermionic p-adic invariant integral on $\mathbb{Z}_p$ under S_4.

1. Introduction

In recent years, many mathematicians have studied on symmetric identities of some well-known special polynomials arising from p-adic q-integral on $\mathbb{Z}_p$. For example, Duran *et al.* [8] on q-Genocchi polynomials under S_4, Duran *et al.* [9] on weighted q-Genocchi polynomials under S_4, Araci *et al.* [3] on q-Frobenious Euler polynomials under S_5, Dolgy *et al.* [6] on q-Euler polynomials under S_3, Dolgy *et al.* [7] on h-extension of q-Euler polynomials under D_3 and furthermore, moreover, Duran *et al.* [10] on Carlitz's twisted (h,q)-Euler polynomials under S_n, furthermore, Rim *et al.* [15] on generalized (h,q)-Euler numbers under D_3 have worked extensively by using p-adic q-integrals on $\mathbb{Z}_p$.

Throughout the present paper we shall make use of the following notations: $\mathbb{Z}_p$ denotes the ring of p-adic rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, and $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$, where p be a fixed odd prime number. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$. The normalized absolute value in accordance with the theory of p-adic analysis is given by $|p|_p = p^{-1}$. The notion "q" can be noted as an indeterminate, a complex number $q \in \mathbb{C}$ with $|q| < 1$, or a p-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ and $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. The q-analog of x is defined as $[x]_q = \frac{1-q^x}{1-q}$. It is obviously that $\lim_{q\to 1} [x]_q = x$. See cf. [3-21] for a systematic work.

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For

$$f\in UD\left(\mathbb{Z}_p\right)=\left\{f\,|f:\mathbb{Z}_p\rightarrow\mathbb{C}_p\text{ is uniformly differentiable function}\right\},$$

the fermionic p-adic invariant integral on $\mathbb{Z}_p$ of a function $f\in UD\left(\mathbb{Z}_p\right)$ is defined by Kim in [12] as follows:

$$I_{-1}(f)=\int_{\mathbb{Z}_p}f\left(x\right)d\mu_{-1}\left(x\right)=\lim_{N\rightarrow\infty}\sum_{x=0}^{p^N-1}f\left(x\right)\left(-1\right)^x. \tag{1.1}$$

Hence, via Eq. (1.1), it follows

$$I_{-1}(f_n)=\left(-1\right)^n I_{-1}(f)+2\sum_{l=0}^{n-1}\left(-1\right)^{n-l-1}f(l)$$

where $f_n(x)$ means $f(x+n)$. For more details, one can take a close look at the references [3], [6], [7], [8], [9], [10], [11], [12], [13], [15], [16], [17].

For $d\in\mathbb{N}$ with $(p,d)=1$ and $d\equiv 1(\mod 2)$, we set

$$X:=X_d=\lim_{\overleftarrow{n}}\mathbb{Z}/dp^n\mathbb{Z}\ \text{ and } X_1=\mathbb{Z}_p,$$

$$X^*=\bigcup_{\substack{0<a<dp\\(a,p)=1}}\left(a+dp\mathbb{Z}_p\right)$$

and

$$a+dp^n\mathbb{Z}_p=\left\{x\in X\mid x\equiv a\left(\mod dp^n\right)\right\}$$

where $a\in\mathbb{Z}$ lies in $0\leq a<dp^n$ cf. [3, 6-13, 15-19].

Note that

$$\int_X f\left(x\right)d\mu_{-1}\left(x\right)=\int_{\mathbb{Z}_p}f\left(x\right)d\mu_{-1}\left(x\right),\text{ for } f\in UD\left(\mathbb{Z}_p\right).$$

As is well-known that the Euler polynomials $E_n(x)$ are defined by means of the following Taylor expansion at $t=0$:

$$\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!}=\frac{2}{e^t+1}e^{xt},\qquad(|t|<\pi). \tag{1.2}$$

If we choose $x=0$ in the Eq. (1.2), it yields $E_n(0):=E_n$ that is called as n-th Euler number. Moreover, the polynomials $E_n\left(x\right)$ can be introduced by the following p-adic integral:

$$E_n(x)=\int_{\mathbb{Z}_p}\left(x+y\right)^n d\mu_{-1}\left(y\right),$$

see, for more details, [2-7, 10-20].

