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BIOPERATIONS ON α-SEMIOPEN SETS

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Abstract. The aim of this paper is to introduce and study the concept of $\alpha_{[\gamma,\gamma']}$-semiopen sets. Using this set, we introduce and study the concept of $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous and $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute functions.

1. Introduction

The notion of semiopen sets is an important concept in general topology. In 1963, Levine [4] defined semiopen sets in a space X and discussed many of its properties. Njastad [3] introduced α-open sets in a topological space and studied some of its properties. Ibrahim [2] defined the concept of an operation on $\alpha O(X,\tau)$ and introduced α_γ-open sets in topological spaces and studied some of their basic properties. Khalaf, et. al. [1] introduced the notion of $\alpha O(X,\tau)_{[\gamma,\gamma']}$, which is the collection of all $\alpha_{[\gamma,\gamma']}$-open sets in a topological space (X,τ). In this paper, we introduce and study the notion of $\alpha SO(X,\tau)_{[\gamma,\gamma']}$ which is the collection of all $\alpha_{[\gamma,\gamma']}$-semiopen by using operations γ and $\gamma^{'}$ on a topological space $\alpha O(X,\tau)$. We also introduce $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous and $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute functions and investigate some important properties of these functions.

2. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure and the interior of a subset A of X are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.1. A subset A of a topological space (X,τ) is called α-open [3] (resp., semiopen [4]) if $A \subseteq Int(Cl(Int(A)))$ (resp., $A \subseteq Cl(Int(A))$). The complement of an α-open (resp., semiopen) set is called α-closed (resp., semiclosed) set.

The family of all α-open (resp., semiopen) sets in a topological space (X,τ) is denoted by $\alpha O(X,\tau)$ or $\alpha O(X)$ (resp.,$SO(X,\tau)$ or $SO(X)$).

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Definition 2.2. [2] Let X be a topological space. An operation γ on the topology $\alpha O(X)$ is a mapping from $\alpha O(X)$ into the power set $P(X)$ of X such that $V \subseteq V^\gamma$ for each $V \in \alpha O(X)$, where V^γ denotes the value of γ at V. It is denoted by $\gamma : \alpha O(X) \to P(X)$.

Definition 2.3. [2] An operation γ on $\alpha O(X)$ is said to be α-regular if for every α-open sets U and V containing $x \in X$, there exists an α-open set W of X containing x such that $W^\gamma \subseteq U^\gamma \cap V^\gamma$.

Definition 2.4. [1] A subset A of X is said to be $\alpha_{[\gamma,\gamma']}$-open if for each $x \in A$, there exist α-open sets U and V of X containing x such that $U^\gamma \cap V^{\gamma'} \subseteq A$. A subset F of (X, τ) is said to be $\alpha_{[\gamma,\gamma']}$-closed if its complement $X \setminus F$ is $\alpha_{[\gamma,\gamma']}$-open.

The family of all $\alpha_{[\gamma,\gamma']}$-open sets of (X, τ) is denoted by $\alpha O(X, \tau)_{[\gamma,\gamma']}$.

Definition 2.5. [1] Let (X, τ) be a topological space and A be a subset of X, then:

(1) The intersection of all $\alpha_{[\gamma,\gamma']}$-closed sets containing A is called the $\alpha_{[\gamma,\gamma']}$-closure of A and denoted by $\alpha_{[\gamma,\gamma']}$-$Cl(A)$.
(2) The union of all $\alpha_{[\gamma,\gamma']}$-open sets contained in A is called the $\alpha_{[\gamma,\gamma']}$-interior of A and denoted by $\alpha_{[\gamma,\gamma']}$-$Int(A)$.

Definition 2.6. [5] A nonempty subset A of (X, τ) is said to be $[\gamma, \gamma']$-open if for each $x \in A$ there exist open sets U and V of X containing x such that $U^\gamma \cap V^{\gamma'} \subseteq A$.

The family of all $[\gamma, \gamma']$-open sets of (X, τ) is denoted by $\tau_{[\gamma,\gamma']}$.

Definition 2.7. [1] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-closed if for $\alpha_{[\gamma,\gamma']}$-closed set A of X, $f(A)$ is $\alpha_{[\beta,\beta']}$-closed in Y.

3. $\alpha_{[\gamma,\gamma']}$-Semiopen Sets

Definition 3.1. Let (X, τ) be a topological space and γ, γ' be two operations on $\alpha O(X, \tau)$. A subset A of X is said to be $\alpha_{[\gamma,\gamma']}$-semiopen, if there exists an $\alpha_{[\gamma,\gamma']}$-open set U of X such that $U \subseteq A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(U)$.

The family of all $\alpha_{[\gamma,\gamma']}$-semiopen sets of a topological space (X, τ) is denoted by $\alpha SO(X, \tau)_{[\gamma,\gamma']}$. Also, the family of all $\alpha_{[\gamma,\gamma']}$-semiopen sets of (X, τ) containing x is denoted by $\alpha SO(X, x)_{[\gamma,\gamma']}$.

Theorem 3.1. *If A is an $\alpha_{[\gamma,\gamma']}$-open set in (X, τ), then it is $\alpha_{[\gamma,\gamma']}$-semiopen set.*

Proof. The proof follows from the definition. □

The following example shows that the converse of the above theorem is not true in general.

Example 3.1. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ be a topology on X. For each $A \in \alpha O(X, \tau)$, we define two operations γ and γ', respectively, by

$$A^\gamma = A^{\gamma'} = \begin{cases} X & \text{if } c \in A \\ A & \text{if } c \notin A. \end{cases}$$

Now, $\alpha O(X, \tau)_{[\gamma,\gamma']} = \{\phi, X, \{a\}, \{a, b\}\}$. Let $A = \{a, c\}$, then there exists an $\alpha_{[\gamma,\gamma']}$-open set $\{a\}$ such that $\{a\} \subseteq A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\{a\}) = X$. Thus, A is $\alpha_{[\gamma,\gamma']}$-semiopen but not $\alpha_{[\gamma,\gamma']}$-open.

Theorem 3.2. *If A is a $[\gamma,\gamma^{'}]$-open set in (X,τ), then it is $\alpha_{[\gamma,\gamma']}$-semiopen set.*

Proof. The proof follows from [[1], Proposition 3.14] and Theorem 3.1. $\square$

The converse of the above theorem need not be true. The subset $\{a,b\}$ in [[1], Example 3.15.], is an $\alpha_{[\gamma,\gamma']}$-semiopen set but it is not $[\gamma,\gamma^{'}]$-open.

Also by Theorem 3.1 and [[1], Proposition 3.14], we obtain the following inclusion
$\tau_{[\gamma,\gamma']} \subseteq \alpha O(X,\tau)_{[\gamma,\gamma']} \subseteq \alpha SO(X,\tau)_{[\gamma,\gamma']}$.

The following examples show that the concept of semiopen and $\alpha_{[\gamma,\gamma']}$-semiopen sets are independent.

Example 3.2. Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\}\}$ be a topology on X. For each $A\in\alpha O(X,\tau)$, we define two operations γ and $\gamma^{'}$, respectively, by

$$A^{\gamma}=A^{\gamma^{'}}=\begin{cases} A & \text{if } a\in A \\ Cl(A) & \text{if } a\notin A. \end{cases}$$

Calculations give $\alpha O(X,\tau)_{[\gamma,\gamma']}=\{\phi,X,\{a\},\{a,b\}\}$. Then, $A=\{a,c\}$ is $\alpha_{[\gamma,\gamma']}$-semiopen but not a semiopen set.

Example 3.3. Let $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$ be a topology on X. For each $A\in\alpha O(X,\tau)$, we define two operations γ and $\gamma^{'}$, respectively, by

$$A^{\gamma}=A^{\gamma^{'}}=\begin{cases} A & \text{if } b\in A \\ Cl(A) & \text{if } b\notin A. \end{cases}$$

Calculations give $\alpha O(X,\tau)_{[\gamma,\gamma']}=\{\phi,X,\{b\},\{a,b\},\{a,c\}\}$. Then, $A=\{a\}$ is semiopen but not an $\alpha_{[\gamma,\gamma']}$-semiopen set.

