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## **PERFORMANCE ANALYSIS OF BRAYTON HEAT ENGINE AT MAXIMUM EFFICIENT POWER USING TEMPERATURE DEPENDENT SPECIFIC HEAT OF WORKING FLUID**

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### **ABSTRACT**

Efficient power optimization of Brayton heat engine with variable specific heat of the working fluid is analyzed from the view of finite time thermodynamics. The efficient power is defined as the multiplication of engine power and engine efficiency. Hence, the proposed method considers not only the power output but also the engine efficiency. Optimizing the efficient power gives a compromise between power and engine efficiency. Results obtained are compared with those obtained by using the maximum power (MP) and maximum power density (MPD) conditions. The results show that the engine designed at maximum efficient power (MEP) criterion is more efficient as compared with those designed at maximum power and maximum power density conditions. The system analysis is done with variable specific heat parameter due to which its performance is comparable to the real systems. Moreover, engine designed at maximum efficient power criterion requires lesser pressure ratio over those designed at maximum power density conditions. Brayton heat engine with variable specific heat of the working fluid gives realistic prediction of engine efficiency and engine power than does the isentropic Brayton heat engine with constant specific heat.

### **1. INTRODUCTION**

Brayton cycles have been widely used in gas power plants and aircrafts. As shown in Fig. 1, the cycle involves reversible adiabatic compression followed by isobaric heat addition. The expansion process takes place isentropically, and finally it has isobaric heat rejection. Leff [2] analysed an endoreversible Brayton heat engine following Curzon & Ahlborn [1] and found that impositions of maximum work put in limits on the Brayton cycle temperatures. Wu [3] optimized the power of an

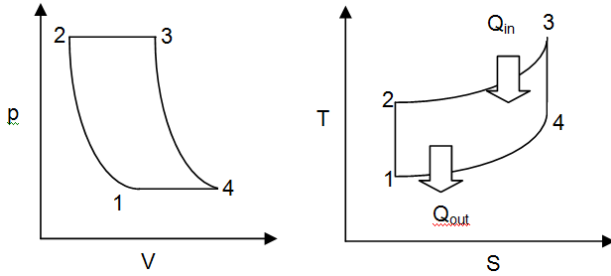
endoreversible Brayton gas heat engine while Wu & Kiang [4] incorporated non-isentropic nature of compressor and turbine to Brayton heat engine and found that engine power and engine efficiency are sturdy functions of the compressor and turbine efficiencies. Sahin et al. [5] analyzed Joule-Brayton cycle at maximum power density. Using the MPD criterion, they found that the efficiency at MPD conditions is greater than that at maximum power output. Sahin et al. [6] also applied the MPD methods to the endoreversible Carnot heat engine while Sahin and Yilmaz [7] extended the work to irreversible Joule-Brayton engine from the view of finite time thermodynamics or entropy generation minimization [8]. Finite time thermodynamics has explored many new fronts since this technique had been used to analyze and optimize the performance of real thermodynamic processes, devices and cycles. In past, many optimization studies for Brayton heat engines based on endoreversible and irreversible mode have been carried out by number of researchers [9-18]. Yilmaz [19] examined the performance of Brayton cycle based on the efficient power criterion and found a compromise between engine power and thermal efficiency on maximizing the efficient power function. Usually constant specific heats of the working fluid were used in their studies. Recently, many researchers [20-22] were involved in performing thermodynamic analysis using variable specific heats of the working fluid. It was proved and verified that variable specific heats give better approximation to actual cycles than using constant specific heat of the working fluid.

In this paper, we have presented a performance analysis of Brayton heat engine with variable specific heat of the working fluid under maximum efficient power conditions as the literature available has not covered the variable specific heat consideration for Brayton cycle performance analysis. So, actual performance of proposed model has not been obtained. The

authors obtained expressions for maximum power output, maximum power density, maximum efficient power and the corresponding thermal efficiency of the cycle. The effect of various efficiencies, isentropic temperature ratio, maximum pressure ratios, maximum cycle temperature ratio, maximum volume ratios have been studied in detail and the results are presented on the graphs. The results obtained with temperature dependent specific heat are also compared with those obtained by using constant specific heat of the working fluid.

## 2. THERMODYNAMIC ANALYSIS

An air standard Joule-Brayton cycle involving two isobaric and two reversible adiabatic processes is shown in Fig. 1. The compression and expansion processes are reversible adiabatic as shown by process (1-2) and process (3-4) respectively on pressure-volume (p-V) and Temperature-Entropy (T-S) diagrams. The heat addition and heat rejection processes are isobaric as shown by process (2-3) and process (4-1) respectively in Fig. 1.



**Fig.1** p-V and T-S diagrams for Brayton Heat Engine Cycle

The employed temperature dependent specific heat model assumes variation of specific heat with temperature in a linear manner.

In the real practice, specific heats of the working fluid are variable and these variations will have great influence on the performance of Brayton heat engine. According to references [14, 18-20], it is assumed that specific heat of working fluid is function of temperature only and the curve for specific heat of working fluid in heat engines is nearly a straight line in the temperature range of 300 – 2200 Kelvin, the variation of specific heat at constant pressure with temperature is given below:

$$C_p = A_1 + A_2 T \quad (1)$$

where  $A_1$ ,  $A_2$  are constants and  $T$  is temperature in Kelvin.

Heat supplied ( $Q_{in}$ ) and heat rejected ( $Q_{out}$ ) during isobaric processes are given as:

$$Q_{in} = m \int_{T_2}^{T_3} C_p dT \quad (2)$$

$$Q_{out} = m \int_{T_1}^{T_4} C_p dT \quad (3)$$

Using Equation (1), one can rewrite Equation (2) and Equation (3) as:

$$Q_{in} = m C_p (T_3 - T_2) = m \left[ A_1 (T_3 - T_2) + \frac{A_2}{2} (T_3^2 - T_2^2) \right] \quad (4)$$

$$Q_{out} = m C_p (T_4 - T_1)$$

$$= m \left[ A_1 (T_4 - T_1) + \frac{A_2}{2} (T_4^2 - T_1^2) \right] \quad (5)$$

Maximum temperature ratio ( $\tau$ ) and isentropic temperature ratio ( $\theta$ ) for Brayton heat engine cycle is defined as:

$$\tau = \frac{T_3}{T_1} \quad (6)$$

$$\theta = \frac{T_2}{T_1} \quad (7)$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{gives } T_4 = \frac{\tau T_1}{\theta} \quad (8)$$

