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RESEARCH ARTICLE

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AN ANALYTICAL SOLUTION FOR THE ELECTROMAGNETIC OSCILLATIONS CAUSED BY A RECTANGULAR PULSE IN A CAVITY WITH LOSSY WALLS*

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ABSTRACT

The purpose of this study is analytical studying Initial-Boundary-Value Problem for the novel format of Maxwell's equations in SI units. A modified version of the Evolutionary Approach to Electromagnetics (EAE) used herein. The problem is considered for the causal electromagnetic oscillations excited by a given external rectangular pulse signal, $\mathcal{J}(\mathbf{r},t)$*, in a hollow cavity with lossy metallic walls. The cavity volume* V *is finite and closed by a singly connected surface* S *with none of its inner angles exceeds* π*. Physically, cavity walls are lossy (completely or partially). Graphical results are exhibited demonstrating that the electromagnetic oscillations inside the cavity with metallic surface satisfy the causality principle.*

Keywords: *Maxwell's Equations, Time-domain Electrodynamics, Cavity, Evolutionary Equations, Matrix Exponentials.*

KAYIPLI YÜZEYLERE SAHİP BİR KAVİTEDEKİ DİKDÖRTGEN DARBE KAYNAKLI ELEKTROMANYETİK OSİLASYONLAR İÇİN BİR ANALİTİK ÇÖZÜM

ÖZ

Bu çalışmanın amacı, SI birim sisteminde yeniden yazılmış Maxwell denklemlerine ilişkin başlangıç-sınır-değer problemine analitik bir çözüm sunmaktır. Çalışmada Elektromanyetik Teoriye Evrimsel Yaklaşım'ın modifiye edilmiş bir versiyonu kullanılmıştır. Problem, kayıplı metalik yüzeylere sahip boş bir kaviteye verilen dikdörtgen $\mathcal{J}(\mathbf{r}, t)$ *darbe sinyalleri, tarafından uyarılan nedensel elektromanyetik osilasyonlar için düşünülmüştür. Kavite hacmi,* V*, sonludur ve* S *yüzeyinin iç açılarından hiçbirinin* π*'den büyük olmadığı pürüzsüz bir* S *yüzeyiyle kapatılmıştır. Fiziksel olarak yüzey (tamamen ya da kısmen) kayıplıdır. Kayıplı yüzeylere sahip kavite içerisindeki elektromanyetik osilasyonların nedensellik prensibini sağladığını gösteren grafiksel sonuçlar sergilenmiştir.*

Anahtar Kelimeler: *Maxwell Denklemleri, Zaman Uzayı Elektrodinamiği, Kavite, Evrimsel Denklemler, Matris Eksponansiyeller.*

1. INTRODUCTION

The goal of the present study is twofold. The first one is to derive an analytical solution for the fields in a hollow cavity with lossy metallic surfaces by making use of the matrix exponential method. The foundations of the approach used in this study, Evolutionary Approach to Electromagnetics (EAE), was proposed at the beginning of 1990s for exact explicit solution of the fields in cavities and waveguides (Tretyakov, 1993). A *new* SI *format* of Maxwell's equations (MEs) presented and acknowledged recently (Tretyakov, 2017; Tretyakov, 2018) where the new electric and magnetic fields have their common physical dimension. The convenience of the *new format*, where the fields have their common dimension, to upgrade the Evolutionary Approach for solving some practical problems was exhibited in the previous studies (Erden, Tretyakov, & Çoşan, 2018; Erden & Tretyakov, 2017; Tretyakov, 2018; Tretyakov, Butrym, & Erden, 2021).

The second goal is to present graphically the evolution of the electromagnetic fields, which can be stimulated in such cavities by a rectangular pulse function. Every rectangular pulse function has a beginning and end, as the digital signals and Walsh functions. This fact requires the involvement of the causality principle at the formulation of our problem (Erden, 2017; Tretyakov, 1993). Since the Walsh functions consist of trains of rectangular pulses, this study can be extended to investigate the evolution of the electromagnetic fields in a cavity excited by digital signals which have been used broadly in telecommunication technology for the last few decades (Aksoy & Tretyakov, 2003; Aksoy & Tretyakov, 2004).