Theorem 3.3. *A subset A is $\alpha_{[\gamma,\gamma']}$-semiopen if and only if $A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.*

Proof. Let $A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Take $U=\alpha_{[\gamma,\gamma']}$-$Int(A)$. Then, by [[1], Proposition 3.44 (1)], U is $\alpha_{[\gamma,\gamma']}$-open and we have $U=\alpha_{[\gamma,\gamma']}$-$Int(A)\subseteq A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$. Hence, A is $\alpha_{[\gamma,\gamma']}$-semiopen.

Conversely, suppose that A is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X. Then, $U\subseteq A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$, for some $\alpha_{[\gamma,\gamma']}$-open sets U in X. Since $U\subseteq\alpha_{[\gamma,\gamma']}$-$Int(A)$. Thus, we have $\alpha_{[\gamma,\gamma']}$-$Cl(U)\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Hence, $A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. $\square$

Theorem 3.4. *Let A be an $\alpha_{[\gamma,\gamma']}$-semiopen set in a space X and B a subset of X. If $A\subseteq B\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(A)$, then B is $\alpha_{[\gamma,\gamma']}$-semiopen.*

Proof. Since A is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X, then there exists an $\alpha_{[\gamma,\gamma']}$-open set U of X such that $U\subseteq A\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$. Since $A\subseteq B$, so $U\subseteq B$. But $\alpha_{[\gamma,\gamma']}$-$Cl(A)\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$, then $B\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$. Hence $U\subseteq B\subseteq\alpha_{[\gamma,\gamma']}$-$Cl(U)$. Thus, B is $\alpha_{[\gamma,\gamma']}$-semiopen. $\square$

Theorem 3.5. *If A_i is $\alpha_{[\gamma,\gamma']}$-semiopen for every $i\in I$, then $\cup\{A_i : i\in I\}$ is $\alpha_{[\gamma,\gamma']}$-semiopen.*

Proof. Since A_i is an $\alpha_{[\gamma,\gamma']}$-semiopen set for every $i \in I$, so there exist an $\alpha_{[\gamma,\gamma']}$-open set U_i of X such that $U_i \subseteq A_i \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(U_i)$ this impies that $\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} A_i \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(\bigcup_{i\in I} U_i)$. By [[1], Proposition 3.2], $\bigcup_{i\in I} U_i$ is $\alpha_{[\gamma,\gamma']}$-open. Therefore, $\cup_{i\in I} A_i$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set of (X, τ). □

If A and B are two $\alpha_{[\gamma,\gamma']}$-semiopen sets in (X, τ), then the following example shows that $A \cap B$ need not be $\alpha_{[\gamma,\gamma']}$-semiopen.

Example 3.4. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ be a topology on X. For each $A \in \alpha O(X, \tau)$, we define two operations γ and γ', by

$$A^{\gamma} = \begin{cases} Cl(A) & \text{if } c \in A, \\ X & \text{if } c \notin A, \end{cases}$$

and

$$A^{\gamma'} = \begin{cases} A & \text{if } A \neq \{a\}, \\ X & \text{if } A = \{a\}. \end{cases}$$

Then, it is obvious that the sets $\{a, b\}$ and $\{a, c\}$ are $\alpha_{[\gamma,\gamma']}$-semiopen, however their intersection $\{a\}$ is not $\alpha_{[\gamma,\gamma']}$-semiopen.

Remark 3.1. From the above example we notice that the family of all $\alpha_{[\gamma,\gamma']}$-semiopen subsets of a space X is a supratopology and need not be a topology in general.

Theorem 3.6. *Let γ and γ' be α-regular operations on $\alpha O(X)$. If A is a subset of X, then for every $\alpha_{[\gamma,\gamma']}$-open set G of X, we have:*

(1) $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$.
(2) $\alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G) = \alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G)$.

Proof. (1) Let $x \in \alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G$ and V be any $\alpha_{[\gamma,\gamma']}$-open set containing x. Then by [[1], Proposition 3.4], $V \cap G$ is also an $\alpha_{[\gamma,\gamma']}$-open set containing x. Since $x \in \alpha_{[\gamma,\gamma']}\text{-}Cl(A)$, implies that $(V \cap G) \cap A \neq \phi$, this implies that $V \cap (A \cap G) \neq \phi$ and hence by [[1], Proposition 3.31], $x \in \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$. Therefore $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$.

(2) By (1), $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$ and so $\alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$. But $A \cap G \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G$ implies that $\alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G)$. Therefore, $\alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G) = \alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \cap G)$.

□

Theorem 3.7. *Let γ and γ' be α-regular operations on $\alpha O(X)$. If A is $\alpha_{[\gamma,\gamma']}$-open and B is $\alpha_{[\gamma,\gamma']}$-semiopen, then $A \cap B$ is $\alpha_{[\gamma,\gamma']}$-semiopen.*

Proof. Since B is $\alpha_{[\gamma,\gamma']}$-semiopen, there exists an $\alpha_{[\gamma,\gamma']}$-open set G such that $G \subseteq B \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(G)$ and so $A \cap G \subseteq A \cap B \subseteq A \cap \alpha_{[\gamma,\gamma']}\text{-}Cl(G)$. By [[1], Proposition 3.4], $A \cap G$ is $\alpha_{[\gamma,\gamma']}$-open and so $A \cap G = \alpha_{[\gamma,\gamma']}\text{-}Int(A \cap G)$. By Theorem 3.6 (1), $A \cap \alpha_{[\gamma,\gamma']}\text{-}Cl(G) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G)$. Therefore, $A \cap B \subseteq A \cap \alpha_{[\gamma,\gamma']}\text{-}Cl(G) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G) = \alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Int(A \cap G)) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(\alpha_{[\gamma,\gamma']}\text{-}Int(A \cap B))$. By Theorem 3.3, $A \cap B$ is $\alpha_{[\gamma,\gamma']}$-semiopen.

□

Proposition 3.1. *The set A is $\alpha_{[\gamma,\gamma']}$-semiopen in X if and only if for each $x \in A$, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set B such that $x \in B \subseteq A$.*

Proof. Suppose that A is an $\alpha_{[\gamma,\gamma']}$-semiopen set in the space X. Then for each $x \in A$, put $B = A$ which is an $\alpha_{[\gamma,\gamma']}$-semiopen set such that $x \in B \subseteq A$.

Conversely, suppose that for each $x \in A$, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set B such that $x \in B \subseteq A$. Thus $A = \cup_{x\in A} B_x$, where $B_x \in \alpha SO(X,\tau)_{[\gamma,\gamma']}$. Therefore, by Theorem 3.5, A is an $\alpha_{[\gamma,\gamma']}$-semiopen set. □

Proposition 3.2. *Let (X,τ) be a topological space and γ, γ' be operations on $\alpha O(X)$. A subset A of X is $\alpha_{[\gamma,\gamma']}$-semiopen if and only if $\alpha_{[\gamma,\gamma']}$-$Cl(A) = \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.*

Proof. Let $A \in \alpha SO(X)_{[\gamma,\gamma']}$. Then, we have $A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$, which implies that $\alpha_{[\gamma,\gamma']}$-$Cl(A) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(A)$ and hence $\alpha_{[\gamma,\gamma']}$-$Cl(A) = \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.

Conversely, since by [[1], Proposition 3.44 (1)] and Theorem 3.1, $\alpha_{[\gamma,\gamma']}$-$Int(A)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set such that $\alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(A) = \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$ and hence A is $\alpha_{[\gamma,\gamma']}$-semiopen.

