http://www.newtheory.org

Journal of **New Theory**

Received: 12.11.2014
Accepted: 24.11.2014

Year: **2015**, *Number*: **1** , *Pages*: **02-16**
Original Article[**]

Connectedness on Intuitionistic Fuzzy Soft Topological Spaces

Serkan Karataş[1,*](serkankaratas@odu.edu.tr)
Mehmet Akif İşleyen[1] (mehmetakifisleyen@msn.com)

[1]*Department of Mathematics, Faculty of Arts and Sciences, Ordu University, 52200 Ordu, Turkey*

Abstract – In this study, we introduce intuitionistic fuzzy soft connected sets in intuitionistic fuzzy soft topological spaces and some properties. Moreover, we extend the notion of C_i connectedness ($i = 1, 2, 3, 4$) to intuitionistic fuzzy soft topological spaces.

Keywords – *Intuitionistic fuzzy soft set, intuitionistic fuzzy soft topological space, intuitionistic fuzzy soft connectedness.*

1 Introduction

Nowadays, several researchers investigate to model the uncertainties. They use different set theories for this, for example fuzzy set theory [1] and intuitionistic fuzzy set theory [2] are the most common. But, such theories have their own difficulties such as constructing membership function. Therefore, Molodtsov [6] proposed a new mathematical tool for uncertainties, called soft set theory. In this theory, it is not necessary which constructing membership function. Soft sets can apply several areas such as Riemann-integration, Perron integration, game theory, operations research, probability theory, etc.

Many researchers study on soft set theory, especially soft topological structures. For example, soft topology and related properties were studied in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Then, several paper were published about fuzzy soft topological spaces [18, 19, 20, 21, 22, 23]. Moreover, recently, some authors have studied over intuitionistic fuzzy soft topological spaces [26, 27, 28, 29].

In this article, we introduce the connectedness on intuitionisitic fuzzy soft topological spaces. Then, we are compare the *ifs* C_i themselves.

2 Preliminary

In this section, we will give basic definitions and theorems with *ifs*-sets, intuitionistic fuzzy soft topology and intuitionistic fuzzy soft continuous functions. Throughout this paper, $\mathcal{P}(X)$, E and $\mathcal{IF}(X)$ denote power set of X, set of parameter and set of all intuitionistic fuzzy sets over X, respectively.

[**]*Edited by* Naim Çağman *(Editors-in-Chief) and* Oktay Muhtaroğlu *(Area Editor).*
[*]*Corresponding Author.*

Definition 2.1. [2] Let X be a nonempty set. An intuitionistic fuzzy set A is defined by

$$A = \Big\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\Big\}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote membership and nonmembership functions respectively. Therefore, $\mu_A(x)$ and $\nu_A(x)$ are membership and nonmembership degree of each element $x \in X$ to the intuitionistic fuzzy set A and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [2] Let $\{A_i\}_{i\in I} \subseteq \mathcal{IF}(X)$, $A = \big\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\big\}$ and $B = \big\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\big\}$ be two intuitionistic fuzzy sets on X. Then, some basic set operations of intuitionistic fuzzy sets are defined as follows.

i. $A \subseteq B \Leftrightarrow \mu_B(x) \geq \mu_A(x)$ and $\nu_B(x) \leq \nu_A(x)$ for all $x \in X$

ii. $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

iii. $\bigcup_{i\in I} A_i = \Big\{\big\langle x, \bigvee_{i\in I} \mu_{A_i}(x), \bigwedge_{i\in I} \nu_{A_i}(x)\big\rangle : x \in X\Big\}$

iv. $\bigcap_{i\in I} A_i = \Big\{\big\langle x, \bigwedge_{i\in I} \mu_{A_i}(x), \bigvee_{i\in I} \nu_{A_i}(x)\big\rangle : x \in X\Big\}$

v. $\Box A = \Big\{\big\langle x, \mu_A(x), 1 - \mu_A(x)\big\rangle : x \in X\Big\}$

vi. $\Diamond A = \Big\{\big\langle x, 1 - \nu_A(x), \nu_A(x)\big\rangle : x \in X\Big\}$

vii. $A^c = \big\{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\big\}$

viii. $\tilde{1} = \big\{\langle x, 1, 0\rangle : x \in X\big\}$ and $\tilde{0} = \big\{\langle x, 0, 1\rangle : x \in X\big\}$.

Theorem 2.3. [3] Let $A, B, C \in \mathcal{IF}(X)$. Then

i. $A \subseteq B$ and $B \subseteq C \Rightarrow\ A \subseteq C$

ii. $A \subseteq B \Rightarrow A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$

iii. $(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c = A^c \cap B^c$

iv. $(A^c)^c = A$, $\tilde{1}^c = \tilde{0}$ and $\tilde{0}^c = \tilde{1}$

v. $A \subseteq B \Rightarrow B^c \subseteq A^c$

Definition 2.4. [6] A pair (F, A) is called a soft set over X, if F is a mapping defined by $F : A \to \mathcal{P}(X)$, where $A \subseteq E$.

Now, we will give a new soft set definition who was given by Çağman [7]. The definition is a new comment for the soft sets.

Definition 2.5. [7] A soft set F over X is a set valued function from E to $\mathcal{P}(X)$. It can be written a set of ordered pairs

$$F = \big\{(e, F(e)) : e \in E\big\}.$$

Note that if $F(e) = \emptyset$, then the element $(e, F(e))$ is not appeared in F. Set of all soft sets over X is denoted by $\mathbb{S}$.

According to Definition 2.5 we will redefine *ifs*-set and its set operations.

Definition 2.6. An intuitionistic fuzzy soft set (or namely *ifs*-set) f over X is a set valued function from E to $\mathcal{IF}(X)$. It can be written a set of ordered pairs

$$f = \Big\{\big(e, \{\langle x, \mu_{f(e)}(x), \nu_{f(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}$$

Note that if $f(e) = \tilde{0}$, then the element $(e, f(e))$ is not appeared in f. Set of all *ifs*-sets over X is denoted by $\mathbb{IFS}_X^E$.