The article is structured as follows. In Sec. II, the formulation is given where the *new format* of MEs and boundary conditions are presented for the problem. In Sec. III, the modal basis, and the modal field expansions available for the time-domain study are presented. In Sec. IV, an ordinary differential equation system for the time-dependent field amplitudes, i.e., the evolutionary equations are derived. In Sec. V and VI, the evolutionary equations are solved by making use of the method of matrix exponential. An analytical method based on Lagrange interpolation is applied therein (Erden & Tretyakov, 2008). In Sec. VII and VIII, we examine our conclusions.

2. FORMULATION OF THE PROBLEM

The central point in rearranging the Maxwell's equations to a new format in SI units (Tretyakov et al., 2021; Tretyakov & Erden, 2021) is based on the *novel definition* of the free-space constants as

$$\varepsilon_0^V = \sqrt{\frac{1N}{\varepsilon_0}}\left\lfloor V = \frac{Nm}{As}\right\rfloor, \quad \mu_0^A = \sqrt{\frac{1N}{\mu_0}}\lfloor A\rfloor \tag{1}$$

where N is a force of one *newton*. Derivations of ε_0^V and μ_0^A are given in Appendix A of the recent paper (Tretyakov & Erden, 2021). One can verify that ε_0^V has the dimension of *volt*, $\lfloor V\rfloor$, with its numerical value of 3.361×10^5, and μ_0^A has the dimension of *ampere*, $\lfloor A\rfloor$, with its numerical value of 8.921×10^2. ε_0^V and μ_0^A can be used as the *scaling coefficients* for the standard electric, $\mathcal{E}$, and magnetic, $\mathcal{H}$, fields to divide the physical dimensions of $\lfloor V/m\rfloor$ and $\lfloor A/m\rfloor$ as

$$\left.\begin{aligned}
\underbrace{\mathcal{E}(\mathbf{r},t)}_{\lfloor V/m\rfloor} &= \underbrace{\varepsilon_0^V}_{\lfloor V\rfloor}\underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m\rfloor} = \underbrace{3.361\times10^5}_{\lfloor V\rfloor}\times\underbrace{\mathbb{E}(\mathbf{r},t)}_{\lfloor 1/m\rfloor}\\
\underbrace{\mathcal{H}(\mathbf{r},t)}_{\lfloor A/m\rfloor} &= \underbrace{\mu_0^A}_{\lfloor A\rfloor}\underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m\rfloor} = \underbrace{8.921\times10^2}_{\lfloor A\rfloor}\times\underbrace{\mathbb{H}(\mathbf{r},t)}_{\lfloor 1/m\rfloor}\\
\underbrace{\mathcal{J}(\mathbf{r},t)}_{\lfloor A/m^2\rfloor} &= \underbrace{\mu_0^A}_{\lfloor A\rfloor}\underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m\rfloor} = \underbrace{8.921\times10^2}_{\lfloor A\rfloor}\times\underbrace{\mathbb{J}(\mathbf{r},t)}_{\lfloor 1/m\rfloor}.
\end{aligned}\right\} \tag{2}$$

The SI dimensions of volt $\lfloor V\rfloor$ and of ampere $\lfloor A\rfloor$ are assigned to the factors ε_0^V and μ_0^A, in our new definition. Meanwhile, novel field vectors, $\mathbb{E}$ and $\mathbb{H}$, have the inverse meter $\lfloor 1/m\rfloor$ physical dimension. So, the *new* SI *format* of the Maxwell's equations is

$$\begin{aligned}\nabla\times\mathbb{H}(\mathbf{r},t)&=\mathbb{J}(\mathbf{r},t)+\frac{1}{c}\frac{\partial}{\partial t}\mathbb{E}(\mathbf{r},t)\\ \nabla\times\mathbb{E}(\mathbf{r},t)&=-\frac{1}{c}\frac{\partial}{\partial t}\mathbb{H}(\mathbf{r},t)\end{aligned}\tag{3}$$

where $\mathbb{J}$ is a current density supplying a given signal to the cavity. Consider the case of S composed of the parts as

$$S=S_1+S_2.\tag{4}$$

In what follows, notation $\mathbf{n}$ and $\mathbf{l}$ are used for the unit vectors outward normal and tangential to the surface S, respectively. The part S_1 is supposed as a lossy surface, over which Leontovich boundary condition (see (Toptygin, 2015)) holds as