□

Proposition 3.3. *If A is a nonempty $\alpha_{[\gamma,\gamma']}$-semiopen set in X, then $\alpha_{[\gamma,\gamma']}$-$Int(A) \neq \phi$.*

Proof. Since A is $\alpha_{[\gamma,\gamma']}$-semiopen, by Proposition 3.2, we have $\alpha_{[\gamma,\gamma']}$-$Cl(A) = \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Suppose that $\alpha_{[\gamma,\gamma']}$-$Int(A) = \phi$. Then, we have $\alpha_{[\gamma,\gamma']}$-$Cl(A) = \phi$ and hence $A = \phi$. This contradicts the hypothesis. Therefore, $\alpha_{[\gamma,\gamma']}$-$Int(A) \neq \phi$. □

Proposition 3.4. *Let (X,τ) be a topological space and γ, γ' be operations on $\alpha O(X)$. Then a subset A of X is $\alpha_{[\gamma,\gamma']}$-semiopen if and only if $A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)))$ and $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.*

Proof. Let A be an $\alpha_{[\gamma,\gamma']}$-semiopen set. Then, we have $A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)))$. Moreover, $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(A) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.

Conversely, since $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Thus, we obtain that $\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A))) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. By hypothesis, we have $A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A))) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Hence, A is an $\alpha_{[\gamma,\gamma']}$-semiopen set. □

Definition 3.2. Let A be a subset of a topological space (X,τ) and γ, γ' be operations on $\alpha O(X)$. Then, a subset A of X is said to be $\alpha_{[\gamma,\gamma']}$-semiclosed if and only if $X \setminus A$ is $\alpha_{[\gamma,\gamma']}$-semiopen. The family of all $\alpha_{[\gamma,\gamma']}$-semiclosed sets of a topological space (X,τ) is denoted by $\alpha SC(X,\tau)_{[\gamma,\gamma']}$.

The following theorem gives characterizations of $\alpha_{[\gamma,\gamma']}$-semiclosed sets.

Theorem 3.8. *Let A be a subset of X and γ, γ' be operations on $\alpha O(X)$. Then, the following statements are equivalent:*

(1) *A is $\alpha_{[\gamma,\gamma']}$-semiclosed.*
(2) $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$.
(3) $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) = \alpha_{[\gamma,\gamma']}$-$Int(A)$.
(4) *There exists an $\alpha_{[\gamma,\gamma']}$-closed set F such that $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq A \subseteq F$.*

Proof. (1) $\Rightarrow$ (2): Since $A \in \alpha SC(X,\tau)_{[\gamma,\gamma']}$, then we have $X \setminus A \in \alpha SO(X,\tau)_{[\gamma,\gamma']}$. Hence, by Theorem 3.3 and [[1], Proposition 3.45], $X \setminus A \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(X \setminus A)) = X \setminus (\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)))$. Therefore, we obtain $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$.

(2) $\Rightarrow$ (3): Since $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$ implies that $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(A)$ but $\alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A))$ and so $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) = \alpha_{[\gamma,\gamma']}$-$Int(A)$.

(3) $\Rightarrow$ (4): Let $F = \alpha_{[\gamma,\gamma']}$-$Cl(A)$, then F is an $\alpha_{[\gamma,\gamma']}$-closed set such that $\alpha_{[\gamma,\gamma']}$-$Int(F) = \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) = \alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq A \subseteq F$, which proves (4).

(4) $\Rightarrow$ (1): If there exists an $\alpha_{[\gamma,\gamma']}$-closed set F such that $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq A \subseteq F$, then $X \setminus F \subseteq X \setminus A \subseteq X \setminus \alpha_{[\gamma,\gamma']}$-$Int(F) = \alpha_{[\gamma,\gamma']}$-$Cl(X \setminus F)$. Since $X \setminus F$ is $\alpha_{[\gamma,\gamma']}$-open, then $X \setminus A$ is $\alpha_{[\gamma,\gamma']}$-semiopen and so A is $\alpha_{[\gamma,\gamma']}$-semiclosed. □

Theorem 3.9. *Let (X,τ) be a topological space and γ, γ' be operations on $\alpha O(X)$. Arbitrary intersection of $\alpha_{[\gamma,\gamma']}$-semiclosed sets is always $\alpha_{[\gamma,\gamma']}$-semiclosed.*

Proof. Follows from Theorem 3.5. □

Lemma 3.1. *Let $A \in \alpha SC(X,\tau)_{[\gamma,\gamma']}$ and suppose that $\alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq B \subseteq A$. Then, $B \in \alpha SC(X,\tau)_{[\gamma,\gamma']}$.*

Proof. Let $A \in \alpha SC(X,\tau)_{[\gamma,\gamma']}$, then by Theorem 3.8, there exists an $\alpha_{[\gamma,\gamma']}$-closed set F such that $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq A \subseteq F$. Since $B \subseteq A$ and $A \subseteq F$. Thus, $B \subseteq F$ also $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(A)$ and $\alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq B$. This implies that $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq B$. Hence, $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq B \subseteq F$, where F is $\alpha_{[\gamma,\gamma']}$-closed in X. This proves that $B \in \alpha SC(X,\tau)_{[\gamma,\gamma']}$.

□

Proposition 3.5. *Let (X,τ) be a topological space and γ, γ' be operations on $\alpha O(X)$. Then, a subset A of X is $\alpha_{[\gamma,\gamma']}$-semiclosed if and only if $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))) \subseteq A$ and $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.*

Proof. Let A be an $\alpha_{[\gamma,\gamma']}$-semiclosed set. Then, by Theorem 3.8 (2), we have $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$. Moreover, by Theorem 3.8 (3), $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) = \alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$.

Conversely, since $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))$. Thus, we obtain that $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A)))$. By hypothesis, we have $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(A))) \subseteq A$. Hence, by Theorem 3.8, A is an $\alpha_{[\gamma,\gamma']}$-semiclosed set. □

Definition 3.3. Let A be a subset of a topological space (X, τ) and γ, γ' be operations on $\alpha O(X)$. Then:

(1) The $\alpha_{[\gamma,\gamma']}$-semiclosure of A is defined as the intersection of all $\alpha_{[\gamma,\gamma']}$-semiclosed sets containing A. That is, $\alpha_{[\gamma,\gamma']}$-$sCl(A) = \bigcap\{F : F$ is $\alpha_{[\gamma,\gamma']}$-semiclosed and $A \subseteq F\}$.

(2) The $\alpha_{[\gamma,\gamma']}$-semiinterior of A is defined as the union of all $\alpha_{[\gamma,\gamma']}$-semiopen sets contained in A. That is, $\alpha_{[\gamma,\gamma']}$-$sInt(A) = \bigcup\{U : U$ is $\alpha_{[\gamma,\gamma']}$-semiopen and $U \subseteq A\}$.

(3) The $\alpha_{[\gamma,\gamma']}$-semiboundary of A, denoted by $\alpha_{[\gamma,\gamma']}$-$sBd(A)$ is defined as $\alpha_{[\gamma,\gamma']}$-$sCl(A) \setminus \alpha_{[\gamma,\gamma']}$-$sInt(A)$.

(4) The set denoted by $\alpha_{[\gamma,\gamma']}$-$sD(A)$ and defined by $\{x :$ for every $\alpha_{[\gamma,\gamma']}$-semiopen set U containing x, $U \cap (A \setminus \{x\}) \neq \phi\}$ is called the $\alpha_{[\gamma,\gamma']}$-semiderived set of A.

The proofs of the following theorems are obvious and therefore are omitted.

Theorem 3.10. *Let A, B be subsets of a topological space (X, τ) and γ, γ' be operations on $\alpha O(X)$. Then:*

(1) $\alpha_{[\gamma,\gamma']}$-$sCl(A)$ *is the smallest* $\alpha_{[\gamma,\gamma']}$-*semiclosed subset of* X *containing* A.
(2) $A \in \alpha SC(X, \tau)_{[\gamma,\gamma']}$ *if and only if* $\alpha_{[\gamma,\gamma']}$-$sCl(A) = A$.
(3) $\alpha_{[\gamma,\gamma']}$-$sCl(\alpha_{[\gamma,\gamma']}$-$sCl(A)) = \alpha_{[\gamma,\gamma']}$-$sCl(A)$.
(4) $A \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(A)$.
(5) *If* $A \subseteq B$, *then* $\alpha_{[\gamma,\gamma']}$-$sCl(A) \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(B)$.
(6) $\alpha_{[\gamma,\gamma']}$-$sCl(A \cap B) \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(A) \cap \alpha_{[\gamma,\gamma']}$-$sCl(B)$.
(7) $\alpha_{[\gamma,\gamma']}$-$sCl(A \cup B) \supseteq \alpha_{[\gamma,\gamma']}$-$sCl(A) \cup \alpha_{[\gamma,\gamma']}$-$sCl(B)$.
(8) $x \in \alpha_{[\gamma,\gamma']}$-$sCl(A)$ *if and only if* $V \cap A \neq \phi$ *for every* $V \in \alpha SO(X, x)_{[\gamma,\gamma']}$.