### 2.1 Analysis for Engine Power

$$W = Q_{in} - Q_{out} \quad (9)$$

$$= m \left[ A_1 (T_3 - T_2) + \frac{A_2}{2} (T_3^2 - T_2^2) - A_1 (T_4 - T_1) - \frac{A_2}{2} (T_4^2 - T_1^2) \right] \quad (10)$$

$$W = m \left[ A_1 (\tau T_1 - \theta T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta^2} - T_1^2 \right) \right] \quad (11)$$

For a given  $\tau$ , taking  $\frac{dW}{d\theta} = 0$ , gives  $\theta$  as  $\theta_{mp}$ . Hence, Maximum Power Output ( $W_{mp}$ ) can be obtained as:

$$W_{mp} = m \left[ A_1 (\tau T_1 - \theta_{mp} T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta_{mp}^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta_{mp}} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta_{mp}^2} - T_1^2 \right) \right] \quad (12)$$

Brayton heat engine efficiency ( $\eta$ ) is defined as:

$$\eta = W / Q_{in} \quad (13)$$

On using Equations (4), (5), (6-8) and (13), Brayton heat engine efficiency at the maximum power condition ( $\eta_{mp}$ ) can be written as:

$$\eta_{mp} = 1 - \frac{A_1 \left( \frac{\tau}{\theta_{mp}} - 1 \right) + \frac{A_2}{2} \left( \frac{\tau^2 T_1}{\theta_{mp}^2} - T_1 \right)}{A_1 (\tau - \theta_{mp}) + \frac{A_2}{2} (\tau^2 T_1 - \theta_{mp}^2 T_1)} \quad (14)$$

### 2.2 Analysis for Power Density

$$W_{pd} = W / v_4 \quad (15)$$

Since 4-1 is isobaric process so one can write,

$$v_4 / v_1 = T_4 / T_1, \text{ So } v_4 = \tau v_1 / \theta \quad (16)$$

On combining equation (11), (15) and (16), one can get:

$$W_{pd} = \frac{W}{\tau v_1} \quad (17)$$

$$W_{pd} = \frac{m\theta}{\tau v_1} \left[ A_1 (\tau T_1 - \theta T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta^2} - T_1^2 \right) \right] \quad (18)$$

For a given  $\tau$ , taking  $\frac{dW_{pd}}{d\theta} = 0$ , gives  $\theta$  as  $\theta_{mpd}$ . Hence, Maximum Power Density ( $W_{mpd}$ ) can be obtained as:

$$W_{mpd} = \frac{m\theta_{mpd}}{\tau v_1} \left[ A_1(\tau T_1 - \theta_{mpd} T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta_{mpd}^2 T_1^2) \right] - A_1 \left( \frac{\tau T_1}{\theta_{mpd}} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta_{mpd}^2} - T_1^2 \right) \quad (19)$$

Consequently, the efficiency at maximum power density ( $\eta_{mpd}$ ) can be expressed as:

$$\eta_{mpd} = 1 - \frac{A_1 \left( \frac{\tau}{\theta_{mpd}} - 1 \right) + \frac{A_2}{2} \left( \frac{\tau^2 T_1}{\theta_{mpd}^2} - T_1 \right)}{A_1 (\tau - \theta_{mpd}) + \frac{A_2}{2} (\tau^2 T_1 - \theta_{mpd}^2 T_1)} \quad (20)$$

### 2.3 Analysis for Efficient Power

The efficient power ( $W_{ep}$ ) of a Joule-Brayton engine is defined as the multiplication of engine power ( $W$ ) and engine efficiency ( $\eta$ ) and is given as:

$$W_{ep} = W \cdot \eta \quad (21)$$

Using equations (11) and (13), one can get;

$$W_{ep} = m \left\{ \left[ A_1(\tau T_1 - \theta T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta^2} - T_1^2 \right) \right] \left\{ 1 - \frac{A_1 \left( \frac{\tau}{\theta} - 1 \right) + \frac{A_2}{2} \left( \frac{\tau^2 T_1}{\theta^2} - T_1 \right)}{A_1 (\tau - \theta) + \frac{A_2}{2} (\tau^2 T_1 - \theta^2 T_1)} \right\} \right\} \quad (22)$$

For a given  $\tau$ , taking  $\frac{dW_{ep}}{d\theta} = 0$ , gives  $\theta$  as  $\theta_{mep}$ . Hence, Maximum Efficient Power ( $W_{mep}$ ) can be obtained as:

$$W_{mep} = m \left\{ \left[ A_1(\tau T_1 - \theta_{mep} T_1) + \frac{A_2}{2} (\tau^2 T_1^2 - \theta_{mep}^2 T_1^2) - A_1 \left( \frac{\tau T_1}{\theta_{mep}} - T_1 \right) - \frac{A_2}{2} \left( \frac{\tau^2 T_1^2}{\theta_{mep}^2} - T_1^2 \right) \right] \left\{ 1 - \frac{A_1 \left( \frac{\tau}{\theta_{mep}} - 1 \right) + \frac{A_2}{2} \left( \frac{\tau^2 T_1}{\theta_{mep}^2} - T_1 \right)}{A_1 (\tau - \theta_{mep}) + \frac{A_2}{2} (\tau^2 T_1 - \theta_{mep}^2 T_1)} \right\} \right\} \quad (23)$$

Consequently, the efficiency at maximum efficient power ( $\eta_{mep}$ ) can be expressed as:

$$\eta_{mep} = 1 - \frac{A_1 \left( \frac{\tau}{\theta_{mep}} - 1 \right) + \frac{A_2}{2} \left( \frac{\tau^2 T_1}{\theta_{mep}^2} - T_1 \right)}{A_1 (\tau - \theta_{mep}) + \frac{A_2}{2} (\tau^2 T_1 - \theta_{mep}^2 T_1)} \quad (24)$$