Definition 2.7. Let $f, g, h \in \mathbb{IFS}_X^E$. Then some basic set operations of *ifs*-sets are defined as follows:

i. (Inclusion) $f \sqsubseteq g$ iff $f(e) \subseteq g(e)$ for all $e \in E$.

ii. (Equality) $f = g$ iff $f \sqsubseteq g$ and $g \sqsubseteq f$

iii. (Union) $h = f \sqcup g$ iff $h(e) = f(e) \cup g(e)$ for all $e \in E$.

iv. (Intersection) $h = f \sqcap g$ iff $h(e) = f(e) \cap g(e)$ for all $e \in E$.

v. (Complement) $h = f^{\tilde{c}}$ iff $h(e) = \big(f(e)\big)^{\tilde{c}}$ for all $e \in E$

vi. (Null *ifs*-set) f is called the null *ifs*-set and denoted by Φ, if $f(e) = \tilde{0}$ for all $e \in E$.

vii. (Universal *ifs*-set) f is called the universal *ifs*-set and denoted by $\tilde{X}$, if $f(e) = \tilde{1}$ for all $e \in E$.

Theorem 2.8. Let $\{f_i\}_{i\in\Lambda} \subseteq \mathbb{IFS}_X^E$ and $g \in \mathbb{IFS}_X^E$. Then

i. $g \sqcap \Big(\bigsqcup_{i\in\Lambda} f_i\Big) = \bigsqcup_{i\in\Lambda}(g \sqcap f_i)$

ii. $g \sqcup \Big(\bigsqcap_{i\in\Lambda} f_i\Big) = \bigsqcap_{i\in\Lambda}(g \sqcup f_i)$

iii. $\Big(\bigsqcup_{i\in\Lambda} f_i\Big)^{\tilde{c}} = \bigsqcap_{i\in\Lambda} f_i^{\tilde{c}}$

iv. $\Big(\bigsqcap_{i\in\Lambda} f_i\Big)^{\tilde{c}} = \bigsqcup_{i\in\Lambda} f_i^{\tilde{c}}$

v. $\Phi \sqsubseteq f \sqsubseteq \tilde{X}$, $\tilde{X}^{\tilde{c}} = \Phi$ and $\Phi^{\tilde{c}} = \tilde{X}$,

vi. $g \sqcup g^{\tilde{c}} = \tilde{X}$ and $\big(g^{\tilde{c}}\big)^{\tilde{c}} = g$.

Definition 2.9. [25, 29] Let $\mathbb{IFS}_X^E$ and $\mathbb{IFS}_Y^K$ be sets of all *ifs*-sets on X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings. Then a mapping $\varphi_\psi : \mathbb{IFS}_X^E \to \mathbb{IFS}_Y^K$ is defined as:

i. For $f \in \mathbb{IFS}_X^E$, the image of f under φ_ψ, denoted $\varphi_\psi(f)$, is an *ifs*-set in $\mathbb{IFS}_Y^K$ given by

$$\mu_{\varphi(f)}(k)(y) = \begin{cases} \sup_{e\in\psi^{-1}(k),\ x\in\varphi^{-1}(y)} \mu_{f(e)}(x), & \text{if } \varphi^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{\varphi(f)}(k)(y) = \begin{cases} \inf_{e\in\psi^{-1}(k),\ x\in\varphi^{-1}(y)} \nu_{f(e)}(x), & \text{if } \varphi^{-1}(y) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

ii. For $g \in \mathbb{IFS}_Y^K$, the inverse image of g under φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is an *ifs*-set in $\mathbb{IFS}_X^E$ given by

$$\mu_{\varphi^{-1}(g)}(e)(x) = \mu_{g(\psi(e))}(\varphi(x)) \text{ and } \nu_{\varphi^{-1}(g)}(e)(x) = \nu_{g(\psi(e))}(\varphi(x))$$

for all $e \in E$ and $x \in X$.

If φ and ψ are injective (surjective) then the *ifs*-mapping φ_ψ is said to be *ifs*-injective (*ifs*-surjective).

Theorem 2.10. [25] Let $\varphi_\psi : \mathbb{IFS}_X^E \to \mathbb{FS}_Y^K$ be a intuitionistic fuzzy soft mapping, $f \in \mathbb{IFS}_X^E$ and $\{f_i\}_{i\in\Lambda} \subseteq \mathbb{IFS}_X^E$. Then

i. If $f_1 \sqsubseteq f_2$, then $\varphi_\psi(f_1) \sqsubseteq \varphi_\psi(f_2)$

ii. $\varphi_\psi\Big(\bigsqcup_{i\in\Lambda} f_i\Big) = \bigsqcup_{i\in\Lambda} \varphi_\psi(f_i)$

iii. $\varphi_\psi\Big(\bigsqcap_{i\in\Lambda} f_i\Big) \sqsubseteq \bigsqcap_{i\in\Lambda} \varphi_\psi(f_i)$

iv. $\big(\varphi_\psi(f)\big)^{\tilde{c}} \sqsubseteq \varphi_\psi\big(f^{\tilde{c}}\big)$

v. If φ_ψ surjective, then $\varphi_\psi(\tilde{X}) = \tilde{Y}$

vi. $f \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(f)\big)$, the equality holds if φ_ψ is *ifs*-injective.

Theorem 2.11. [25] Let $\varphi_\psi : \mathbb{IFS}_X^E \to \mathbb{IFS}_Y^K$ be a intuitionistic fuzzy soft mapping, $g \in \mathbb{IFS}_Y^K$ and $\{g_j\}_{j\in J} \subseteq \mathbb{IFS}_Y^K$. Then

i. If $g_1 \sqsubseteq g_2$, then $\varphi_\psi^{-1}(g_1) \sqsubseteq \varphi_\psi^{-1}(g_2)$

ii. $\varphi_\psi^{-1}\Big(\bigsqcup_{i\in J} g_j\Big) = \bigsqcup_{j\in J} \varphi_\psi^{-1}(g_j)$

iii. $\varphi_\psi^{-1}\Big(\bigsqcap_{i\in J} g_j\Big) = \bigsqcap_{j\in J} \varphi_\psi^{-1}(g_j)$

iv. $\big(\varphi_\psi^{-1}(g)\big)^{\tilde{c}} = \varphi_\psi^{-1}\big(g^{\tilde{c}}\big)$

v. $\varphi_\psi^{-1}(\tilde{Y}) = \tilde{X}$ and $\varphi_\psi^{-1}(\Phi) = \Phi$

vi. $\varphi_\psi\big(\varphi_\psi^{-1}(g)\big) \sqsubseteq g$, the equality holds if φ_ψ is *ifs*-surjective.

Definition 2.12. [26] An *ifs*-topological space is a triplet (X, τ, E) where X is a nonempty set and τ a family of *ifs*-sets over X satisfying the following properties:

i. $\Phi, \tilde{X} \in \tau$,

ii. If $f, g \in \tau$, then $f \sqcap g \in \tau$,

iii. If $\{f_i\}_{i\in\Lambda} \subseteq \tau$, then $\bigsqcup_{i\in\Lambda} f_i \in \tau$.