Let χ be a primitive Dirichlet's character with conductor $d\in\mathbb{N}$ with $d\equiv 1($ mod 2). Details on the Dirichlet's character χ can be found in [1].

As a generalization of $E_n(x)$, the generalized Euler polynomials attached to χ, $E_{n,\chi}(x)$, are defined by

$$\int_X \chi(y) e^{(x+y)t} d\mu_{-1}(y) = \left(\frac{2\sum_{a=0}^{d-1} \chi(a)(-1)^a e^{at}}{e^{dt+1}} \right) e^{xt} = \sum_{n=0}^{\infty} E_{n,\chi}(x) \frac{t^n}{n!}. \tag{1.3}$$

Thus, by virtue of Eq. (1.3), we have

$$E_{n,\chi}(x) = \int_X \chi(y)(x+y)^n d\mu_{-1}(y), \, n \geq 0.$$

Substituting with $x = 0$ in the Eq. (1.3) yields $E_{n,\chi}(0) := E_{n,\chi}$ known as n-th generalized Euler number attached to χ, see [12] and [18].

Let $T_p = \bigcup_{N\geq 1} C_{p^N} = \lim_{N\to\infty} C_{p^N}$, in which $C_{p^N} = \left\{ w : w^{p^N} = 1 \right\}$ is the cyclic group of order p^N. For $w \in T_p$, we denote by $\phi_w : \mathbb{Z}_p \to C_p$ the locally constant function $\ell \to w^\ell$. For $q \in C_p$ with $|1-q|_p < 1$ and $w \in T_p$, in [17], Ryoo introduced the Carlitz's twisted q-Euler polynomials by the following fermionic p-adic invariant integral on $\mathbb{Z}_p$:

$$\mathcal{E}_{n,q,w}(x) = \int_{\mathbb{Z}_p} w^y [x+y]_q^n d\mu_{-1}(y) \quad (n \geq 0). \tag{1.4}$$

Letting $x = 0$ into the Eq. (1.4) gives $\mathcal{E}_{n,q,w}(0) := \mathcal{E}_{n,q,w}$ called n-th Carlitz's twisted q-Euler numbers.

From (1.4), we can derive the generating function of the generalized Carlitz's twisted q-Euler polynomials attached to χ as follows:

$$\sum_{n=0}^{\infty} \mathcal{E}_{n,\chi,q,w}(x) \frac{t^n}{n!} = \int_{\mathbb{Z}_p} \chi(y) w^y e^{[x+y]_q t} d\mu_{-1}(y). \tag{1.5}$$

When $x = 0$, we have $\mathcal{E}_{n,\chi,q,w}(0) := \mathcal{E}_{n,\chi,q,w}$ are called generalized Carlitz's twisted q-Euler numbers attached to χ.

The present paper is organized as follows. In the following section, we consider the generalized Carlitz's twisted q-Euler polynomials attached to χ and present some not only new but also interesting symmetric identities for these polynomials associated with the fermionic p-adic invariant integral on $\mathbb{Z}_p$ under symmetric group of degree four. Furthermore, some speacial cases of our results in this paper are examined in the Corollary.

2. Novel symmetric Identities for $\mathcal{E}_{n,\chi,q,w}(x)$ under S_4

Let $w_i \in \mathbb{N}$ be fixed natural number which satisfies the condition $w_i \equiv 1($ mod $2)$, where $i \in \mathbb{Z}$ lies in $1 \leq i \leq 4$ and χ be the trivial character. Then, we observe

$$\int_{\mathbb{Z}_p} \chi(y)\, w^{w_1w_2w_3y} e^{[w_1w_2w_3y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t} d\mu_{-1}(y)$$
$$= \lim_{N\to\infty} \sum_{y=0}^{dp^N-1} (-1)^y \chi(y)\, w^{w_1w_2w_3y} e^{[w_1w_2w_3y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}$$
$$= \lim_{N\to\infty} \sum_{l=0}^{dw_4-1} \sum_{y=0}^{p^N-1} (-1)^{l+y} \chi(l)\, w^{w_1w_2w_3(l+dw_4y)} \times e^{[w_1w_2w_3(l+dw_4y)+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}.$$