### 3. ANALYSIS OF RESULTS

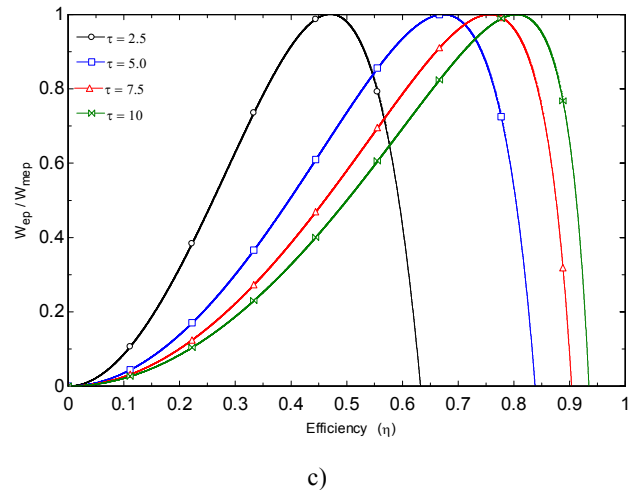
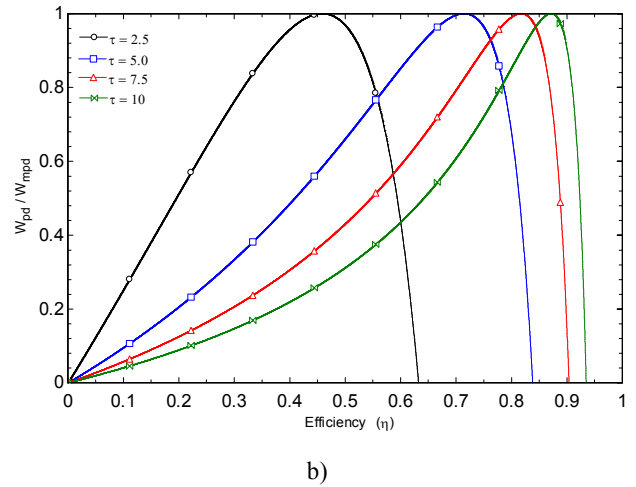
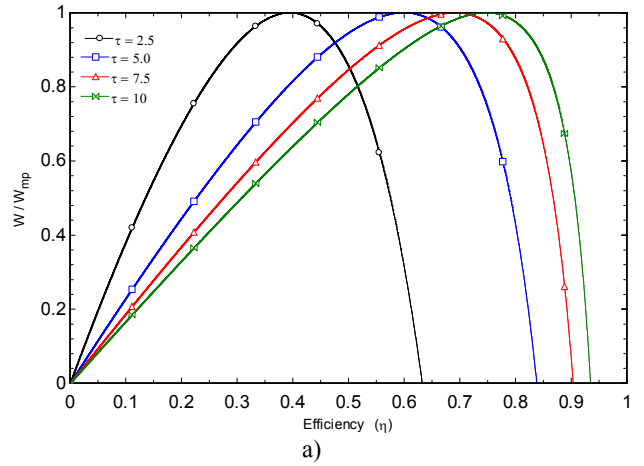
The above derived formulae are used and plotted in order to analyze performance of Brayton heat engine with variable specific heats of the working fluid.

According to references [18-20], the following constants and ranges of parameters are used in the calculation:

$A_1=0.9521$  kJ/kg-K,  $A_2=0.0002$  kJ/kg-K,  $T_1 = 298$  K,  $\gamma = 1.4$ ,  $\tau = 1-25$

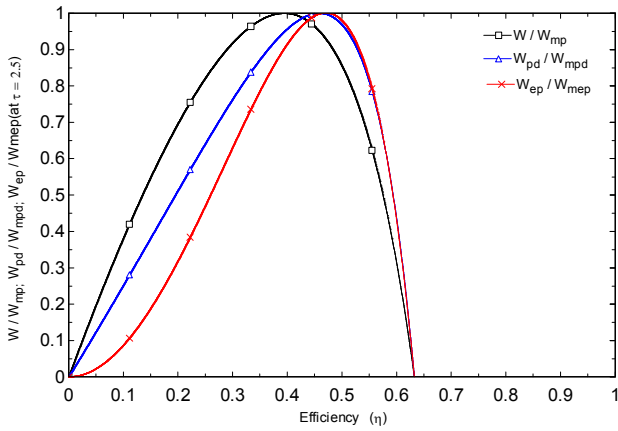
Variations of the normalized power ( $W / W_{mpd}$ ), normalized power density ( $W_{pd} / W_{mpd}$ ) and normalized efficient power ( $W_{ep} / W_{mep}$ ) with respect to thermal efficiency are shown in Figs 2(a)

to 2(c), respectively in variations of the maximum cycle temperature ratio,  $\tau$ .

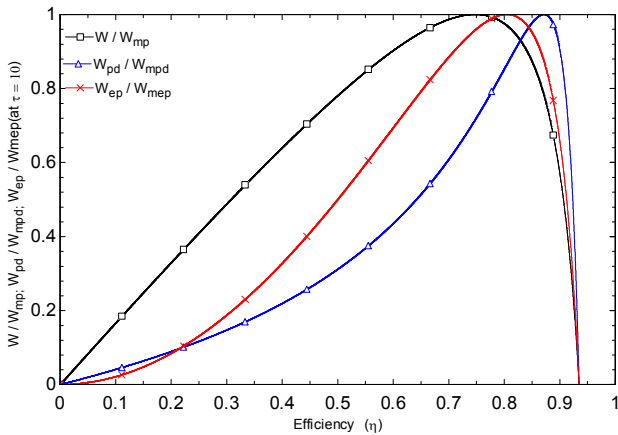


**Fig. 2** Variations of the (a) normalized power, (b) normalized power density and (c) normalized efficient power with respect to thermal efficiency

As one can see from Fig. 2 that, thermal efficiency at maximum normalized power density ( $\eta_{mpd}$ ) and thermal efficiency at maximum normalized efficient power ( $\eta_{mep}$ ) are always greater than the thermal efficiency at maximum normalized power conditions ( $\eta_{mp}$ ). It can also be observed that the value of thermal efficiency at above maximum normalized conditions increase with an increasing value of maximum cycle temperature ratio ( $\tau$ ). Normalized power, normalized power density and normalized efficient power are plotted together for  $\tau = 2.5$  and  $\tau = 10$  in Fig. 3 (a) and Fig. 3 (b), respectively.



a)



b)

**Fig. 3** Normalized power, normalized power density and normalized efficient power are plotted together for  $\tau = 2.5$  and (b)  $\tau = 10$