$$\mathbf{n}\times\mathbb{E}(\mathbf{r},t)=\alpha\,\mathbf{l}\cdot\mathbb{H}(\mathbf{r},t),\quad \mathbf{r}\in S_1\tag{5}$$

where $\alpha=\varsigma\rho$ is a small parameter, and ς is the *impedance* of the lossy metallic surface. The constant $\rho=\mu_0^A/\varepsilon_0^V=\sqrt{\varepsilon_0/\mu_0}$ is numerically very small, i.e., 2.654×10^{-3}. The ρ appears in α when Maxwell's equations are in the new format. But ρ is *absent* (and $\alpha\equiv\varsigma$ becomes large) if Maxwell's equations are standard. The Leontovich *approximate* boundary condition (5), relates the *tangential* components of the electric field, $\mathbf{n}\times\mathbb{E}(\mathbf{r},t)$, to magnetic field, $\mathbf{l}\cdot\mathbb{H}(\mathbf{r},t)$, over the surface of well-conducting bodies. The Leontovich impedance boundary condition is *accurate* for most metals while the impedance ς is *large*, but finite.

The part S_2 is perfect electric conducting where the boundary conditions are

$$\mathbf{n}\times\mathbb{E}(\mathbf{r},t)=0,\quad \mathbf{n}\cdot\mathbb{H}(\mathbf{r},t)=0,\quad \mathbf{r}\in S_2.\tag{6}$$

The initial conditions for the fields are

$$\mathbb{E}(\mathbf{r},t)\big|_{t=0}=0,\quad \mathbb{H}(\mathbf{r},t)\big|_{t=0}=0,\quad \mathbf{r}\in V.\tag{7}$$

3. MODAL BASIS AND FIELD DECOMPOSITONS

The space of solutions is chosen as Hilbert space L_2 where the inner product of the vectors are defined as

$$\langle \mathbf{A},\mathbf{B} \rangle = \frac{1}{V}\int_V \mathbf{A}\cdot\mathbf{B}^* dV. \tag{8}$$

The modal basis has been derived *without* postulating fields as time-harmonic in L_2 and presented herein in the form of the boundary-eigenvalue problems as

$$\left.\begin{aligned} \nabla\times\mathbf{H}_n &= -ik_n\mathbf{E}_n, \quad \nabla\cdot\mathbf{H}_n = 0, \quad \mathbf{n}\cdot\mathbf{H}_n\big|_S = 0 \\ \nabla\times\mathbf{E}_n &= ik_n\mathbf{H}_n, \quad \nabla\cdot\mathbf{E}_n = 0, \quad \mathbf{n}\times\mathbf{E}_n\big|_S = 0 \end{aligned}\right\} \tag{9}$$

where the eigenvalues, k_n, $(n = 1,2,...)$ have $\lfloor 1/m \rfloor$ physical dimension. The elements of basis satisfy the orthonormal conditions as

$$\left.\begin{aligned} \langle \mathbf{E}_{n'},\mathbf{E}_n \rangle &= \frac{1}{V}\int_V \mathbf{E}_{n'}\cdot\mathbf{E}_n^* dV = \delta_{n'n} \\ \langle \mathbf{H}_{n'},\mathbf{H}_n \rangle &= \frac{1}{V}\int_V \mathbf{H}_{n'}\cdot\mathbf{H}_n^* dV = \delta_{n'n} \end{aligned}\right\} \tag{10}$$

where $\delta_{n'n}$ is Kronecker delta. The modal field decompositions for $\mathbb{E}$ and $\mathbb{H}$ fields are presentable as

$$\mathbb{E}(\mathbf{r},t) = \sum_{n'=1}^{\infty} e_{n'}(t)\mathbf{E}_{n'}(\mathbf{r}), \quad \mathbb{H}(\mathbf{r},t) = \sum_{n'=1}^{\infty} h_{n'}(t)\mathbf{H}_{n'}(\mathbf{r}) \tag{11}$$

where the modal basis vectors $\mathbf{E}_{n'}$ and $\mathbf{H}_{n'}$ have the same physical dimension of inverse meter as the new fields $\mathbb{E}$ and $\mathbb{H}$, and the time-dependent modal amplitudes are dimension-free.