Theorem 3.11. *Let A, B be subsets of a topological space (X, τ) and γ, γ' be operations on $\alpha O(X)$. Then:*

(1) $\alpha_{[\gamma,\gamma']}$-$sInt(A)$ *is the largest* $\alpha_{[\gamma,\gamma']}$-*semiopen subset of* X *contained in* A.
(2) A *is* $\alpha_{[\gamma,\gamma']}$-*semiopen if and only if* $A = \alpha_{[\gamma,\gamma']}$-$sInt(A)$.
(3) $\alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$sInt(A)) = \alpha_{[\gamma,\gamma']}$-$sInt(A)$.
(4) $\alpha_{[\gamma,\gamma']}$-$sInt(A) \subseteq A$.
(5) *If* $A \subseteq B$, *then* $\alpha_{[\gamma,\gamma']}$-$sInt(A) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(B)$.
(6) $\alpha_{[\gamma,\gamma']}$-$sInt(A \cup B) \supseteq \alpha_{[\gamma,\gamma']}$-$sInt(A) \cup \alpha_{[\gamma,\gamma']}$-$sInt(B)$.
(7) $\alpha_{[\gamma,\gamma']}$-$sInt(A \cap B) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(A) \cap \alpha_{[\gamma,\gamma']}$-$sInt(B)$.
(8) $X \setminus \alpha_{[\gamma,\gamma']}$-$sInt(A) = \alpha_{[\gamma,\gamma']}$-$sCl(X \setminus A)$.
(9) $X \setminus \alpha_{[\gamma,\gamma']}$-$sCl(A) = \alpha_{[\gamma,\gamma']}$-$sInt(X \setminus A)$.
(10) $\alpha_{[\gamma,\gamma']}$-$sInt(A) = X \setminus \alpha_{[\gamma,\gamma']}$-$sCl(X \setminus A)$.
(11) $\alpha_{[\gamma,\gamma']}$-$sCl(A) = X \setminus \alpha_{[\gamma,\gamma']}$-$sInt(X \setminus A)$.

Theorem 3.12. *Let A, B be subsets of a topological space (X, τ) and γ, γ' be operations on $\alpha O(X)$. Then:*

(1) $\alpha_{[\gamma,\gamma']}$-$sCl(A) = \alpha_{[\gamma,\gamma']}$-$sInt(A) \cup \alpha_{[\gamma,\gamma']}$-$sBd(A)$.
(2) $\alpha_{[\gamma,\gamma']}$-$sInt(A) \cap \alpha_{[\gamma,\gamma']}$-$sBd(A) = \phi$.
(3) $\alpha_{[\gamma,\gamma']}$-$sBd(A) = \alpha_{[\gamma,\gamma']}$-$sCl(A) \cap \alpha_{[\gamma,\gamma']}$-$sCl(X \setminus A)$.
(4) $\alpha_{[\gamma,\gamma']}$-$sBd(A) = \alpha_{[\gamma,\gamma']}$-$sBd(X \setminus A)$.

(5) $\alpha_{[\gamma,\gamma']}$-$sBd(A)$ *is an* $\alpha_{[\gamma,\gamma']}$-*semiclosed set.*

Theorem 3.13. *Let* A, B *be subsets of a topological space* (X,τ) *and* γ, γ' *be operations on* $\alpha O(X)$. *Then:*

(1) *If* $x \in \alpha_{[\gamma,\gamma']}$-$sD(A)$, *then* $x \in \alpha_{[\gamma,\gamma']}$-$sD(A \setminus \{x\})$.
(2) $\alpha_{[\gamma,\gamma']}$-$sD(A \cup B) \supseteq \alpha_{[\gamma,\gamma']}$-$sD(A) \cup \alpha_{[\gamma,\gamma']}$-$sD(B)$.
(3) $\alpha_{[\gamma,\gamma']}$-$sD(A \cap B) \subseteq \alpha_{[\gamma,\gamma']}$-$sD(A) \cap \alpha_{[\gamma,\gamma']}$-$sD(B)$.
(4) $\alpha_{[\gamma,\gamma']}$-$sD(\alpha_{[\gamma,\gamma']}$-$sD(A)) \setminus A \subseteq \alpha_{[\gamma,\gamma']}$-$sD(A)$.
(5) $\alpha_{[\gamma,\gamma']}$-$sD(A \cup \alpha_{[\gamma,\gamma']}$-$sD(A)) \subseteq A \cup \alpha_{[\gamma,\gamma']}$-$sD(A)$.
(6) $\alpha_{[\gamma,\gamma']}$-$sCl(A) = A \cup \alpha_{[\gamma,\gamma']}$-$sD(A)$.
(7) A *is* $\alpha_{[\gamma,\gamma']}$-*semiclosed if and only if* $\alpha_{[\gamma,\gamma']}$-$sD(A) \subseteq A$.

Remark 3.2. Let A be subset of a topological space (X,τ) and γ, γ' be operations on $\alpha O(X)$. Then:
$\alpha_{[\gamma,\gamma']}$-$Int(A) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(A) \subseteq A \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(A) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(A)$.

Theorem 3.14. *Let* (X,τ) *be a topological space,* γ, γ' *operations on* $\alpha O(X)$ *and* A *a subset of* X. *Then, the following statements are equivalent:*

(1) $A = \alpha_{[\gamma,\gamma']}$-$sCl(A)$.
(2) $\alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$sCl(A)) \subseteq A$.
(3) $(\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))) \setminus (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))) \supseteq (\alpha_{[\gamma,\gamma']}$-$Cl(A) \setminus A)$.

Proof. (1) $\Rightarrow$ (2): If $A = \alpha_{[\gamma,\gamma']}$-$sCl(A)$, then $\alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$sCl(A)) = \alpha_{[\gamma,\gamma']}$-$sInt(A) \subseteq A$.

(2) $\Rightarrow$ (1): Suppose that $\alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$sCl(A)) \subseteq A$. Now, by Theorem 3.10 (1), $\alpha_{[\gamma,\gamma']}$-$sCl(A)$ is an $\alpha_{[\gamma,\gamma']}$-semiclosed set and so, by Theorem 3.8, there is an $\alpha_{[\gamma,\gamma']}$-closed set F such that $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(A) \subseteq F$. Since $\alpha_{[\gamma,\gamma']}$-$Int(F)$ is $\alpha_{[\gamma,\gamma']}$-semiopen, then $\alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$Int(F)) = \alpha_{[\gamma,\gamma']}$-$Int(F)$. Therefore, $\alpha_{[\gamma,\gamma']}$-$Int(F) = \alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$Int(F)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(\alpha_{[\gamma,\gamma']}$-$sCl(A))$ and hence $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq A$. But $A \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(A) \subseteq F$. Thus, $\alpha_{[\gamma,\gamma']}$-$Int(F) \subseteq A \subseteq F$, where F is $\alpha_{[\gamma,\gamma']}$-closed. Hence by Theorem 3.8, A is $\alpha_{[\gamma,\gamma']}$-semiclosed and by Theorem 3.10 (2), $A = \alpha_{[\gamma,\gamma']}$-$sCl(A)$.