It can be concluded from these figures that,  $\eta_{mep} > \eta_{mpd}$  for  $\tau = 2.5$  but  $\eta_{mpd}$  becomes greater than  $\eta_{mep}$  for  $\tau = 10$ . These findings can be seen more clearly from Fig. 4, which shows the comparison of three maximum efficiencies with respect to various values of  $\tau$ . It can be seen from Fig. 4 that  $\eta_{mep} > \eta_{mpd}$  for  $1 < \tau < 2.7$  and after this value  $\eta_{mpd}$  becomes greater than  $\eta_{mep}$ . For example, at  $\tau = 2.5$ ,  $\eta_{mp} = 0.3939$ ,  $\eta_{mpd} = 0.4578$ ,  $\eta_{mep} = 0.4625$  while at  $\tau = 10$ ,  $\eta_{mp} = 0.7469$ ,  $\eta_{mpd} = 0.8706$ ,  $\eta_{mep} = 0.8021$

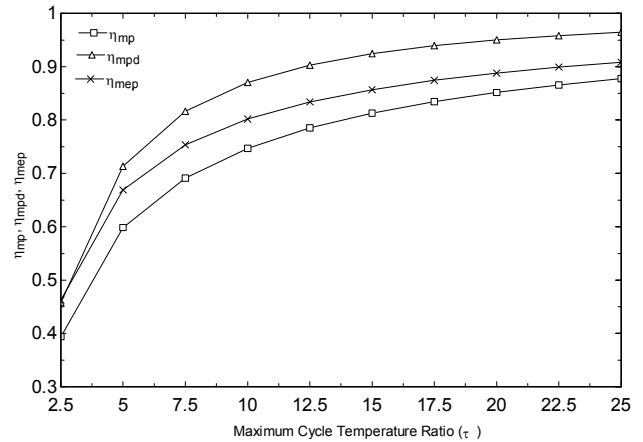
The size of a Brayton heat engine can be characterized by the maximum volume in the cycle i.e.  $V_4$ . Using equation (16), one can write

$$\frac{(V_4)_{mpd}}{(V_4)_{mp}} = \frac{v_1 \tau / \theta_{mpd}}{v_1 \tau / \theta_{mp}} = \frac{\theta_{mp}}{\theta_{mpd}} \quad (25)$$

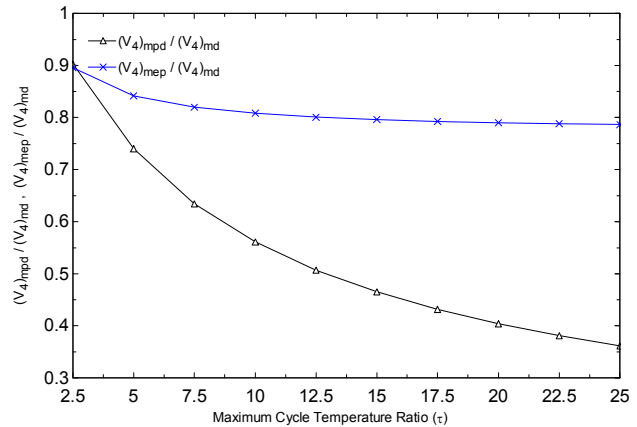
where  $(V_4)_{mpd}$  is volume at maximum power density and  $(V_4)_{mp}$  is volume at maximum power criterion.

Similarly, the ratio of maximum volume at maximum efficient power to that at maximum power can be written as:

$$\frac{(V_4)_{mep}}{(V_4)_{mp}} = \frac{v_1 \tau / \theta_{mep}}{v_1 \tau / \theta_{mp}} = \frac{\theta_{mp}}{\theta_{mep}} \quad (26)$$



**Fig. 4** Variations of the various efficiencies ( $\eta_{mp}$ ,  $\eta_{mpd}$ ,  $\eta_{mep}$ ) with respect to  $\tau$



**Fig. 5** Maximum volume ratios variations with respect to cycle temperature ratio ( $\tau$ )

Maximum volume ratio variations with respect to cycle temperature ratio are shown in Fig. 5. It can be concluded that Brayton heat engine size designed at maximum power density and maximum efficient power criterion is smaller compared with engine designed at maximum power criterion. It can also be observed that increasing value of maximum cycle temperature ratio, results in further reduction in size of Brayton heat engine designed at maximum power density conditions

while in maximum efficient power conditions, size of Brayton engine remains more or less the same.

Maximum pressure ratio at maximum power density to pressure ratio at maximum power is defined as:

$$\frac{(P_3)_{mpd}}{(P_3)_{mp}} = \left[ \frac{\theta_{mpd}}{\theta_{mp}} \right]^{\frac{\gamma}{\gamma-1}} \quad (27)$$

where  $(P_3)_{mpd}$  is pressure at maximum power density and  $(P_3)_{mp}$  is the pressure at maximum power criterion.

Similarly,  $\frac{(P_3)_{mep}}{(P_3)_{mp}}$  can be written as:

$$\frac{(P_3)_{mep}}{(P_3)_{mp}} = \left[ \frac{\theta_{mep}}{\theta_{mp}} \right]^{\frac{\gamma}{\gamma-1}} \quad (28)$$

where  $(P_3)_{mep}$  is the maximum pressure at maximum efficient power conditions.

Maximum pressure ratios variations with respect to maximum cycle temperature ratio are shown in Fig. 6. It has been observed that increased value of  $\tau$  results in increase of maximum pressure ratios and Fig. 6 also reflects that engine designed at maximum power density conditions operates at higher pressure ratios than those designed at maximum efficient power conditions. Therefore, tougher material is required for Brayton heat engine designed at maximum power density conditions to with stand higher pressure ratio than those designed at maximum efficient power conditions.