The current density, $\mathbb{J}$, in equation (3) is responsible for excitation of forced oscillations in the cavity. $\mathbb{J}$ is decomposable as $\mathbb{J} = j(t)\mathbf{I}(\mathbf{r})$ where

$j(t)$ is a given signal. The vector $\mathbf{I}$ is specified by configuration and position within V of an item supplying $j(t)$ to the cavity. Anyway, $\mathbf{I}$ is presentable as

$$\mathbf{I}(\mathbf{r}) = \sum\nolimits_{n'=1}^{\infty} g_{n'} k_{n'} \mathbf{E}_{n'}(\mathbf{r}) \tag{12}$$

where $g_{n'}$ are constant dimension-free coefficients.

4. EVOLUTIONARY EQUATIONS

Projecting Maxwell's equations (3) onto the modal basis results in

$$\begin{cases} \dfrac{d}{d\tau} e_n(\tau) + i h_n(\tau) = -j(\tau) g_n \\ \dfrac{d}{d\tau} h_n(\tau) + 2\beta h_n(\tau) + i e_n(\tau) = -\alpha I'_n(\tau) \\ e_n(\tau)\big|_{\tau=0} = 0, \quad h_n(\tau)\big|_{\tau=0} = 0 \end{cases} \tag{13}$$

where $n = 1, 2, \ldots$. To make formulas compact and observable in what follows, introduce a set of notations:

$$\left.\begin{aligned} \tau = k_n ct, \quad \beta_n = \alpha\gamma_n, \; \gamma_n = \frac{1}{s}\int_{S_1} \mathbf{H}_n \mathbf{H}_n^* dS \\ I'_n(\tau) = \sum\nolimits_{n' \neq n}^{\infty} h'_{n'}(\tau)(k_{n'}/k_n)\gamma_{n'n} \\ \gamma_{n'n} = \frac{1}{s}\int_{S_1} \mathbf{H}_{n'} \mathbf{H}_n^* dS. \end{aligned}\right\} \tag{14}$$

5. METHOD OF SUCCESSIVE SUBSTITUTION

To apply the method of successive substitution to problem (13), the modal amplitudes should be presented as consisting of two parts. Each part is sought for. Thus,

$$e_n(\tau) = e'_n(\tau) + e''_n(\tau), \quad h_n(\tau) = h'_n(\tau) + h''_n(\tau). \tag{15}$$

The problem for e'_n and h'_n is selected from (13) as

$$\begin{cases} \dfrac{d}{d\tau} e_n'(\tau) + i h_n'(\tau) = -j(\tau) g_n \\ \dfrac{d}{d\tau} h_n'(\tau) + 2\beta h_n'(\tau) + i e_n'(\tau) = 0 \\ e_n'(\tau)\big|_{\tau=0} = 0, \quad h_n'(\tau)\big|_{\tau=0} = 0. \end{cases} \tag{16}$$

The remainder of original problem (13) yields

$$\begin{cases} \dfrac{d}{d\tau} e_n''(\tau) + i h_n''(\tau) = 0 \\ \dfrac{d}{d\tau} h_n''(\tau) + 2\beta h_n''(\tau) + i e_n''(\tau) = -\alpha I_n'(\tau) \end{cases} \tag{17}$$

Cauchy problem (16) is solved analytically in the next Section. A quick look at problem (17) suggests that the parts e_n'' and h_n'' are of order of the small parameter α.

6. ANALYTICAL SOLUTION FOR FIELD EXPANSION

Introducing matrix Q_n and two vectors, Y_n' and F_n, as

$$Q_n = \begin{pmatrix} 0 & i \\ i & 2\beta_n \end{pmatrix}$$
$$Y_n'(\tau) = \begin{pmatrix} e_n' \\ h_n' \end{pmatrix}, \quad F_n(\tau) = -\begin{pmatrix} j(\tau) g_n \\ 0 \end{pmatrix} \tag{18}$$

rearranges problem (16) into simple "vector" equation as

$$\frac{d}{d\tau} Y_n'(\tau) + Q_n Y_n'(\tau) = F_n(\tau). \tag{19}$$

The method of matrix exponential (Tretyakov et al., 2021) yields solution as

$$Y_n'(\tau) = e^{-\tau Q_n} \int_0^{\tau} e^{\tau' Q_n} F_n(\tau') d\tau' \tag{20}$$