Then, the family τ is called an *ifs*-topology on X. Every member of τ is called *ifs*-open. g is called *ifs*-closed in (X, τ, E) if $g^{\tilde{c}} \in \tau$.

If f is *ifs*-open and *ifs*-closed, then it is called *ifs*-clopen set. In case $f \neq \tilde{X}$ and $f \neq \Phi$, f is called *ifs*-proper set.

Example 2.13. $\tau^0 = \{\tilde{X}, \Phi\}$ and $\tau^1 = \mathbb{IFS}_X^E$ are *ifs*-topologies on X.

Definition 2.14. [26] Let (X, τ, E) be a *ifs*-topological space and $f \in \mathbb{IFS}_X^E$. Then, *ifs*-interior of f denoted by f° is the union of all *ifs*-open subsets of f. So, we can write the *ifs*-interior of f as

$$f^\circ = \bigsqcup_{\substack{g \sqsubseteq f \\ g \in \tau}} g.$$

Definition 2.15. [26] Let (X, τ, E) be a *ifs*-topological space and $f \in \mathbb{IFS}_X^E$. Then, *ifs*-closure of f denoted by $\overline{f}$ is the intersection of all *ifs*-closed supersets of f. So, we can write the *ifs*-closure of f as

$$\overline{f} = \bigsqcap_{\substack{f \sqsubseteq h \\ h^{\tilde{c}} \in \tau}} h.$$

It can be seen clearly that f° and $\overline{f}$ are the largest *ifs*-open set which contained in f and the smallest *ifs*-closed set which contains f over X, respectively.

Definition 2.16. Let (X, τ, E) be a *ifs*-topological space and $f \in \mathbb{IFS}_X^E$. If $f = \big(\overline{f}\big)^\circ$, then f is called *ifs*-regular open set. If If $f = \overline{f^\circ}$, then f is called *ifs*-regular closed set.

Theorem 2.17. [26] Let (X, τ, E) be a *ifs*-topological space and f, $g \in \mathbb{IFS}_X^E$. Then,

i. If $f \sqsubseteq g$, then $f^\circ \sqsubseteq g^\circ$ and $\overline{f} \sqsubseteq \overline{g}$

ii. f is a soft open set iff $f^\circ = f$

iii. f is a soft closed set iff $\overline{f} = f$

iv. $\big(\overline{f}\big)^{\tilde{c}} = \big(f^{\tilde{c}}\big)^\circ$ and $\overline{\big(f^{\tilde{c}}\big)} = \big(f^\circ\big)^{\tilde{c}}$

Definition 2.18. [29] Let (X, τ, E) and (Y, σ, K) be two *ifs*-topological spaces. An *ifs*-mapping $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ is called an *ifs*-continuous mapping if $\varphi_\psi^{-1}(g) \in \tau$ for all $g \in \sigma$.

Example 2.19. [29] In Example 2.13, every *ifs*-mapping $\varphi_\psi : (X, \tau^1, E) \to (Y, \sigma, K)$ is an *ifs*-continuous mapping.

3 Intuitionistic Fuzzy Soft Connectedness

In this section, we will give definition of *ifs*-connected spaces and their some properties. Further, we will introduce *ifs* C_i-connectedness $(i = 1, 2, 3, 4)$ and *ifs*-super connectedness.

Definition 3.1. Let (X, τ, E) be a *ifs*-topological space and $f \in \mathbb{IFS}_X^E$. If there are two *ifs*-proper open sets g_1 and g_2 such that $f \sqsubseteq g_1 \sqcup g_2$ and $g_1 \sqcap g_2 = \Phi$, then the *ifs*-set f is called *ifs*-disconnected set. If there does not exist such two *ifs*-proper open sets, then the *ifs*-set f is called *ifs*-connected set. If we take $\tilde{X}$ instead of f, then the (X, τ, E) is called *ifs*-disconnected (connected) space.

Example 3.2. Let consider the *ifs*-topological spaces (X, τ^0, E) and (X, τ^1, E) in Example 2.13, (X, τ^0, E) is an *ifs*-connected topological space, but (X, τ^1, E) is an *ifs*-disconnected topological space.

Theorem 3.3. Let (X, τ, E) be a *ifs*-topological space. (X, τ, E) *ifs*-connected if and only if there does not exist a *ifs*-proper clopen set f in (X, τ, E).

Proof. $(\Rightarrow)$: Let (X, τ, E) be a *ifs*-connected space. Suppose that there exist a *ifs*-proper clopen set f in (X, τ, E) such that $f \sqcup f^{\tilde{c}} = \tilde{X}$ and $f \sqcap f^{\tilde{c}} = \Phi$. It is a contradiction.

$(\Leftarrow)$: It is clear.

Theorem 3.4. Let (X, τ, E) be a *ifs*-topological space and $\sigma \subseteq \tau$. Then, (X, σ, E) is a connected *ifs*-topological space.

Proof. It is clear.

Theorem 3.5. Let (X, τ, E) and (Y, σ, K) be two *ifs*-topological spaces, $f \in \mathbb{IFS}_X^E$ and $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be an *ifs*-continuous mapping. If f is an *ifs*-connected set, then $\varphi_\psi(f)$ is an *ifs*-connected set.

Proof. Assume that $\varphi_\psi(f)$ is an *ifs*-disconnected set. Therefore, there exist two *ifs*-proper open sets g and h such that $\varphi_\psi(f) \sqsubseteq g \sqcup h$ and $g \sqcap h = \Phi$. By Theorem 2.11, we have

$$f \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(f)\big) \sqsubseteq \varphi_\psi^{-1}(g) \sqcup \varphi_\psi^{-1}(h)$$

and

$$\varphi_\psi^{-1}(g) \sqcap \varphi_\psi^{-1}(h) = \varphi_\psi^{-1}(\Phi) = \Phi.$$

It is a contradiction and this complete the proof.

Theorem 3.6. Let (X, τ, E) and (Y, σ, K) be two *ifs*-topological spaces and $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be an *ifs*-continuous and *ifs*-surjective mapping. If (X, τ, E) is an *ifs*-connected space, then (Y, σ, K) is also an *ifs*-connected space.