Hence, we discover

$$I = \sum_{i=0}^{dw_1-1} \sum_{j=0}^{dw_2-1} \sum_{k=0}^{dw_3-1} (-1)^{i+j+k} \chi(i)\chi(j)\chi(k)\, w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} \times \int_{\mathbb{Z}_p} \chi(y)\, w^{w_1w_2w_3y} e^{[w_1w_2w_3y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t} d\mu_{-1}(y)$$
$$= \lim_{N\to\infty} \sum_{i=0}^{dw_1-1} \sum_{j=0}^{dw_2-1} \sum_{k=0}^{dw_3-1} \sum_{l=0}^{dw_4-1} \sum_{y=0}^{p^N-1} (-1)^{i+j+k+y+l} \chi(ijkl)\, w^{w_1w_2w_3(l+dw_4y)+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} \times e^{[w_1w_2w_3(l+dw_4y)+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}. \tag{2.1}$$

Notice that Eq. (2.1) is invariant for any permutation $\sigma \in S_4$. Therefore, we state the following theorem.

Theorem 2.1. *Let $w_i \in \mathbb{N}$ be any natural number which satisfies the condition $w_i \equiv 1(\mod 2)$, in which $i \in \mathbb{Z}$ lies in $1 \le i \le 4$, χ be the trivial character and $n \ge 0$. Then the following*

$$I = \sum_{i=0}^{dw_{\sigma(1)}-1} \sum_{j=0}^{dw_{\sigma(2)}-1} \sum_{k=0}^{dw_{\sigma(3)}-1} (-1)^{i+j+k} \chi(i)\chi(j)\chi(k)\, w^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k} \times \int_{\mathbb{Z}_p} \chi(y)\, w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}(l+w_{\sigma(4)}y)} \times e^{\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}y+w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}x+w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k\right]_q t} d\mu_{-1}(y)$$

holds true for any $\sigma \in S_4$.

On account of the definition of $[x]_q$, we readily find that

$$[w_1w_2w_3y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q = [w_1w_2w_3]_q \left[y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]_{q^{w_1w_2w_3}},$$

which gives

(2.2)

$$\int_{\mathbb{Z}_p} \chi(y)\, w^{w_1w_2w_3y}\left[w_1w_2w_3y + w_1w_2w_3w_4x + w_4w_2w_3i + w_4w_1w_3j + w_4w_1w_2k\right]_q^n d\mu_{-1}(y)$$
$$= [w_1w_2w_3]_q^n\, \mathcal{E}_{n,\chi,q^{w_1w_2w_3},w^{w_1w_2w_3}}\left(w_4x + \frac{w_4}{w_1}i + \frac{w_4}{w_2}j + \frac{w_4}{w_3}k\right), \text{ for } n \geq 0.$$

So, by the Theorem 2.1 and Eq. (2.2), we procure the following theorem.

Theorem 2.2. *Let $w_i \in \mathbb{N}$ be any natural number which satisfies the condition $w_i \equiv 1(\ mod 2)$, in which $i \in \mathbb{Z}$ lies in $1 \leq i \leq 4$ and χ be the trivial character. For $n \geq 0$, the following expression*

$$\begin{aligned} I &= \left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}\right]_q^n \sum_{i=0}^{dw_{\sigma(1)}-1}\sum_{j=0}^{dw_{\sigma(2)}-1}\sum_{k=0}^{dw_{\sigma(3)}-1}(-1)^{i+j+k}\chi(i)\,\chi(j)\,\chi(k) \\ &\quad \times w^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k} \\ &\quad \times \mathcal{E}_{n,\chi,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}},w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}}}\left(w_{\sigma(4)}x + \frac{w_{\sigma(4)}}{w_{\sigma(1)}}i + \frac{w_{\sigma(4)}}{w_{\sigma(2)}}j + \frac{w_{\sigma(4)}}{w_{\sigma(3)}}k\right) \end{aligned}$$

holds true for any $\sigma \in S_4$.