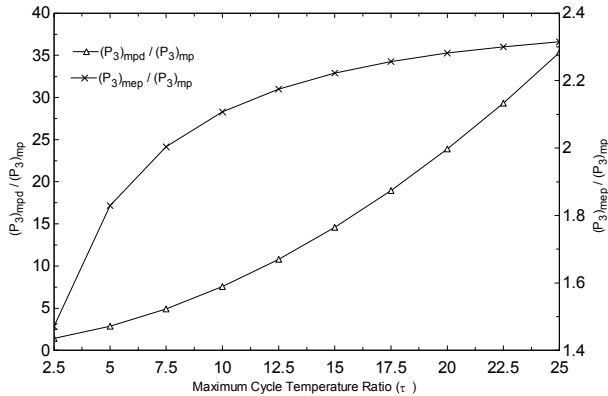


Fig. 6 Maximum pressure ratios variations with respect to cycle temperature ratio ( $\tau$ )

Fig. 7 shows the variations of the isentropic temperature ratios at maximum power ( $\theta_{mp}$ ), maximum power density ( $\theta_{mpd}$ ), and maximum efficient power ( $\theta_{mep}$ ) for variable specific heat of the working fluid with cycle temperature ratio ( $\tau$ ). As can be seen from Fig. 7, that the values of various isentropic temperature ratios go on increasing with an increasing value of the cycle temperature ratio ( $\tau$ ). It can also be observed that  $\theta_{mep} = \theta_{mpd}$  at  $\tau = 2.7$ .

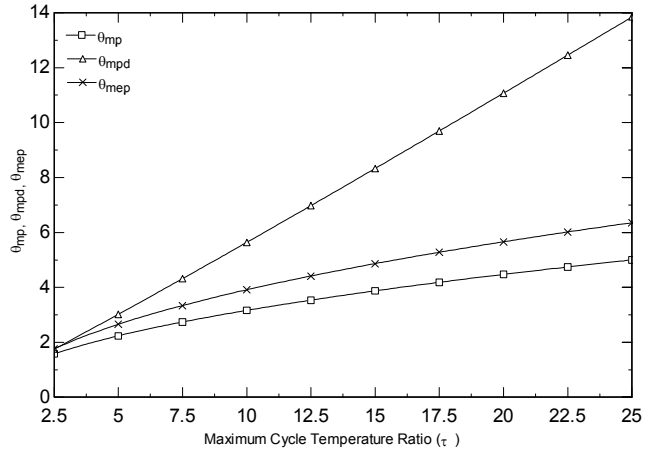


Fig. 7 Isentropic temperature ratios variations at maximum power, maximum power density, max. efficient power with respect to maximum cycle temperature ratio

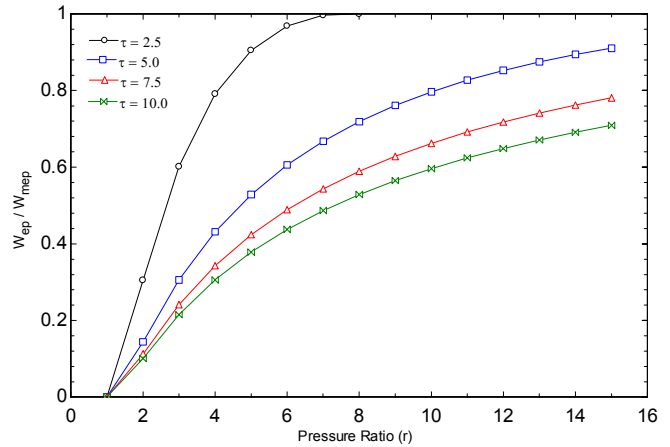


Fig. 8 Normalized efficient power ( $W_{ep}/W_{mep}$ ) variations with respect to pressure ratio ( $r$ )

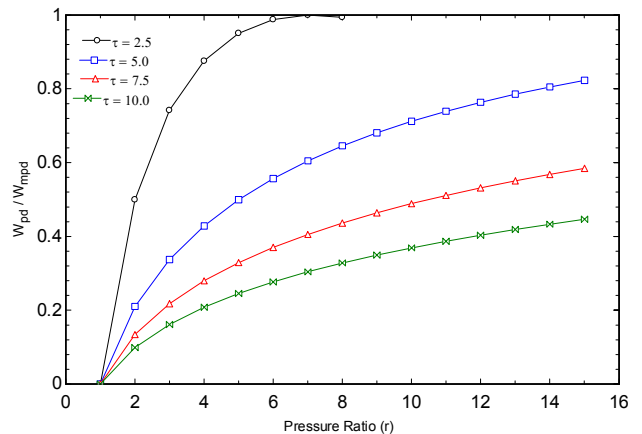
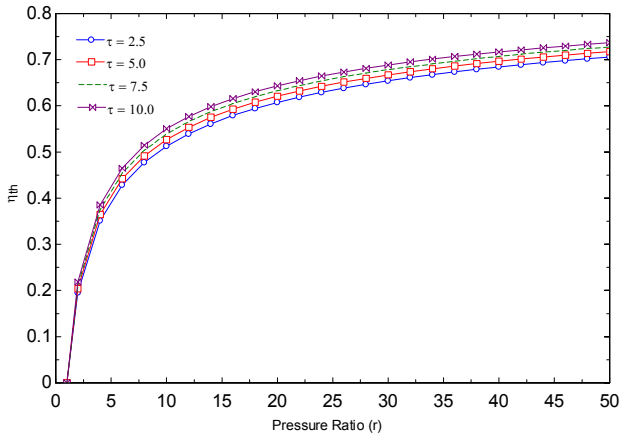


Fig. 9 Normalized power density ( $W_{pd}/W_{mpd}$ ) variations with respect to pressure ratio ( $r$ )

Variations of normalized efficient power ( $W_{ep}/W_{mep}$ ) and normalized power density ( $W_{pd}/W_{mpd}$ ) with respect to pressure ratio ( $r$ ) are shown in Fig. 8 and Fig. 9 respectively. The following observations are made from these figures:

- i) For a given value of pressure ratio, normalized power density and normalized efficient power increases with decreasing value of maximum cycle temperature ratio.
- ii) For any value of  $\tau$  and  $r$  (less than 40), the normalized efficient power is more as compared with normalized power density.
- iii) For a value of  $\tau = 2.5$ , the normalized power density and normalized efficient power touches its maximum value at a pressure ratio of 8. But normalized efficient power always remains more as compared with normalized power density.



**Fig. 10** Thermal efficiency variations with respect to pressure ratio ( $r$ )

Variations of thermal efficiency with pressure ratio for various maximum cycle temperature ratios are shown in Fig. 10. It has been observed that thermal efficiency increases with an increasing value of maximum cycle temperature ratio and pressure ratio. The value increases sharply at pressure ratio of ( $r \leq 5$ ) compared with at pressure ratio of ( $r > 5$ ).