Proof. Assume that (Y, σ, K) is an *ifs*-disconnected space. So, there exist two *ifs*-proper open sets g_1 and g_2 such that $g_1 \sqcup g_2 = \tilde{Y}$, $g_1 \sqcap g_2 = \Phi$. By Theorem 2.11 $\varphi_\psi^{-1}(g_1) \sqcup \varphi_\psi^{-1}(g_2) = \tilde{X}$ and $\varphi_\psi^{-1}(g_1) \sqcap \varphi_\psi^{-1}(g_2) = \Phi$. This contradiction completes the proof.

Definition 3.7. Let (X, τ, E) be an *ifs*-topological space. If there exist $f, g \in \mathbb{IFS}_X^E$ which are *ifs*-proper, such that $\overline{f} \sqcap g = \Phi$ and $f \sqcap \overline{g} = \Phi$ then the *ifs*-sets f and g are called *ifs*-separated sets.

Theorem 3.8. Let (X, τ, E) be a *ifs*-topological space, f and g be two *ifs*-open sets. If $f \sqcap g = \Phi$, then f and g are *ifs*-separated sets.

Proof. Let $f, g \in \tau$ and $f \sqcap g = \Phi$. Then, $f^{\tilde{c}} \sqcup g^{\tilde{c}} = \tilde{X}$. So, $f \sqsubseteq g^{\tilde{c}}$ and $g \sqsubseteq f^{\tilde{c}}$. $f^{\tilde{c}}$ and $g^{\tilde{c}}$ are *ifs*-closed sets. By 2.17, we have

$$\overline{f} \sqsubseteq \overline{g^{\tilde{c}}} = g^{\tilde{c}} \text{ and } \overline{g} \sqsubseteq \overline{f^{\tilde{c}}} = f^{\tilde{c}}.$$

Therefore, $\overline{f} \sqcap g = \Phi$ and $f \sqcap \overline{g} = \Phi$.

Theorem 3.9. Let (X, τ, E) be an *ifs*-topological space, f and g be two *ifs*-closed sets. If $f \sqcap g = \Phi$, then f and g are *ifs*-separated sets.

Proof. From Theorem 2.17, it is clear.

Theorem 3.10. An *ifs*-topological space (X, τ, E) is connected if and only if $\tilde{X}$ cannot be written as union of *ifs*-separated sets.

Proof. $(\Rightarrow)$: Assume that $\tilde{X}$ can be written as union of *ifs*-separated sets f and g. Thus, $\tilde{X} = f \sqcup g$, $\overline{f} \sqcap g = \Phi$ and $f \sqcap \overline{g} = \Phi$. So, we have $f \sqcap g = \Phi$, $f = g^{\tilde{c}}$ and $g = f^{\tilde{c}}$. Furthermore

$$\begin{aligned} \overline{f} &= \overline{f} \sqcap \tilde{X} \\ &= \overline{f} \sqcap (f \sqcup g) \\ &= (\overline{f} \sqcap f) \sqcup (\overline{f} \sqcap g) \\ &= f. \end{aligned}$$

Thus, f is an *ifs*-closed set. With similar way, it can be seen clearly that g is also an *ifs*-closed set. This is a contradiction because $f = g^{\tilde{c}}$ and $g = f^{\tilde{c}}$, f and g are *ifs*-open sets.

$(\Leftarrow)$: Assume that (X, τ, E) is not an *ifs*-connected space. Thus, there exist an *ifs*-proper clopen set f. But it contradicts by hypothesis.

Theorem 3.11. Let (X, τ, E) be an *ifs*-topological space and $f \in \mathbb{IFS}_X^E$ be an *ifs*-open connected set. If $f \sqsubseteq g \sqsubseteq \overline{f}$, then g is an *ifs*-connected set.

Proof. Suppose that g is an *ifs*-disconnected set. Then, there exist two *ifs*-open proper sets h_1 and h_2 such that

$$h_1 \sqcap h_2 = \Phi \text{ and } g \sqsubseteq h_1 \sqcup h_2.$$

So,

$$f = [f \sqcap h_1] \sqcup [f \sqcap h_2]$$

and

$$[f \sqcap h_1] \sqcap [f \sqcap h_2] = \Phi.$$

But it is a contradiction. Thus g is an *ifs*-connected set.

Remark 3.12. Let (X, τ, E) be an *ifs*-topological space and $f \in \mathbb{IFS}_X^E$ be an *ifs*-open set. If f is an *ifs*-connected set, then $\overline{f}$ is an *ifs*-connected set.

Definition 3.13. Let (X, τ, E) be an *ifs*-topological space. If there exist an *ifs*-regular open proper set f, then (X, τ, E) is called *ifs*-super disconnected.

Example 3.14. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then, for

$$\begin{aligned} f &= \Big\{ \big(e_1, \{\langle x_1, 0.4, 0.6\rangle, \langle x_2, 0.6, 0.3\rangle, \langle x_3, 0.2, 0.3\rangle\}\big), \\ &\quad \big(e_2, \{\langle x_1, 0.6, 0.4\rangle, \langle x_2, 0.3, 0.6\rangle, \langle x_3, 0.3, 0.2\rangle\}\big) \Big\} \\ g &= \Big\{ \big(e_1, \{\langle x_1, 0.5, 0.2\rangle, \langle x_2, 0.3, 0.6\rangle, \langle x_3, 0.4, 0.3\rangle\}\big), \\ &\quad \big(e_2, \{\langle x_1, 0.2, 0.5\rangle, \langle x_2, 0.6, 0.3\rangle, \langle x_3, 0.3, 0.4\rangle\}\big) \Big\} \\ h &= \Big\{ \big(e_1, \{\langle x_1, 0.5, 0.4\rangle, \langle x_2, 0.4, 0.5\rangle, \langle x_3, 0.2, 0.4\rangle\}\big), \\ &\quad \big(e_2, \{\langle x_1, 0.4, 0.5\rangle, \langle x_2, 0.5, 0.4\rangle, \langle x_3, 0.4, 0.2\rangle\}\big) \Big\} \end{aligned}$$

$\tau = \{\tilde{X}, \Phi, f, g, h\}$ is an *ifs*-topology on X and (X, τ, E) is an *ifs*-super connected space.