where Lagrange interpolation of $e^{-\tau Q_n}$, see (Tretyakov et al., 2021), results in

$$e^{-\tau Q_n} = e^{-\tau\beta_n}\begin{pmatrix} \dfrac{\cos(\tau\eta_n - \theta_n)}{\cos(\theta_n)} & i\dfrac{\sin(\tau\eta_n)}{\cos(\theta_n)} \\ i\dfrac{\sin(\tau\eta_n)}{\cos(\theta_n)} & \dfrac{\cos(\tau\eta_n + \theta_n)}{\cos(\theta_n)} \end{pmatrix} \tag{21}$$

$$\beta_n = \alpha\gamma_n, \quad \lambda_n = \sqrt{1-\beta_n^2}, \quad \theta_n = \cos^{-1}\lambda_n.$$

Notice that the matrix $e^{-\tau Q_n}$ turns into the *identity matrix* for time $\tau = 0$. Mathematicians call the matrices with this property as the *evolutionary matrices*. At the integrand in (20), the inverse matrix $(e^{-\tau Q_n})^{-1}$ stands. That one is defined as $e^{-\tau Q_n}$ with replacement τ by $-\tau'$ what yields

$$e^{\tau' Q_n} = e^{\tau'\beta_n}\begin{pmatrix} \dfrac{\cos(\tau'\eta_n + \theta_n)}{\cos(\theta_n)} & -i\dfrac{\sin(\tau'\eta_n)}{\cos(\theta_n)} \\ -i\dfrac{\sin(\tau'\eta_n)}{\cos(\theta_n)} & \dfrac{\cos(\tau'\eta_n - \theta_n)}{\cos(\theta_n)} \end{pmatrix}. \tag{22}$$

Calculation of the integrals in (20) results in

$$e^{-\tau\beta_n}\int_0^{\tau} e^{\tau' Q_n} F_n(\tau') = g_n\begin{pmatrix} -A_n \\ iB_n \end{pmatrix} \tag{23}$$

where $e^{-\tau\beta_n}$ is transferred from $e^{-\tau Q_n}$ (see (21)), and

$$\begin{aligned} A_n &= \int_0^{\tau} e^{-(\tau-\tau')\beta_n} j(\tau')\frac{\cos(\tau'\eta_n + Q_n)}{\cos(Q_n)}d\tau' \\ B_n &= \int_0^{\tau} e^{-(\tau-\tau')\beta_n} j(\tau')\frac{\sin(\tau'\eta_n)}{\cos(Q_n)}d\tau'. \end{aligned} \tag{24}$$

Multiplying the matrix $e^{-\tau Q_n}$ (from (21) without $e^{-\tau\beta_n}$) and vector (23) results in a vector as

$$\begin{pmatrix} e_n' \\ h_n' \end{pmatrix} = -g_n \begin{pmatrix} \left[A_n \dfrac{\cos(\tau\eta_n - \theta_n)}{\cos(\theta_n)} + B_n \dfrac{\sin(\tau\eta_n)}{\cos(\theta_n)} \right] \\ i\left[A_n \dfrac{\sin(\tau\eta_n)}{\cos(\theta_n)} + B_n \dfrac{\cos(\tau\eta_n + \theta_n)}{\cos(\theta_n)} \right] \end{pmatrix}. \tag{25}$$

Finally, observation of modal field expansions (11) and vector (25) results in the *analytical* solution as

$$\begin{aligned} \mathbb{E}(\mathbf{r},\tau) &= \sum\nolimits_{n'=1}^{\infty} e_{n'}'(\tau)\tilde{\mathbf{E}}_{n'}(\mathbf{r}) \\ \mathbb{H}(\mathbf{r},\tau) &= \sum\nolimits_{n'=1}^{\infty} h_{n'}'(\tau)\tilde{\mathbf{H}}_{n'}(\mathbf{r}) \end{aligned} \tag{26}$$