(3) $\Leftrightarrow$ (1): We have $(\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))) \setminus (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))) \supseteq (\alpha_{[\gamma,\gamma']}$-$Cl(A) \setminus A)$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Cl(A) \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))) \setminus (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))) \subseteq A$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Cl(A) \cap [X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))) \setminus (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))))] \subseteq A$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Cl(A) \cap [X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))) \cap (\alpha_{[\gamma,\gamma']}$-$Cl(A)))] \subseteq A$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Cl(A) \cap [(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A))))) \cup (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))] \subseteq A$
$\Leftrightarrow [\alpha_{[\gamma,\gamma']}$-$Cl(A) \cap (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))))] \cup [\alpha_{[\gamma,\gamma']}$-$Cl(A) \cap (X \setminus (\alpha_{[\gamma,\gamma']}$-$Cl(A)))] \subseteq A$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Cl(A) \cap \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$
$\Leftrightarrow \alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(A)) \subseteq A$
$\Leftrightarrow A$ is $\alpha_{[\gamma,\gamma']}$-semiclosed
$\Leftrightarrow A = \alpha_{[\gamma,\gamma']}$-$sCl(A)$. $\square$

Theorem 3.15. *If A is a subset of a nonempty space X and $\gamma, \gamma^{'}$ are operations on $\alpha O(X)$, then the following statements are equivalent:*

(1) $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) = X$.
(2) $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) = X$.
(3) *If B is any $\alpha_{[\gamma,\gamma']}$-semiclosed subset of X such that $A \subseteq B$, then $B = X$.*
(4) *Every nonempty $\alpha_{[\gamma,\gamma']}$-semiopen set has a nonempty intersection with A.*
(5) $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) = \phi$.

Proof. (1) $\Rightarrow$ (2): Suppose $x \notin \alpha_{[\gamma,\gamma']}\text{-}sCl(A)$. Then, by Theorem 3.10 (8), there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set G containing x such that $G \cap A = \phi$. Since G is a nonempty $\alpha_{[\gamma,\gamma']}$-semiopen set, then there is a nonempty $\alpha_{[\gamma,\gamma']}$-open set H such that $H \subseteq G$ and so $H \cap A = \phi$ which implies that $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) \neq X$, a contradiction. Hence $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) = X$.

(2) $\Rightarrow$ (3): If B is any $\alpha_{[\gamma,\gamma']}$-semiclosed set such that $A \subseteq B$, then $X = \alpha_{[\gamma,\gamma']}\text{-}sCl(A) \subseteq \alpha_{[\gamma,\gamma']}\text{-}sCl(B) = B$ and so $B = X$.

(3) $\Rightarrow$ (4): If G is any nonempty $\alpha_{[\gamma,\gamma']}$-semiopen set such that $G \cap A = \phi$, then $A \subseteq X \setminus G$ and $X \setminus G$ is $\alpha_{[\gamma,\gamma']}$-semiclosed. By hypothesis, $X \setminus G = X$ and so $G = \phi$, a contradiction. Therefore, $G \cap A \neq \phi$.

(4) $\Rightarrow$ (5): Suppose that $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) \neq \phi$. Then, by Theorem 3.11 (1), $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A)$ is a nonempty $\alpha_{[\gamma,\gamma']}$-semiopen set such that $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) \cap A = \phi$, a contradiction. Therefore, $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) = \phi$.

(5) $\Rightarrow$ (1): Since $\alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) = \phi$ implies that $X \setminus \alpha_{[\gamma,\gamma']}\text{-}sInt(X \setminus A) = X$ by Theorem 3.11 (11), implies that $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) = X$. By Remark 3.2, $\alpha_{[\gamma,\gamma']}\text{-}sCl(B) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(B)$ for every subset B of X. Therefore, $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) = X$ implies that $\alpha_{[\gamma,\gamma']}\text{-}Cl(A) = X$. $\square$

Proposition 3.6. *Let γ and $\gamma^{'}$ be α-regular operations on $\alpha O(X)$. If A is a subset of X and $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) = X$, then for every $\alpha_{[\gamma,\gamma']}$-open set G of X, we have $\alpha_{[\gamma,\gamma']}\text{-}Cl(A \cap G) = \alpha_{[\gamma,\gamma']}\text{-}Cl(G)$.*

Proof. The proof follows from Theorem 3.15 and Theorem 3.6 (2). $\square$

Definition 3.4. Let (X, τ) be a topological space and $\gamma, \gamma^{'}$ be operations on $\alpha O(X)$. A subset B_x of X is said to be an $\alpha_{[\gamma,\gamma']}$-semineighborhood (resp. $\alpha_{[\gamma,\gamma']}$-neighborhood) of a point $x \in X$ if there exists an $\alpha_{[\gamma,\gamma']}$-semiopen (resp. $\alpha_{[\gamma,\gamma']}$-open) set U such that $x \in U \subseteq B_x$.

Theorem 3.16. *Let (X, τ) be a topological space and $\gamma, \gamma^{'}$ be operations on $\alpha O(X)$. A subset G of X is $\alpha_{[\gamma,\gamma']}$-semiopen if and only if it is an $\alpha_{[\gamma,\gamma']}$-semineighborhood of each of its points.*

Proof. Let G be an $\alpha_{[\gamma,\gamma']}$-semiopen set of X. Then, by Definition 3.4, it is clear that G is an $\alpha_{[\gamma,\gamma']}$-semineighborhood of each of its points, since for every $x \in G, x \in G \subseteq G$ and G is $\alpha_{[\gamma,\gamma']}$-semiopen.

Conversely, suppose that G is an $\alpha_{[\gamma,\gamma']}$-semineighborhood of each of its points. Then, for each $x \in G$, there exists $S_x \in \alpha SO(X,x)_{[\gamma,\gamma']}$ such that $S_x \subseteq G$. Then, $G = \bigcup\{S_x : x \in G\}$. Since each S_x is $\alpha_{[\gamma,\gamma']}$-semiopen, hence by Theorem 3.5, G is $\alpha_{[\gamma,\gamma']}$-semiopen in (X,τ). □

Proposition 3.7. *For any two subsets A, B of a topological space (X,τ) and $A \subseteq B$, if A is an $\alpha_{[\gamma,\gamma']}$-semineighborhood of a point $x \in X$, Then, B is also $\alpha_{[\gamma,\gamma']}$-semineighborhood of the same point x.*

Proof. Obvious. □

4. Some New Functions

Throughout this section, let $\gamma, \gamma^{'} : \alpha O(X) \to P(X)$ and $\beta, \beta^{'} : \alpha O(Y) \to P(Y)$ be operations on $\alpha O(X)$ and $\alpha O(Y)$, respectively.

Definition 4.1. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous if for each $x \in X$ and each $\alpha_{[\beta,\beta']}$-open set V of Y containing $f(x)$, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set U of X such that $x \in U$ and $f(U) \subseteq V$.

Theorem 4.1. *For a function $f : (X,\tau) \to (Y,\sigma)$ the following statements are equivalent:*

1. *f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous.*
2. *The inverse image of each $\alpha_{[\beta,\beta']}$-open set in Y is $\alpha_{[\gamma,\gamma']}$-semiopen in X.*
3. *The inverse image of each $\alpha_{[\beta,\beta']}$-closed set in Y is $\alpha_{[\gamma,\gamma']}$-semiclosed in X.*
4. *For each subset A of X, $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A)) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(f(A))$.*
5. *For each subset B of Y, $\alpha_{[\gamma,\gamma']}\text{-}sCl(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(B))$.*
6. *For each subset B of Y, $f^{-1}(\alpha_{[\beta,\beta']}\text{-}Int(B)) \subseteq \alpha_{[\gamma,\gamma']}\text{-}sInt(f^{-1}(B))$.*

Proof. (1) ⇒ (2): Let f be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. Let V be any $\alpha_{[\beta,\beta']}$-open set in Y. To show that $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X, if $f^{-1}(V) = \phi$, then $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X, if $f^{-1}(V) \neq \phi$, then there exists $x \in f^{-1}(V)$ which implies $f(x) \in V$. Since f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set U in X containing x such that $f(U) \subseteq V$. This implies that $x \in U \subseteq f^{-1}(V)$. This shows $f^{-1}(V)$ is $\alpha_{[\gamma,\gamma']}$-semiopen.

(2) ⇒ (3): Let F be any $\alpha_{[\beta,\beta']}$-closed set of Y. Then $Y \setminus F$ is an $\alpha_{[\beta,\beta']}$-open set of Y. By (2), $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X and hence $f^{-1}(F)$ is an $\alpha_{[\gamma,\gamma']}$-semiclosed set in X.