Theorem 3.15. The followings are equivalent.

i. (X, τ, E) is an *ifs*-super connected space

ii. For each f such that $f \neq \Phi$, $\overline{f} = \tilde{X}$

iii. For each f such that $f \neq \Phi$, $f^\circ = \Phi$

iv. There exist no *ifs*-open sets f and g such that $f \neq \Phi$, $g \neq \Phi$ and $f \sqsubseteq g^{\tilde{c}}$

v. There exist no *ifs*-open sets $f$and g such that $f \neq \Phi$, $g \neq \Phi$, $g = (\overline{f})^{\tilde{c}}$ and $f = (\overline{g})^{\tilde{c}}$

vi. There exist no *ifs*-closed sets $f$and g such that $f \neq \tilde{X}$, $g \neq \tilde{X}$, $g = (f^\circ)^{\tilde{c}}$ and $f = (g^\circ)^{\tilde{c}}$

Proof. ($i. \Rightarrow ii.$): Suppose that there exists an *ifs*-open f such that $f \neq \Phi$ and $\overline{f} \neq \tilde{X}$. If we take $g = (\overline{f})^\circ$, then g is an *ifs*-proper and regular open set. But it is a contradiction.

($ii. \Rightarrow iii.$): Let $f \neq \tilde{X}$ be an *ifs*-closed set. If we take $g = f^{\tilde{c}}$, then g is an *ifs*-open and $g \neq \Phi$. For $\overline{g} = \tilde{X}$, we have $(g^\circ)^{\tilde{c}} = \Phi$ and $(\overline{g})^\circ = \Phi$. So, $f^\circ = \Phi$.

($iii. \Rightarrow iv.$): Let f and g be *ifs*-open sets such that $f \neq \Phi$, $g \neq \Phi$ and $f \sqsubseteq g^{\tilde{c}}$. Thus, $g^{\tilde{c}}$ is an *ifs*-closed set and because of $g \neq \Phi$, $g^{\tilde{c}} \neq \tilde{X}$. So, we obtain $(g^{\tilde{c}})^\circ = \Phi$. But, with $f \sqsubseteq g^{\tilde{c}}$, we can write $\Phi \neq f = f^\circ \sqsubseteq (g^{\tilde{c}})^\circ = \Phi$. It is a contradiction

($iv. \Rightarrow i.$): Let f be an *ifs*-regular open proper. If we take $g = (\overline{f})^{\tilde{c}}$, we obtain $g \neq \Phi$. (Otherwise, $(\overline{f})^{\tilde{c}} = \Phi \Rightarrow \overline{f} = \tilde{X}$ and so, $f = (\overline{f})^\circ = \tilde{X}$. But it contradicts the fact $f \neq \tilde{X}$.)

($i. \Rightarrow v.$): Let f and g be *ifs*-open sets such that $f \neq \Phi$, $g \neq \Phi$, $g = (\overline{f})^{\tilde{c}}$ and $f = (\overline{g})^{\tilde{c}}$. Then we have $(\overline{f})^\circ = (g^{\tilde{c}})^\circ = (\overline{g})^{\tilde{c}} = f$ where $f \neq \Phi$ and $f \neq \tilde{X}$. (Otherwise, if $f = \tilde{X}$, then $\tilde{X} = (\overline{g})^{\tilde{c}}$ and thus $\Phi = \overline{g}$.) But it is a contradiction.

($v. \Rightarrow i.$): Let f be an *ifs*-open proper set such that $f = (\overline{f})^\circ$. If we take $g = (\overline{f})^{\tilde{c}}$, then we have $g \neq \Phi$, $g \in \tau$, $g = (\overline{f})^{\tilde{c}}$ and so

$$(\overline{g})^{\tilde{c}} = \left(\overline{(\overline{f})^{\tilde{c}}}\right)^{\tilde{c}} = \left(((\overline{f})^\circ)^{\tilde{c}}\right)^{\tilde{c}} = (\overline{f})^\circ = f$$

but it is a contradiction.

($v. \Rightarrow vi.$): Let f and g be *ifs*-closed sets such that $f \neq \tilde{X}$, $g \neq \tilde{X}$, $g = (f^\circ)^{\tilde{c}}$ and $f = (g^\circ)^{\tilde{c}}$. If we take $h_1 = f^{\tilde{c}}$ and $h_2 = g^{\tilde{c}}$, then h_1 and h_2 become *ifs*-open sets such that $h_1 \neq \Phi$ and $h_2 \neq \Phi$. Thus $(\overline{h_1})^{\tilde{c}} = (\overline{f^{\tilde{c}}})^{\tilde{c}} = \left((f^\circ)\right)^{\tilde{c}} = f^\circ = g^{\tilde{c}} = h_2$ and similarly $(\overline{h_2})^{\tilde{c}} = h_1$. But this is a contradiction, clearly.

($vi. \Rightarrow v.$): It can be proved similar way in ($v. \Rightarrow vi.$)

Now, we will introduce *ifs* C_i-connected spaces ($i = 1, 2, 3, 4$) by helping of fuzzy C_i-connectedness in intuitionistic fuzzy sets [4]. Definitions of *ifs* C_i-connected spaces can be seen as an extension of intuitionistic fuzzy connected space.

Definition 3.16. Let (X, τ, E) be a *ifs*-topological space and $f \in \mathbb{IFS}_X^E$. f is called

i. *ifs* C_1-connected iff does not exist two non null *ifs*-open sets g and h such that $f \sqsubseteq g \sqcup h$, $g \sqcap h \sqsubseteq f^{\tilde{c}}$, $f \sqcap g \neq \Phi$ and $f \sqcap h \neq \Phi$.

ii. *ifs* C_2-connected iff does not exist two non null *ifs*-open sets g and h such that $f \sqsubseteq g \sqcup h$, $f \sqcap g \sqcap h = \Phi$ $f \sqcap g \neq \Phi$ and $f \sqcap h \neq \Phi$.

iii. *ifs* C_3-connected iff does not exist two non null *ifs*-open sets g and h such that $f \sqsubseteq g \sqcup h$, $g \sqcap h \sqsubseteq f^{\tilde{c}}$, $g \not\sqsubseteq f^{\tilde{c}}$ and $h \not\sqsubseteq f^{\tilde{c}}$.

iv. *ifs* C_4-connected iff does not exist two non null *ifs*-open sets g and h such that $f \sqsubseteq g \sqcup h$, $f \sqcap g \sqcap h = \Phi$, $g \not\sqsubseteq f^{\tilde{c}}$ and $h \not\sqsubseteq f^{\tilde{c}}$.

From Definition 3.16, relations between *ifs* C_i-connectedness ($i = 1, 2, 3, 4$) can be described by the following diagram:

$$\begin{array}{ccc} \textit{ifs } C_1 \text{ connectedness} & \longrightarrow & \textit{ifs } C_2 \text{ connectedness} \\ \downarrow & & \downarrow \\ \textit{ifs } C_3 \text{ connectedness} & \longrightarrow & \textit{ifs } C_4 \text{ connectedness} \end{array}$$

In the following examples, we illustrate all reverse implications.