**4. COMPARISON WITH IDEAL BRAYTON CYCLE USING CONSTANT SPECIFIC HEAT OF THE WORKING FLUID:**

The formulae derived above also apply to an isentropic Brayton heat engine with constant specific heat of working fluid provided one sets  $A_1$  equals to  $C_p$  and  $A_2$  equals to zero. Thus, for Brayton heat engine cycle with constant specific heat of the working fluid, maximum power output, maximum power density, maximum efficient power and the corresponding efficiencies can be found as:

$$W_{mp} = mC_p T_1 (\tau - 2\sqrt{\tau} + 1) \tag{29}$$

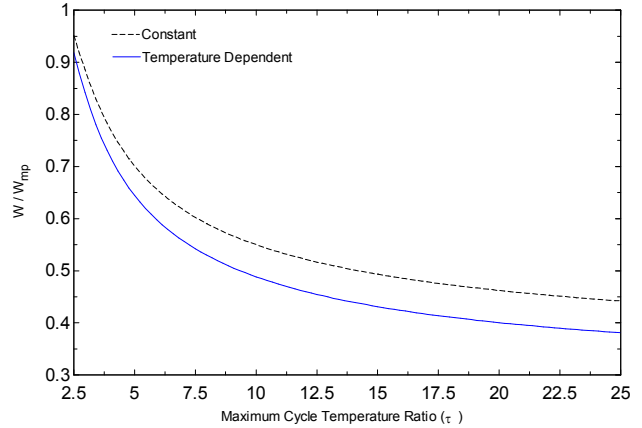
$$W_{mpd} = \frac{mC_p T_1 (\tau - 1)^2}{4\tau v_1} \tag{30}$$

$$W_{mep} = \frac{8\tau^2 + 20\tau - (1+8\tau)^{1.5} - 1}{8\tau} \tag{31}$$

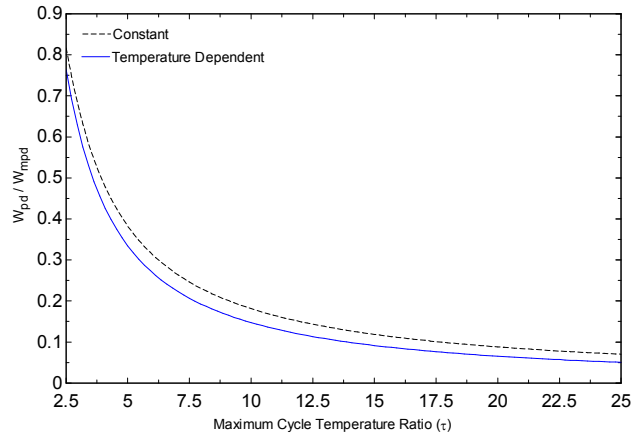
$$\eta_{mp} = 1 - \frac{1}{\sqrt{\tau}} \tag{32}$$

$$\eta_{mpd} = 1 - \frac{2}{\tau + 1} \tag{33}$$

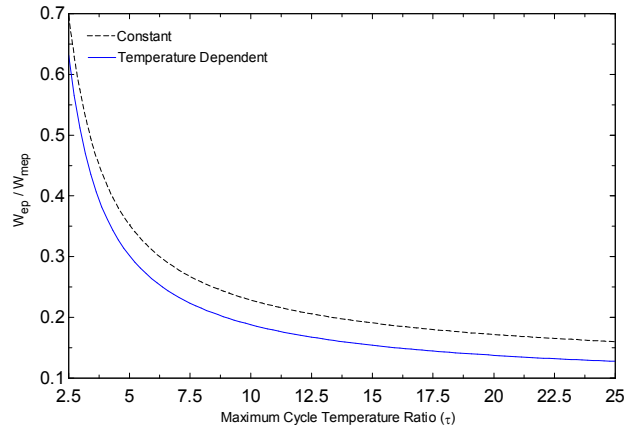
$$\eta_{mep} = 1 - \frac{2}{\sqrt{8\tau + 1} - 1} \tag{34}$$



**Fig. 11** Normalized power output variations with respect to maximum cycle temperature ratio



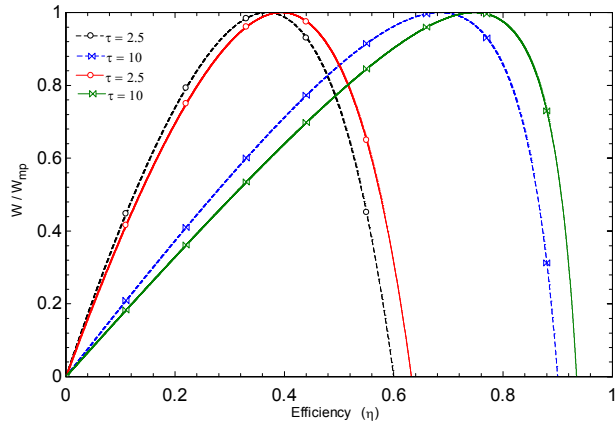
**Fig. 12** Variations of normalised power density with respect to maximum cycle temperature ratio



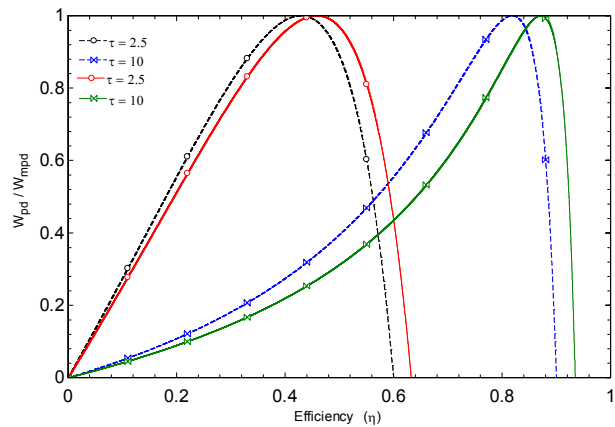
**Fig. 13** Variations of normalised efficient power with respect to maximum cycle temperature ratio

The above equations (29)-(34) replicate the results obtained by Yilmaz [17] in which Joule-Brayton engine with constant specific heat was examined. These equations are plotted in the view of comparison of Brayton heat engine with temperature dependent specific heat with those results