(3) ⇒ (4): Let A be any subset of X. Then, $f(A) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(f(A))$ and $\alpha_{[\beta,\beta']}\text{-}Cl(f(A))$ is an $\alpha_{[\beta,\beta']}$-closed set in Y. Hence $A \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(f(A)))$. By (3), we have $f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(f(A)))$ is an $\alpha_{[\gamma,\gamma']}$-semiclosed set in X. Therefore, $\alpha_{[\gamma,\gamma']}\text{-}sCl(A) \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(f(A)))$. Hence, $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A)) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(f(A))$.

(4) ⇒ (5): Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (4), we have $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(f^{-1}(B))) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(f(f^{-1}(B))) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(B)$. Hence, $\alpha_{[\gamma,\gamma']}\text{-}sCl(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(B))$.

(5) $\Leftrightarrow$ (6): Let B be any subset of Y. Then apply (5) to $Y \setminus B$ we obtain $\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(Y \setminus B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(Y \setminus B)) \Leftrightarrow \alpha_{[\gamma,\gamma']}$-$sCl(X \setminus f^{-1}(B)) \subseteq f^{-1}(Y \setminus \alpha_{[\beta,\beta']}$-$Int(B)) \Leftrightarrow X \setminus \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B)) \subseteq X \setminus f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \Leftrightarrow f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$. Therefore, $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$.

(6) $\Rightarrow$ (1): Let $x \in X$ and V be any $\alpha_{[\beta,\beta']}$-open set of Y containing $f(x)$. Then, $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By (6), we have $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(V)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(V))$. Since V is an $\alpha_{[\beta,\beta']}$-open set, then $f^{-1}(V) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set in X which contains x and clearly $f(f^{-1}(V)) \subseteq V$. Hence, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. □

Theorem 4.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous function. Then, for each subset B of Y, $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(B)))$.*

Proof. Let B be any subset of Y. Then, $\alpha_{[\beta,\beta']}$-$Int(B)$ is $\alpha_{[\beta,\beta']}$-open in Y and so by Theorem 4.1, $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B))$ is $\alpha_{[\gamma,\gamma']}$-semiopen in X. Hence, Theorem 3.3, we have $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)))) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(B)))$. □

Corollary 4.1. *Let $f : (X,\tau) \to (Y,\sigma)$ be an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous function. Then, for each subset B of Y, $\alpha_{[\gamma,\gamma']}$-$Int(\alpha_{[\gamma,\gamma']}$-$Cl(f^{-1}(B))) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(B))$.*

Proof. The proof is obvious. □

Theorem 4.3. *Let $f : (X,\tau) \to (Y,\sigma)$ a bijective function. Then, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous if and only if $\alpha_{[\beta,\beta']}$-$Int(f(A)) \subseteq f(\alpha_{[\gamma,\gamma']}$-$sInt(A))$ for each subset A of X.*

Proof. Let A be any subset of X. Then, by Theorem 4.1, $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(f(A))) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(f(A)))$. Since f is a bijective function, then $\alpha_{[\beta,\beta']}$-$Int(f(A)) = f(f^{-1}(\alpha_{[\beta,\beta']}$-$Int(f(A)))) \subseteq f(\alpha_{[\gamma,\gamma']}$-$sInt(A))$.

Conversely, let B be any subset of Y. Then, $\alpha_{[\beta,\beta']}$-$Int(f(f^{-1}(B))) \subseteq f(\alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B)))$. Since f is a bijection, so, $\alpha_{[\beta,\beta']}$-$Int(B) = \alpha_{[\beta,\beta']}$-$Int(f(f^{-1}(B))) \subseteq f(\alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B)))$. Hence, $f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$. Therefore, by Theorem 4.1, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. □

Proposition 4.1. *A function $f : (X,\tau) \to (Y,\sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous if and only if $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(B) \setminus \alpha_{[\beta,\beta']}$-$Int(B))$, for each subset B in Y.*

Proof. Let B be any subset of Y. By Theorem 4.1 (2) and (5), we have $f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(B) \setminus \alpha_{[\beta,\beta']}$-$Int(B)) = f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(B)) \setminus f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) \supseteq \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \setminus f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B)) = \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \setminus \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(\alpha_{[\beta,\beta']}$-$Int(B))) \supseteq \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \setminus \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B)) = \alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B))$, and hence $f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(B) \setminus \alpha_{[\beta,\beta']}$-$Int(B)) \supseteq \alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B))$.

Conversely, let V be $\alpha_{[\beta,\beta']}$-open in Y and $F = Y \backslash V$. Then by (2), we obtain $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(F)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(F) \backslash \alpha_{[\beta,\beta']}$-$Int(F)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(F)) = f^{-1}(F)$ and hence by Theorem 3.12 (1), $\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(F)) = \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(F)) \cup \alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(F)) \subseteq f^{-1}(F)$. Thus, $f^{-1}(F)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed and hence $f^{-1}(V)$ is $\alpha_{[\gamma,\gamma']}$-semiopen in X. Therefore, by Theorem 4.1 (2), f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. □

Proposition 4.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous if and only if $f(\alpha_{[\gamma,\gamma']}$-$sD(A)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(f(A))$, for any subset A of X.*

Proof. Let A be any subset of X. By Theorem 4.1 (4), and by the fact that $\alpha_{[\gamma,\gamma']}$-$sCl(A) = A \cup \alpha_{[\gamma,\gamma']}$-$sD(A)$, we get $f(\alpha_{[\gamma,\gamma']}$-$sD(A)) \subseteq f(\alpha_{[\gamma,\gamma']}$-$sCl(A)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(f(A))$.

Conversely, let F be any $\alpha_{[\beta,\beta']}$-closed set in Y. By (2), we obtain $f(\alpha_{[\gamma,\gamma']}$-$sD(f^{-1}(F))) \subseteq \alpha_{[\beta,\beta']}$-$Cl(f(f^{-1}(F))) \subseteq \alpha_{[\beta,\beta']}$-$Cl(F) = F$. This implies $\alpha_{[\gamma,\gamma']}$-$sD(f^{-1}(F)) \subseteq f^{-1}(F)$. Hence, by Theorem 3.13 (7), $f^{-1}(F)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed in X. Therefore, by Theorem 4.1 (3), f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. □

Definition 4.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen if and only if for each $\alpha_{[\gamma,\gamma']}$-open set U in X, $f(U)$ is $\alpha_{[\beta,\beta']}$-semiopen set in Y.

Theorem 4.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen if and only if for every subset $E \subseteq X$, we have $f(\alpha_{[\gamma,\gamma']}$-$Int(E)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(f(E)))$.*

Proof. Let f be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen. Since $f(\alpha_{[\gamma,\gamma']}$-$Int(E)) \subseteq f(E)$, and $f(\alpha_{[\gamma,\gamma']}$-$Int(E))$ is $\alpha_{[\beta,\beta']}$-semiopen. Then, $f(\alpha_{[\gamma,\gamma']}$-$Int(E)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(f(\alpha_{[\gamma,\gamma']}$-$Int(E)))) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(f(E)))$.