Example 3.17. Let $X = [0,1]$ and $E = \{a,b\}$. Moreover, define soft sets f, g and h as following:

$$\begin{array}{rcl} f &=& \Big\{\big(a, \{\langle x, \mu_{f(a)}(x), \nu_{f(a)}(x)\rangle : x \in X\}\big), \\ && \big(b, \{\langle x, \mu_{f(b)}(x), \nu_{f(b)}(x)\rangle : x \in X\}\big)\Big\} \\ g &=& \Big\{\big(a, \{\langle x, \mu_{g(a)}(x), \nu_{g(a)}(x)\rangle : x \in X\}\big), \\ && \big(b, \{\langle x, \mu_{g(b)}(x), \nu_{g(b)}(x)\rangle : x \in X\}\big)\Big\} \\ h &=& \Big\{\big(a, \{\langle x, \mu_{h(a)}(x), \nu_{h(a)}(x)\rangle : x \in X\}\big), \\ && \big(b, \{\langle x, \mu_{h(b)}(x), \nu_{h(b)}(x)\rangle : x \in X\}\big)\Big\} \end{array}$$

where

$$\mu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 1, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases}$$

$$\nu_{g(a)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases} \quad \text{and} \quad \nu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 1, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 1, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\nu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 1, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \quad \text{and} \quad \nu_{h(b)}(x) = \begin{cases} 1, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$\mu_{f(a)}(x) = \mu_{f(b)}(x) = \nu_{f(a)}(x) = \nu_{f(b)}(x) = 3/4$ for all $x \in [0,1]$. $\tau = \{\Phi, \tilde{X}, g, h, g \sqcap h\}$ is a *ifs*-topology on X. It can be see clearly that f is *ifs* C_4-connected but *ifs* C_3-disconnected.

Example 3.18. Let $X = [0,1]$ and $E = \{a,b\}$. Moreover, define soft sets g, h and f as following:

$$\begin{array}{rcl} g &=& \Big\{\big(a, \{\langle x, \mu_{g(a)}(x), \nu_{g(a)}(x)\rangle : x \in X\}\big), \\ && \big(b, \{\langle x, \mu_{g(b)}(x), \nu_{g(b)}(x)\rangle : x \in X\}\big)\Big\} \\ h &=& \Big\{\big(a, \{\langle x, \mu_{h(a)}(x), \nu_{h(a)}(x)\rangle : x \in X\}\big), \\ && \big(b, \{\langle x, \mu_{h(b)}(x), \nu_{h(b)}(x)\rangle : x \in X\}\big)\Big\} \\ f &=& g \sqcup h \end{array}$$

where

$$\mu_{g(a)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \quad \text{and} \quad \mu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases}$$

$$\nu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases} \quad \text{and} \quad \nu_{g(b)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases} \quad \text{and} \quad \mu_{h(b)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\nu_{h(a)}(x) = \begin{cases} 0, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \quad \text{and} \quad \nu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases}$$

$\tau = \{\Phi, \tilde{X}, g, h, g \sqcup h\}$ is a *ifs*-topology on X. It can be seen clearly that f is *ifs* C_4-connected but *ifs* C_2-disconnected.

Example 3.19. Let $X = [0,1]$ and $E = \{a, b\}$. Moreover, define soft sets f, g and h as following:

$$\begin{aligned} f &= \Big\{\big(a, \{\langle x, \mu_{f(a)}(x), \nu_{f(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{f(b)}(x), \nu_{f(b)}(x)\rangle : x \in X\}\big)\Big\} \\ g &= \Big\{\big(a, \{\langle x, \mu_{g(a)}(x), \nu_{g(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{g(b)}(x), \nu_{g(b)}(x)\rangle : x \in X\}\big)\Big\} \\ h &= \Big\{\big(a, \{\langle x, \mu_{h(a)}(x), \nu_{h(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{h(b)}(x), \nu_{h(b)}(x)\rangle : x \in X\}\big)\Big\} \end{aligned}$$

where

$$\mu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \text{ and } \mu_{g(b)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases}$$

$$\nu_{g(a)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases} \text{ and } \nu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases} \text{ and } \mu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases}$$

$$\nu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{1}{3} \end{cases} \text{ and } \nu_{h(b)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{1}{3} < x \le 1 \\ \frac{1}{3}, & \text{if } 0 \le x \le \frac{1}{3}, \end{cases}$$

$\mu_{f(a)}(x) = \mu_{f(b)}(x) = \nu_{f(a)}(x) = \nu_{f(b)}(x) = 1/3$ for all $x \in [0,1]$. $\tau = \{\Phi, \tilde{X}, g, h, g \sqcap h, g \sqcup h\}$ is a *ifs*-topology on X. It can be seen clearly that f is *ifs* C_3-connected and *ifs* C_2-connected but *ifs* C_1-disconnected.

Example 3.20. Let $X = [0,1]$ and $E = \{a, b\}$. Moreover, define soft sets f, g and h as following:

$$\begin{aligned} f &= \Big\{\big(a, \{\langle x, \mu_{f(a)}(x), \nu_{f(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{f(b)}(x), \nu_{f(b)}(x)\rangle : x \in X\}\big)\Big\} \\ g &= \Big\{\big(a, \{\langle x, \mu_{g(a)}(x), \nu_{g(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{g(b)}(x), \nu_{g(b)}(x)\rangle : x \in X\}\big)\Big\} \\ h &= \Big\{\big(a, \{\langle x, \mu_{h(a)}(x), \nu_{h(a)}(x)\rangle : x \in X\}\big), \\ &\quad \big(b, \{\langle x, \mu_{h(b)}(x), \nu_{h(b)}(x)\rangle : x \in X\}\big)\Big\} \end{aligned}$$

where

$$\mu_{g(a)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{2}{3} \end{cases} \text{ and } \mu_{g(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{2}{3}, \end{cases}$$

$$\nu_{g(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{2}{3}, \end{cases} \text{ and } \nu_{g(b)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{2}{3} \end{cases}$$

$$\mu_{h(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{2}{3}, \end{cases} \text{ and } \mu_{h(b)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{2}{3} \end{cases}$$