By using the definitions of $[x]_q$ and binomial theorem, we see that

$$\begin{aligned} &\left[y + w_4x + \frac{w_4}{w_1}i + \frac{w_4}{w_2}j + \frac{w_4}{w_3}k\right]_{q^{w_1w_2w_3}}^n \\ &= \sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_4]_q}{[w_1w_2w_3]_q}\right)^{n-m}[w_2w_3i + w_1w_3j + w_1w_2k]_{q^{w_4}}^{n-m} \\ &\quad \times q^{m(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k)}\,[y + w_4x]_{q^{w_1w_2w_3}}^m, \end{aligned} \tag{2.3}$$

which yields

(2.4)

$$\begin{aligned} &[w_1w_2w_3]_q^n \sum_{i=0}^{dw_1-1}\sum_{j=0}^{dw_2-1}\sum_{k=0}^{dw_3-1}(-1)^{i+j+k}w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}\chi(i)\,\chi(j)\,\chi(k) \\ &\quad \times \int_{\mathbb{Z}_p}\chi(y)\,w^{w_1w_2w_3y}\left[y + w_4x + \frac{w_4}{w_1}i + \frac{w_4}{w_2}j + \frac{w_4}{w_3}k\right]_{q^{w_1w_2w_3}}^n d\mu_{-1}(y) \\ &= \sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3]_q^m\,[w_4]_q^{n-m}\,\mathcal{E}_{m,\chi,q^{w_1w_2w_3},w^{w_1w_2w_3}}(w_4x)\sum_{i=0}^{dw_1-1}\sum_{j=0}^{dw_2-1} \\ &\quad \sum_{k=0}^{dw_3-1}(-1)^{i+j+k}\chi(i)\,\chi(j)\,\chi(k)\,w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} \\ &\quad \times q^{m(w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k)}\,[w_2w_3i + w_1w_3j + w_1w_2k]_{q^{w_4}}^{n-m} \\ &= \sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3]_q^m\,[w_4]_q^{n-m}\,\mathcal{E}_{m,\chi,q^{w_1w_2w_3},w^{w_1w_2w_3}}(w_4x)\,\widetilde{U}_{n,m,q^{w_4},w^{w_4}}(w_1, w_2, w_3 \mid \chi), \end{aligned}$$

where

$$\begin{aligned}
(2.5) \qquad & \widetilde{U}_{n,m,q,w}(w_1, w_2, w_3 \mid \chi) \\
= & \sum_{i=0}^{dw_1-1} \sum_{j=0}^{dw_2-1} \sum_{k=0}^{dw_3-1} (-1)^{i+j+k} \chi(i) \chi(j) \chi(k) w^{w_2 w_3 i + w_1 w_3 j + w_1 w_2 k} \\
& \times q^{m(w_2 w_3 i + w_1 w_3 j + w_1 w_2 k)} [w_2 w_3 i + w_1 w_3 j + w_1 w_2 k]_q^{n-m}.
\end{aligned}$$

Consequently, from Eqs. (2.5) and (2.5), we present the following theorem.

Theorem 2.3. *Let $w_i \in \mathbb{N}$ be any natural number which satisfies the condition $w_i \equiv 1(\mod 2)$, where $i \in \mathbb{Z}$ lies in $1 \leq i \leq 4$, χ be the trivial character and $n \in \mathbb{N}$. Hence, the following expression*

$$\sum_{m=0}^{n} \binom{n}{m} \left[w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)}\right]_q^m \left[w_{\sigma(4)}\right]_q^{n-m}$$
$$\times \mathcal{E}_{n,\chi,q^{w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)}}, w^{w_{\sigma(1)} w_{\sigma(2)} w_{\sigma(3)}}} \left(w_{\sigma(4)} x\right) \widetilde{U}_{n,m,q^{w_{\sigma(4)}}, w^{w_{\sigma(4)}}} (w_{\sigma(1)}, w_{\sigma(2)}, w_{\sigma(3)} \mid \chi)$$

holds true for some $\sigma \in S_4$.

3. Conclusion

In this study, we have obtained some new symmetric identities for generalized Carlitz's twisted q-Euler polynomials attached to χ associated with the p-adic invariant integral on $\mathbb{Z}_p$ under the symmetric group of degree four. We note that for $w = 1$, all our results in this paper reduce to the results of the generalized q-Euler polynomials attached to χ under S_4 in [18]. Moreover while $q \to 1$, all our results in this paper reduce to the results of the generalized twisted Euler polynomials attached to χ under S_4. Furthermore, for $w = 1$ and $q \to 1$, all our results in this paper reduce to the results of the generalized Euler polynomials attached to χ under S_4.

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