Conversely, let G be any $\alpha_{[\gamma,\gamma']}$-open set in X. Then, $\alpha_{[\beta,\beta']}$-$Int(f(G)) \subseteq f(G) \subseteq f(\alpha_{[\gamma,\gamma']}$-$Int(G)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(f(G)))$. Therefore, $f(G)$ is $\alpha_{[\beta,\beta']}$-semiopen and consequently f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen. □

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen function, then for every subset G of Y, $\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(G)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(G)))$.*

Proof. Let f be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen. By Theorem 4.4, we have $f(\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(G))) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(f(f^{-1}(G)))) \subseteq \alpha_{[\beta,\beta']}$-$Cl(\alpha_{[\beta,\beta']}$-$Int(G)) \subseteq \alpha_{[\beta,\beta']}$-$Cl(G)$ implies that $\alpha_{[\gamma,\gamma']}$-$Int(f^{-1}(G)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(G)) \subseteq \alpha_{[\gamma,\gamma']}$-$Cl(f^{-1}(\alpha_{[\beta,\beta']}$-$Cl(G)))$. □

Theorem 4.6. *A function $f : (X, \tau) \to (Y, \sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen if and only if for every $x \in X$ and for every $\alpha_{[\gamma,\gamma']}$-neighborhood U of x, there exists an $\alpha_{[\beta,\beta']}$-semineighborhood V of $f(x)$ such that $V \subseteq f(U)$.*

Proof. Let U be an $\alpha_{[\gamma,\gamma']}$-neighborhood of $x \in X$. Then, there exists an $\alpha_{[\gamma,\gamma']}$-open set O such that $x \in O \subseteq U$. By hypothesis, $f(O)$ is $\alpha_{[\beta,\beta']}$-semineighborhood in Y such that $f(x) \in f(O) \subseteq f(U)$.

Conversely, let U be any $\alpha_{[\gamma,\gamma']}$-open set in X. For each $y \in f(U)$, by hypothesis there exists an $\alpha_{[\beta,\beta']}$-semineighborhood V_y of y in Y such that $V_y \subseteq f(U)$. Since V_y is $\alpha_{[\beta,\beta']}$-semineighbourhood of y, there exists an $\alpha_{[\beta,\beta']}$-semiopen set A_y in Y such that $y \in A_y \subseteq V_y$. Therefore, $f(U) = \cup\{A_y : y \in f(U)\}$ is an $\alpha_{[\beta,\beta']}$-semiopen in Y. This shows that f is an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen function. □

Theorem 4.7. *The following statements are equivalent for a bijective function $f : (X, \tau) \to (Y, \sigma)$:*

(1) *f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen.*
(2) *$f(\alpha_{[\gamma,\gamma']}\text{-}Int(A)) \subseteq \alpha_{[\beta,\beta']}\text{-}sInt(f(A))$, for every $A \subseteq X$.*
(3) *$\alpha_{[\gamma,\gamma']}\text{-}Int(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}sInt(B))$, for every $B \subseteq Y$.*
(4) *$f^{-1}(\alpha_{[\beta,\beta']}\text{-}sCl(B)) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(f^{-1}(B))$, for every $B \subseteq Y$.*
(5) *$\alpha_{[\beta,\beta']}\text{-}sCl(f(A)) \subseteq f(\alpha_{[\gamma,\gamma']}\text{-}Cl(A))$, for every $A \subseteq X$.*
(6) *$\alpha_{[\beta,\beta']}\text{-}sD(f(A)) \subseteq f(\alpha_{[\gamma,\gamma']}\text{-}Cl(A))$, for every $A \subseteq X$.*

Proof. (1) $\Rightarrow$ (2): Let A be any subset of X. Since $f(\alpha_{[\gamma,\gamma']}\text{-}Int(A))$ is $\alpha_{[\beta,\beta']}$-semiopen and $f(\alpha_{[\gamma,\gamma']}\text{-}Int(A)) \subseteq f(A)$, and thus $f(\alpha_{[\gamma,\gamma']}\text{-}Int(A)) \subseteq \alpha_{[\beta,\beta']}\text{-}sInt(f(A))$.
The proof of the other implications are obvious. □

Theorem 4.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous and $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen and let $A \in \alpha SO(X)_{[\gamma,\gamma']}$. Then, $f(A) \in \alpha SO(Y)_{[\beta,\beta']}$.*

Proof. Since A is $\alpha_{[\gamma,\gamma']}$-semiopen, then there exists an $\alpha_{[\gamma,\gamma']}$-open set O in X such that $O \subseteq A \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(O)$. Therefore, $f(O) \subseteq f(A) \subseteq f(\alpha_{[\gamma,\gamma']}\text{-}Cl(O)) \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(f(O))$. Thus, by Theorem 3.4, $f(A) \in \alpha SO(Y)_{[\beta,\beta']}$. □

Theorem 4.9. *Let π and π' be operations on $\alpha O(Z)$. If $f : X \to Y$ is a function, $g : Y \to Z$ is $(\alpha_{[\beta,\beta']}, \alpha_{[\pi,\pi']})$-semiopen and injective, and $gof : X \to Z$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\pi,\pi']})$-semicontinuous. Then, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous.*

Proof. Let V be an $\alpha_{[\beta,\beta']}$-open subset of Y. Since g is $(\alpha_{[\beta,\beta']}, \alpha_{[\pi,\pi']})$-semiopen, $g(V)$ is $\alpha_{[\pi,\pi']}$-semiopen subset of Z. Since gof is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\pi,\pi']})$-semicontinuous and g is injective, then $f^{-1}(V) = f^{-1}(g^{-1}(g(V))) = (gof)^{-1}(g(V))$ is $\alpha_{[\gamma,\gamma']}$-semiopen in X, which proves that f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous. □

Definition 4.3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if the inverse image of every $\alpha_{[\beta,\beta']}$-semiopen set of Y is $\alpha_{[\gamma,\gamma']}$-semiopen in X.

Proposition 4.3. *Every $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute function is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous.*

Proof. Straightforward. □

The converse of the above proposition need not be true in general as it is shown below.

Example 4.1. Let $X = \{a, b, c\}$ and $\tau = \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ be a topology on X. For each $A \in \alpha O(X)$, define the operations $\gamma : \alpha O(X, \tau) \to P(X)$, γ' :

$\alpha O(X,\tau) \to P(X)$, $\beta : \alpha O(X,\sigma) \to P(X)$ and $\beta^{'} : \alpha O(X,\sigma) \to P(X)$, respectively, by

$$A^{\gamma} = A^{\gamma^{'}} = \begin{cases} A & \text{if } A = \{a,b\} \\ X & \text{if } A \neq \{a,b\} \end{cases}$$

and

$$A^{\beta} = A^{\beta^{'}} = \begin{cases} A & \text{if } A = \{b\} \\ X & \text{if } A \neq \{b\}. \end{cases}$$

Define a function $f : (X,\tau) \to (X,\sigma)$ as follows:

$$f(x) = \begin{cases} a & \text{if } x = a \\ a & \text{if } x = b \\ c & \text{if } x = c. \end{cases}$$

Then, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous, but not $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute because $\{b,c\}$ is an $\alpha_{[\beta,\beta']}$-semiopen set of Y but $f^{-1}(\{b,c\}) = \{c\}$ is not $\alpha_{[\gamma,\gamma']}$-semiopen in X.

Theorem 4.10. *If $f : (X,\tau) \to (Y,\sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous and $f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(V)) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(f^{-1}(V))$ for each subset $V \in \alpha O(Y)_{[\beta,\beta']}$, then f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute.*

Proof. Let B be any $\alpha_{[\beta,\beta']}$-semiopen subset of Y. Then, there exists $V \in \alpha O(Y)_{[\beta,\beta']}$ such that $V \subseteq B \subseteq \alpha_{[\beta,\beta']}\text{-}Cl(V)$. Therefore, we have $f^{-1}(V) \subseteq f^{-1}(B) \subseteq f^{-1}(\alpha_{[\beta,\beta']}\text{-}Cl(V)) \subseteq \alpha_{[\gamma,\gamma']}\text{-}Cl(f^{-1}(V))$. Since f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semicontinuous and $V \in \alpha O(Y)_{[\beta,\beta']}$, then $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set of X. Hence, by Theorem 3.4, $f^{-1}(B)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen set of X. This shows that f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute. $\square$

Theorem 4.11. *A function $f : (X,\tau) \to (Y,\sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if and only if for each $x \in X$ and each $\alpha_{[\beta,\beta']}$-semiopen set V of Y containing $f(x)$, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set U of X containing x such that $f(U) \subseteq V$.*

Proof. Let $x \in X$ and V be any $\alpha_{[\beta,\beta']}$-semiopen set of Y containing $f(x)$. Set $U = f^{-1}(V)$, then by f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute, U is an $\alpha_{[\gamma,\gamma']}$-semiopen subset of X containing x and $f(U) \subseteq V$.