$$\nu_{h(a)}(x) = \begin{cases} 0, & \text{if } \frac{2}{3} < x \le 1 \\ \frac{2}{3}, & \text{if } 0 \le x \le \frac{2}{3} \end{cases} \text{ and } \nu_{h(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \le 1 \\ 0, & \text{if } 0 \le x \le \frac{2}{3}, \end{cases}$$

$$\mu_{f(a)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \leq 1 \\ \frac{2}{3}, & \text{if } 0 \leq x \leq \frac{2}{3} \end{cases} \quad \text{and} \quad \mu_{f(b)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{2}{3} < x \leq 1 \\ \frac{1}{3}, & \text{if } 0 \leq x \leq \frac{2}{3} \end{cases}$$

$$\nu_{f(a)}(x) = \begin{cases} \frac{2}{3}, & \text{if } \frac{2}{3} < x \leq 1 \\ \frac{1}{3}, & \text{if } 0 \leq x \leq \frac{2}{3} \end{cases} \quad \text{and} \quad \nu_{f(b)}(x) = \begin{cases} \frac{1}{3}, & \text{if } \frac{2}{3} < x \leq 1 \\ \frac{2}{3}, & \text{if } 0 \leq x \leq \frac{2}{3} \end{cases}$$

$\tau = \{\Phi, \tilde{X}, g, h, g \sqcup h\}$ is a *ifs*-topology on X. It can be seen clearly that f is *ifs* C_3-connected but *ifs* C_2-disconnected and *ifs* C_1-disconnected.

Example 3.21. In the Example 3.19, if we take $\mu_{f(a)}(x) = \mu_{f(b)}(x) = \nu_{f(a)}(x) = \nu_{f(b)}(x) = \frac{2}{3}$ for all $x \in [0, 1]$, then f is *ifs* C_2-connected but *ifs* C_3-disconnected.

Theorem 3.22. Let $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a *ifs*-surjective continuous mapping and $f \in \mathbb{IFS}_X^E$. If f is a *ifs* C_1-connected, then $\varphi_\psi(f)$ is *ifs* C_1-connected.

Proof. Suppose that $\varphi_\psi(f)$ is not *ifs* C_1-connected. Then, there exist two non null *ifs*-open sets g and h in (Y, σ, K) such that

$$\begin{aligned} \varphi_\psi(f) &\sqsubseteq g \sqcup h, \\ g \sqcap h &\sqsubseteq \left(\varphi_\psi(f)\right)^{\tilde{c}}, \\ \varphi_\psi(f) \sqcap g &\neq \Phi, \\ \varphi_\psi(f) \sqcap h &\neq \Phi. \end{aligned}$$

Thus, by Theorem 2.11 we have

$$\begin{aligned} f &\sqsubseteq \varphi_\psi^{-1}(g) \sqcup \varphi_\psi^{-1}(h), \\ \varphi_\psi^{-1}(g) \sqcap \varphi_\psi^{-1}(h) &\sqsubseteq f^{\tilde{c}} \\ \varphi_\psi^{-1}(g) \sqcap f &\neq \Phi, \\ \varphi_\psi^{-1}(h) \sqcap f &\neq \Phi. \end{aligned}$$

But this contradict by hypothesis. So, $\varphi_\psi(f)$ is an *ifs* C_1-connected.

Theorem 3.23. Let $\varphi_\psi : (X, \tau, E) \to (Y, \sigma, K)$ be a *ifs*-surjective continuous mapping and $f \in \mathbb{IFS}_X^E$. If f is a *ifs* C_2-connected, then $\varphi_\psi(f)$ is *ifs* C_2-connected.

Proof. it can be proved similar way to above theorem.

Theorem 3.24. Let $\varphi_\psi : (X, \tau) \to (Y, \sigma)$ be *ifs*-continuous surjective mapping and $f \in \mathbb{IFS}_X^E$. If f is a *ifs* C_3-connected, then $\varphi_\psi(f)$ is a *ifs* C_3-connected.

Proof. Assume that, $\varphi_\psi(f)$ is not *ifs* C_3-connected. Then, there exist two non null *ifs*-open sets g and h in (Y, σ, K) such that

$$\begin{aligned} \varphi_\psi(f) &\sqsubseteq g \sqcup h, \\ g \sqcap h &\sqsubseteq \left(\varphi_\psi(f)\right)^{\tilde{c}}, \\ g &\not\sqsubseteq \left(\varphi_\psi(f)\right)^{\tilde{c}}, \\ h &\not\sqsubseteq \left(\varphi_\psi(f)\right)^{\tilde{c}}. \end{aligned}$$

By Theorem 2.11,

$$f \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(f)\big) \sqsubseteq \varphi_\psi^{-1}\big(g \sqcup h\big) = \varphi_\psi^{-1}(g) \sqcup \varphi_\psi^{-1}(h)$$

and

$$\varphi_\psi^{-1}\big(g \sqcap h\big) = \varphi_\psi^{-1}(g) \sqcap \varphi_\psi^{-1}(h) \sqsubseteq f^{\tilde{c}}.$$

Since, $f \sqsubseteq \varphi_\psi^{-1}\big(\varphi_\psi(f)\big)$ implies $\big(\varphi_\psi^{-1}(\varphi_\psi(f))\big)^{\tilde{c}} \sqsubseteq f^{\tilde{c}}$ and φ_ψ is a *ifs*-continuous function, so $\varphi_\psi^{-1}(g), \varphi_\psi^{-1}(h) \in \tau$. Moreover, from $g \not\sqsubseteq \big(\varphi_\psi(f)\big)^{\tilde{c}}$ and $h \not\sqsubseteq \big(\varphi_\psi(f)\big)^{\tilde{c}}$, there exist $y_1, y_2 \in Y$ such that

$$g_e(y_1) \geq 1 - \varphi_\psi(f)(k)(y_1) \tag{1}$$

$$h_e(y_2) \geq 1 - \varphi_\psi(f)(k)(y_2) \tag{2}$$

We claim that $\varphi_\psi^{-1}(g) \not\sqsubseteq f^{\tilde{c}}$ and $\varphi_\psi^{-1}(h) \not\sqsubseteq f^{\tilde{c}}$. To prove the claim, we suppose $\varphi_\psi^{-1}(g) \sqsubseteq f^{\tilde{c}}$. Clearly, this claim contradicts by (1). Similarly, $\varphi_\psi^{-1}(h) \sqsubseteq f^{\tilde{c}}$ contradicts by (2). So, $\varphi_\psi(f)$ is *ifs* C_3-connected.