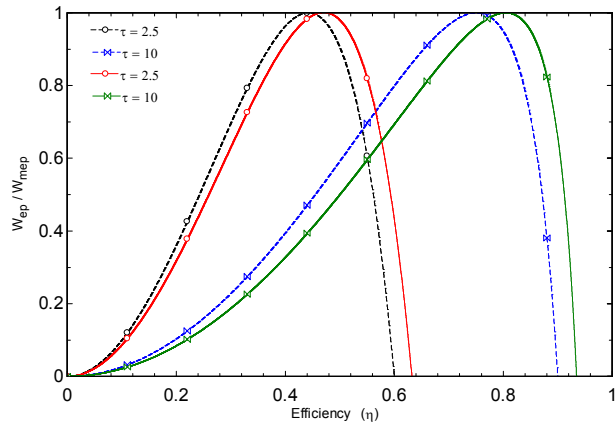
assuming a constant specific heat as shown in Figures 11 through 17.



a)



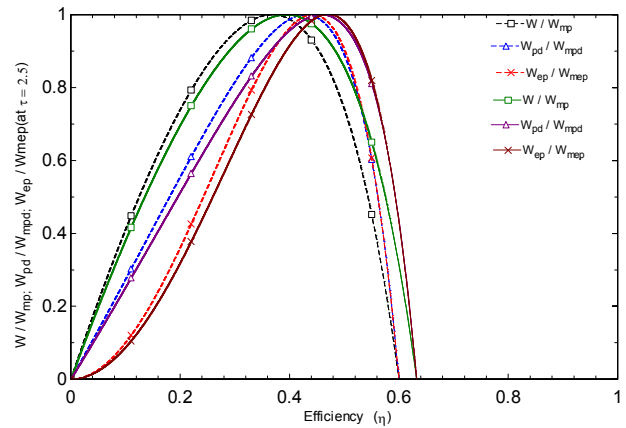
b)



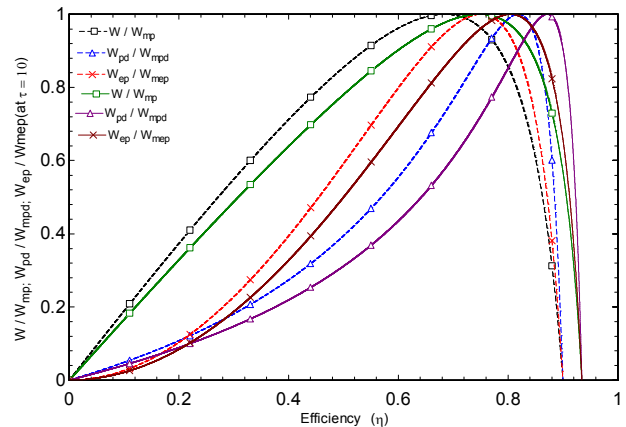
c)

**Fig. 14** Variations of the (a) normalized power ( $W / W_{mp}$ ), (b) normalized power density ( $W_{pd} / W_{mpd}$ ) and normalized efficient power ( $W_{ep} / W_{mep}$ ) with respect to thermal efficiency for various  $\tau$  values

The following constants and range of  $\tau$  are selected:  $\gamma = 1.4$ ,  $T_1 = 298$ ,  $\tau = 1-25$ . Fig. 11 – Fig. 13 show the variations of normalised power output, normalised power density, normalised efficient power for variable and constant specific heats with maximum cycle temperature ratio ( $\tau$ ). The dotted lines represent the various normalised powers with constant specific heat while dark lines represents with temperature dependent specific heat of the working fluid. The deviation between  $W / W_{mp}$ ,  $W_{pd} / W_{mpd}$ ,  $W_{ep} / W_{mep}$  using the temperature dependent and constant specific heat increases with increasing  $\tau$ . The values of  $W / W_{mp}$ ,  $W_{pd} / W_{mpd}$ ,  $W_{ep} / W_{mep}$  using constant specific heat are superior than using temperature dependent specific heat of working fluid.



a)



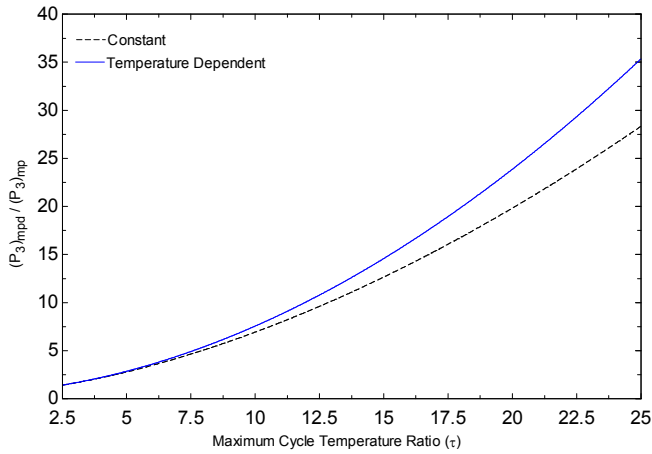
b)

**Fig. 15** Normalized power, Normalized power density and normalized efficient power are plotted together for (a)  $\tau = 2.5$  and (b)  $\tau = 10$

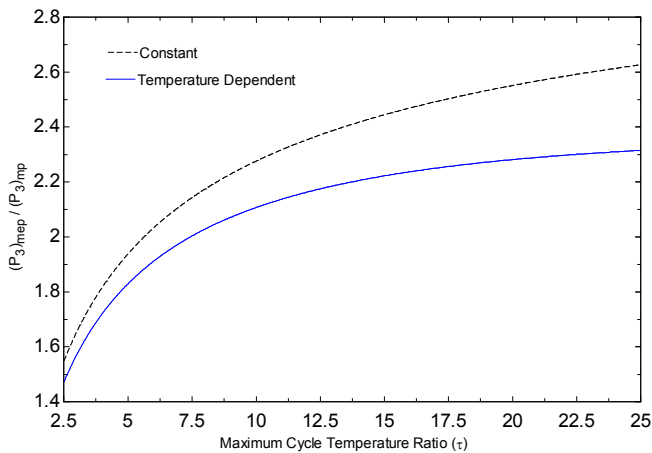
Normalised power, normalised power density and normalised efficient power variations with efficiency at  $\tau = 2.5$  and  $\tau = 10$  are shown in Fig. 14. The dotted lines represent cycle with constant specific heat while the dark lines represent with variable specific heat of the working fluid. The values of  $W / W_{mp}$ ,  $W_{pd} / W_{mpd}$ ,  $W_{ep} / W_{mep}$  are different at the same efficiency. Both constant and variable specific heat curves of  $W / W_{mp}$ ,  $W_{pd} / W_{mpd}$ ,  $W_{ep} / W_{mep}$  have parabolic trend at  $\tau = 2.5$ . Similar trend for  $\tau = 10$



but larger divergence appears at superior values of  $\tau$ . Fig. 14, exhibits a point where both temperature dependent and constant specific heat of working fluid give approximately identical result at a given value of efficiency.