Conversely, let V be any $\alpha_{[\beta,\beta']}$-semiopen set of Y and $x \in f^{-1}(V)$. By hypothesis, there exists an $\alpha_{[\gamma,\gamma']}$-semiopen set U of X containing x such that $f(U) \subseteq V$. Thus, we have $x \in U \subseteq f^{-1}(f(U)) \subseteq f^{-1}(V)$. By Proposition 3.1, $f^{-1}(V)$ is $\alpha_{[\gamma,\gamma']}$-semiopen of X. Therefore, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute. $\square$

Theorem 4.12. *A function $f : (X,\tau) \to (Y,\sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if and only if for every $\alpha_{[\beta,\beta']}$-semiclosed subset H of Y, $f^{-1}(H)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed in X.*

Proof. Let f be $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute, then for every $\alpha_{[\beta,\beta']}$-semiopen subset Q of Y, $f^{-1}(Q)$ is $\alpha_{[\gamma,\gamma']}$-semiopen in X. Let H be any $\alpha_{[\beta,\beta']}$-semiclosed subset of Y, then $Y \setminus H$ is $\alpha_{[\beta,\beta']}$-semiopen. Thus, $f^{-1}(Y \setminus H)$ is $\alpha_{[\gamma,\gamma']}$-semiopen, but $f^{-1}(Y \setminus H) = X \setminus f^{-1}(H)$ so that $f^{-1}(H)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed.

Conversely, suppose that for all $\alpha_{[\beta,\beta']}$-semiclosed subset H of Y, $f^{-1}(H)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed in X and let Q be any $\alpha_{[\beta,\beta']}$-semiopen subset of Y, then $Y \setminus Q$ is $\alpha_{[\beta,\beta']}$-semiclosed. By hypothesis, $X \setminus f^{-1}(Q) = f^{-1}(Y \setminus Q)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed. Thus, $f^{-1}(Q)$ is $\alpha_{[\gamma,\gamma']}$-semiopen. □

Theorem 4.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be function. Then, the following statements are equivalent:*

(1) *f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute.*
(2) *$\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B))$, for each subset B of Y.*
(3) *$f(\alpha_{[\gamma,\gamma']}$-$sCl(A)) \subseteq \alpha_{[\beta,\beta']}$-$sCl(f(A))$, for each subset A of X.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Then, $B \subseteq \alpha_{[\beta,\beta']}$-$sCl(B)$ and $f^{-1}(B) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B))$. Since f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute, so, $f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B))$ is an $\alpha_{[\gamma,\gamma']}$-semiclosed subset of X. Hence, $\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B))) = f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B))$

(2) $\Rightarrow$ (3): Let A be any subset of X. Then, $f(A) \subseteq \alpha_{[\beta,\beta']}$-$sCl(f(A))$ and $\alpha_{[\gamma,\gamma']}$-$sCl(A) \subseteq \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(f(A))) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(f(A)))$. This implies that $f(\alpha_{[\gamma,\gamma']}$-$sCl(A)) \subseteq f(f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(f(A)))) \subseteq \alpha_{[\beta,\beta']}$-$sCl(f(A))$.

(3) $\Rightarrow$ (1): Let V be an $\alpha_{[\beta,\beta']}$-semiclosed subset of Y. Then, $f(\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(V))) \subseteq \alpha_{[\beta,\beta']}$-$sCl(f(f^{-1}(V))) \subseteq \alpha_{[\beta,\beta']}$-$sCl(V) = V$. This implies that $\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(V)) \subseteq f^{-1}(f(\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(V)))) \subseteq f^{-1}(V)$. Thus, $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiclosed subset of X and consequently f is an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute function. □

Theorem 4.14. *A function $f : (X, \tau) \to (Y, \sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if and only if $f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(B)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$ for each subset B of Y.*

Proof. Let B be any subset of Y. Then, $\alpha_{[\beta,\beta']}$-$sInt(B) \subseteq B$. Since f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute, $f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(B))$ is an $\alpha_{[\gamma,\gamma']}$-semiopen subset of X. Hence, $f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(B)) = \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(B))) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$.

Conversely, let V be an $\alpha_{[\beta,\beta']}$-semiopen subset of Y. Then, $f^{-1}(V) = f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(V)) \subseteq \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is an $\alpha_{[\gamma,\gamma']}$-semiopen subset of X and consequently f is an $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute function. □

Proposition 4.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if and only if $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sBd(B))$, for each subset B of Y.*

Proof. Let B be any subset of Y. Then, $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B)) = \alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(B)) \setminus \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B)) \setminus \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(B))$ used Theorem 4.13. Therefore, by Theorem 4.14, we have $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(B)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B)) \setminus f^{-1}(\alpha_{[\beta,\beta']}$-$sInt(B)) = f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(B)) \setminus \alpha_{[\beta,\beta']}$-$sInt(B)) = f^{-1}(\alpha_{[\beta,\beta']}$-$sBd(B))$.

Conversely, let V be $\alpha_{[\beta,\beta']}$-semiopen in Y and $F = Y \setminus V$. Then, by hypothesis, we obtain $\alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(F)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sBd(F)) = f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(F) \setminus \alpha_{[\beta,\beta']}$-$sInt(F)) \subseteq f^{-1}(\alpha_{[\beta,\beta']}$-$sCl(F)) = f^{-1}(F)$ and hence by Theorem 3.12 (1), $\alpha_{[\gamma,\gamma']}$-$sCl(f^{-1}(F)) = \alpha_{[\gamma,\gamma']}$-$sInt(f^{-1}(F)) \cup \alpha_{[\gamma,\gamma']}$-$sBd(f^{-1}(F)) \subseteq$

$f^{-1}(F)$. Thus, $f^{-1}(F)$ is $\alpha_{[\gamma,\gamma']}$-semiclosed and hence $f^{-1}(V)$ is $\alpha_{[\gamma,\gamma']}$-semiopen in X. Therefore, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute. □

Corollary 4.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-closed and $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute, then $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A)) = \alpha_{[\beta,\beta']}\text{-}sCl(f(A))$ for every subset A of X.*

Proof. Since for any subset A of X, $A \subseteq \alpha_{[\gamma,\gamma']}\text{-}sCl(A)$. Therefore, $f(A) \subseteq f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A))$. Since f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-closed, then $\alpha_{[\beta,\beta']}\text{-}sCl(f(A)) \subseteq \alpha_{[\beta,\beta']}\text{-}sCl(f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A))) = f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A))$. Hence, $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A)) \supseteq \alpha_{[\beta,\beta']}\text{-}sCl(f(A))$ and by Theorem 4.13, we have $f(\alpha_{[\gamma,\gamma']}\text{-}sCl(A)) = \alpha_{[\beta,\beta']}\text{-}sCl(f(A))$. □

Corollary 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then, f is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-semiopen and $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute if $f^{-1}(\alpha_{[\beta,\beta']}\text{-}sCl(V)) = \alpha_{[\gamma,\gamma']}\text{-}sCl(f^{-1}(V))$ for every subset V of Y.*

Proof. The proof is follows from Remark 3.2, Theorems 4.7 and 4.13. □

Theorem 4.15. *If $f : X \to Y$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\beta,\beta']})$-irresolute and $g : Y \to Z$ is $(\alpha_{[\beta,\beta']}, \alpha_{[\delta,\delta']})$-irresolute, then $g(f) : X \to Z$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\delta,\delta']})$-irresolute.*

Proof. If $A \subseteq Z$ is $\alpha_{[\delta,\delta']}$-semiopen, then $g^{-1}(A)$ is $\alpha_{[\beta,\beta']}$-semiopen and $f^{-1}(g^{-1}(A))$ is $\alpha_{[\gamma,\gamma']}$-semiopen. Thus, $(g(f))^{-1}(A) = f^{-1}(g^{-1}(A))$ is $\alpha_{[\gamma,\gamma']}$-semiopen and hence $g(f)$ is $(\alpha_{[\gamma,\gamma']}, \alpha_{[\delta,\delta']})$-irresolute. □

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