Theorem 3.25. Let $\varphi_\psi : (X, \tau) \to (Y, \sigma)$ be *ifs*-continuous surjevtive mapping and $f \in \mathbb{IFS}_X^E$. If f is a *ifs* C_4-connected, then $\varphi_\psi(f)$ is a SC_4 connected.

Proof. It can be proved similarly way in Theorem 3.24.

Theorem 3.26. Let (X, τ, E) be a *ifs*-topological space, f_1 and f_2 be two *ifs* C_1-connected *ifs*-sets such that $f_1 \sqcap f_2 \neq \Phi$. Then, $f_1 \sqcup f_2$ is *ifs* C_1-connected.

Proof. It is easy.

Remark 3.27. From Theorem 3.26, we can say easily that if f_1 and f_2 be two *ifs* C_2-connected *ifs*-sets such that $f_1 \sqcap f_2 \neq \Phi$, then $f_1 \sqcup f_2$ is *ifs* C_2-connected.

Theorem 3.28. Let (X, τ, E) be a *ifs*-topological space and $\{f_k\}_{k\in\Lambda} \subseteq \mathbb{IFS}_X^E$ be family of *ifs* C_1-connected *ifs*-sets such that $f_i \sqcap f_j \neq \Phi$ for $i, j \in \Lambda$ $(i \neq j)$. Then, $\bigsqcup_{k\in\Lambda} f_k$ is is a *ifs* C_1-connected *ifs*-set.

Proof. It can be proved by using Theorem 3.26.

4 Conclusion

In this paper we introduced *ifs*-connectedness which super *ifs* connectedness and *ifs* C_i $(i = 1, 2, 3, 4)$ connectedness and presented fundamentals properties. For future works, we consider to study on *ifs* C_M and C_5 connected sets in *ifs* topological spaces.

References

[1] L.A. Zadeh, *Fuzzy sets*, Information and Control 8, 338.353, 1965.

[2] K.T. Atanassov, *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems 20(1986)87-96.

[3] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems 88(1997)81-89.

[4] S. Özçağ and D. Çoker, *On connectedness in intuitionistic fuzzy special topological spaces*, Int. J. Math. & Math. Sci. Vol. 21, No. 1 33-40. CMP 1 486 955. Zbl 892.54005.

[5] N. Turanli and D. Çoker, *Fuzzy connectedness in intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, 116 (2000) 369-375.

[6] D. Molodtsov, *Soft set theory-first results*, Computers Math. Appl. 37 (4/5), 19.31, 1999.

[7] N. Çağman, *Contributions to the theory of soft sets*, Journal of New Results in Science (4)(2014), 33-41.

[8] M. Shabir and M. Naz, *On soft topological spaces*, Computers and Mathematics with Applications 61, 1786.1799, 2011.

[9] N. Çağman, S. Karataş and S. Enginoğlu, *Soft topology*, Computers and Mathematics with Applications, 62 (2011), 351-358.

[10] A. Aygünoğlu and H. Aygün, *Some notes on soft topological spaces*, Neural Comput and Applic, (2011), 1-7.

[11] S. Hussain and B. Ahmad, *Some properties of soft topological spaces*, Computers and Mathematics with Applications, 62 (2011) 4058–4067.

[12] S. Hussain, *A note on soft connectedness*, Journal of Egyptian Mathematical Society DOI: 10.1016/j.joems.2014.02.003.

[13] İ. Zorlutuna, M. Akdağ, W.K. Min, S. Atmaca, *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, Volume 3, No. 2, pp. 171-185.

[14] E. Peyghan, B. Samadi and A. Tayebi, *About soft topological spaces* Journal of New Results in Science Number: 2, Year: 2013, Pages: 60-75.

[15] D. N. Georgiou and A. C. Megaritis, *Soft set theory and topology*, Applied General Topology, 14, (2013).

[16] D. N. Georgiou, A. C. Megaritis and V. I. Petropoulos, *On Soft Topological Spaces*, Applied Mathematics & Information Sciences, 7, No. 5, 1889-1901 (2013).

[17] B. P. Varol, A. Shostak and H. Aygün, *A new approach to soft topology*, Hacettepe Journal of Mathematics and Statistics Volume 41(5) (2012), 731-741.

[18] B. Tanay and M.B. Kandemir, *Topological structures of fuzzy soft sets*, Computers and Mathematics with Applications 61, 412.418, 2011.

[19] S. Roy and T.K. Samanta, *A note on fuzzy soft topological spaces*, Annals of Fuzzy Mathematics and Informatics Volume 3, No. 2, (April 2012), pp. 305- 311 ISSN 2093–9310

[20] B.P. Varol and H. Aygün, *Fuzzy soft topology*, Hacettepe Journal of Mathematics and Statistics Volume 41 (3)(2012), 407–419.

[21] P.K. Gain, P. Mukherjee, R.P. Chakraborty and M. Pal, *On some structural properties of fuzzy soft topological spaces*, Intern. J. Fuzzy Mathematical Archive Vol. 1, 2013, 1-15 ISSN: 2320–3242 (P), 2320–3250.

[22] T. Şimşekler and Ş. Yüksel, *Fuzzy soft topological spaces*, Annals of Fuzzy Mathematics and Informatics Volume 5, No. 1, (January 2013), pp. 8796.

[23] Ç. Gündüz and S. Bayramov *Some results on fuzzy soft topological spaces* Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2013, Article ID 835308.

[24] P.K. Maji, R. Biswas, and A.R. Roy, *Intuitionistic fuzzy soft sets*, The Journal of Fuzzy Mathematics 9(3), 677–692 (2001)

[25] Y. Yin, H. Li, Y.B. Jun, *On algebraic structures of intuitionistic fuzzy soft sets*, Computers and Mathematics with Applications 61 (2012) 2896-2911.

[26] Z. Li and R. Cui, *On the topological structure of intuitionistic fuzzy soft set*, Annals of Fuzzy Mathematics and Informatics, Volume 5, No. 1 (2013), pp. 229-239.

[27] İ. Osmanoğlu and D. Tokat, *On Intuitionistic Fuzzy Soft Topology*, General Mathematics Notes, 19(2), 59-70, 2013.

[28] N. Turanlı and A. H. Es, *A note on compactness in intuitionistic fuzzy soft topological spaces*, World Applied Sciences Journals, 9(9) (2012),1355-1359.

[29] S. Karataş and M. Akdağ, *On intuitionistic fuzzy soft continuous mappings*, Journal of New Results in Science, Number: 4, Year: 2014, Pages: 55-70.