a)



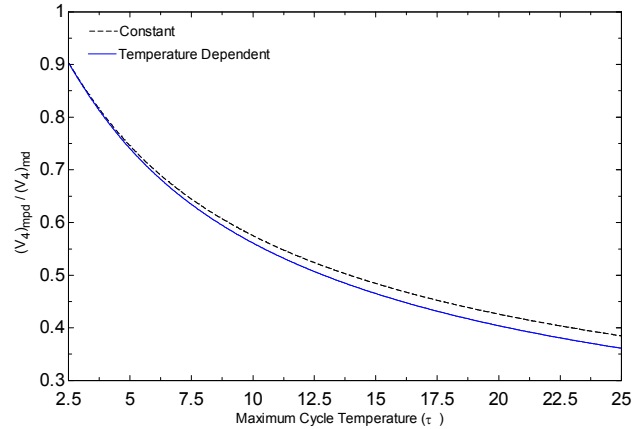
b)

**Fig. 16** Variations of maximum pressure ratios (a)  $(P_3)_{mpd} / (P_3)_{mp}$  (b)  $(P_3)_{mep} / (P_3)_{mp}$  with respect to maximum cycle temperature ratio ( $\tau$ )

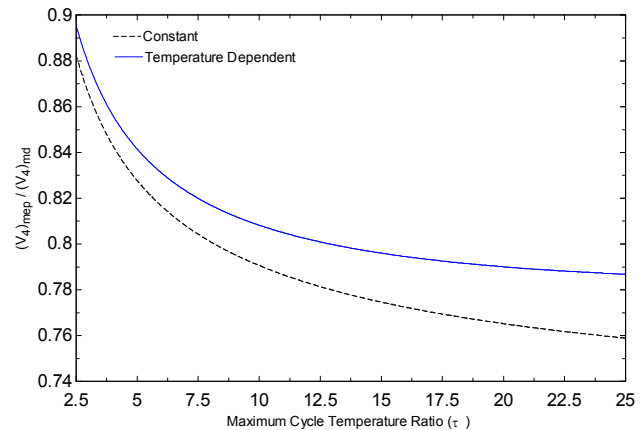
Fig. 15 shows the variations of normalised power, normalised power density, normalised efficient power with thermal efficiency at various values of maximum cycle temperature ratio. The dotted lines represent cycle with constant specific heat while the dark lines represent with temperature dependent specific heat of working fluid. Both curves are not alike. It can be concluded from these figures that,  $\eta_{mep} > \eta_{mpd}$  for  $\tau = 2.5$  but  $\eta_{mpd}$  becomes greater than  $\eta_{mep}$  for  $\tau > 2.5$ .

Fig. 16 shows the variations of  $(P_3)_{mpd} / (P_3)_{mp}$  and  $(P_3)_{mep} / (P_3)_{mp}$  with respect to maximum cycle temperature ratio ( $\tau$ ). The dotted lines represent cycle with constant specific heat while the dark lines represent with variable specific heat of the working fluid. It has been observed that increased value of cycle temperature ratio results in increase of maximum pressure ratios

with both constant and temperature dependent specific heat of the working fluid and Fig. 16 also reflects that engine designed at maximum power density conditions operates at lower pressure ratios with constant specific heat when compared with variable specific heat while engine designed at maximum efficient power conditions operates at higher pressure ratio with constant specific heat when compared with variable specific heat.



a)



b)

**Fig. 17** Variations of maximum volume ratios (a)  $(V_4)_{mpd} / (V_4)_{mp}$ , (b)  $(V_4)_{mep} / (V_4)_{mp}$  with respect to maximum cycle temperature ratio ( $\tau$ )

Variations of the maximum volume ratio with respect to maximum cycle temperature ratio are shown in Fig. 17. It can be concluded that engine size designed at maximum efficient power condition is smaller with constant specific heat than those designed at temperature dependent specific heat of the working fluid. It can also be observed that increasing value of cycle temperature ratio, results in further reduction in Brayton heat engine size designed at maximum power density conditions while in maximum efficient power conditions, size of engine remains more or less the same with both constant and temperature dependent specific heat of the working fluid.



**5. CONCLUSION**

A comparative performance analysis for Brayton heat engine with temperature dependent specific heat based on maximum power, maximum power density and maximum efficient power conditions has been done by introducing new criterion. The results show that more efficient engines are designed at maximum efficient power conditions than those at maximum power and maximum power density conditions. Although the Brayton heat engine designed at maximum power density conditions requires smaller size but higher pressure ratio in comparison with those designed at maximum efficient power conditions. Brayton heat engine with temperature dependent specific heat of the working fluid gives realistic prediction of engine efficiency and power output than does the reversible Brayton heat engine with constant specific heat. The results obtained from this work can be useful in the optimal design and operation of real Brayton heat engines. This work can further be extended by taking both internal and external irreversibilities and optimizing the system using evolutionary techniques.

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**NOMENCLATURE**

|                                 |  |
|---------------------------------|--|
| 1, 2, 3, 4                      | state points   |
| A <sub>1</sub> , A <sub>2</sub> | constants  |
| C <sub>p</sub>                  | specific heat at constant pressure (kJkg <sup>-1</sup> K <sup>-1</sup> ) |
| m                               | mass flow (kgs <sup>-1</sup> )   |
| P                               | pressure (kPa)   |
| Q                               | heat transfer rate (kW)  |
| r                               | pressure ratio   |
| T                               | temperature (K)  |

V volume (m<sup>3</sup>)  
W power output (kW)

**Subscripts**

ep efficient power  
mp maximum power  
pd power density  
mpd maximum power density  
mep maximum efficient power

**Greek letters**

$\tau$  maximum cycle temperature ratio  
 $\theta$  isentropic temperature ratio  
 $\gamma$  specific heat ratio  
 $\eta$  efficiency