where $\tilde{\mathbf{E}}_{n'}(\mathbf{r})$, $\tilde{\mathbf{H}}_{n'}(\mathbf{r})$ are the *real-valued* elements of the basis. Modal field $\mathbf{E}_n$ can be obtained as a *real-valued* vector. Denote that as $\tilde{\mathbf{E}}_n(\mathbf{r})$. The modal field $\tilde{\mathbf{H}}_n$ is specified via $\tilde{\mathbf{E}}_n(\mathbf{r})$ by formula $\nabla \times \tilde{\mathbf{E}}_n = ik_n\mathbf{H}_n$ what yields $\mathbf{H}_n = (-i)\nabla \times \tilde{\mathbf{E}}_n / k_n = (-i)\tilde{\mathbf{H}}_n$ where $\tilde{\mathbf{H}}_n$ is real-valued, also. This $(-i)$ cancels later that i, which is present in h_n' in (25). $e_{n'}'$ and $h_{n'}'$ are the *real-valued* amplitudes as

$$\begin{aligned} e_n'(\tau) &= -g_n \left[A_n \frac{\cos(\tau\eta_n - \theta_n)}{\cos(\theta_n)} + B_n \frac{\sin(\tau\eta_n)}{\cos(\theta_n)} \right] \\ h_n'(\tau) &= -g_n \left[A_n \frac{\sin(\tau\eta_n)}{\cos(\theta_n)} + B_n \frac{\cos(\tau\eta_n + \theta_n)}{\cos(\theta_n)} \right]. \end{aligned} \tag{27}$$

The graphical results for the modal amplitudes of the electromagnetic oscillations, $e_n'(\tau)$ and $h_n'(\tau)$, caused by a rectangular pulse, $j(\tau')$, in a cavity with lossless surfaces, $\beta_n = 0$, and lossy surfaces, $\beta_n = 0.2$, are exhibited below in Figure 1 and Figure 2, respectively. The dimensionless time, τ; is specified as $\tau = tck_n$ where c is the light speed.

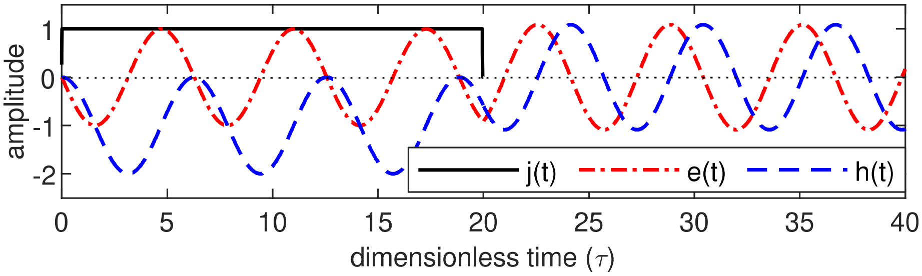

Figure 1. Modal amplitudes for the lossless case: $\beta_n = 0, \quad t \geq 0$.

In Figure 1, electric and magnetic fields' modal amplitudes, $e_n'(\tau)$ and $h_n'(\tau)$, excited by a rectangular pulse, $j_n(\tau)$, can be seen evolving sinusoidally. It can also be seen $\pi/2$ phase shift between electric field and magnetic fields.

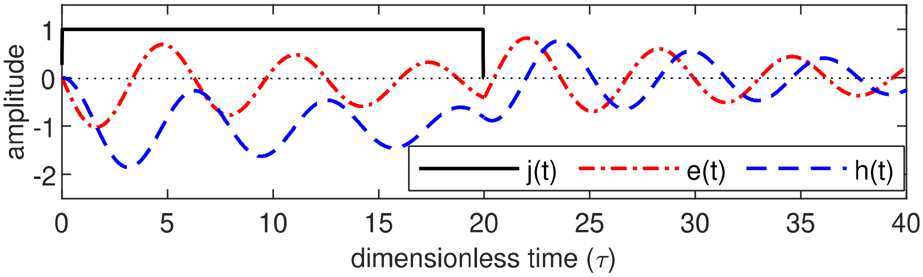

Figure 2. Modal amplitudes for the lossy case: $\beta_n = 0.2, \quad t \geq 0$.

In Figure 2, decaying in time sinusoidal oscillations can be seen due to lossy walls of the cavity. When studying digital signals, duration of this rectangular pulses will be very short.

7. CONCLUSION

The solution given in (26)-(27) satisfies the initial conditions at $\tau = 0$ automatically. The solution is *casual*. Physically, this solution exhibits how the amplitudes of the modes are *evolving* from their initial value (at $\tau = 0$) to the state of observation τ.

The solution is analytical and “pliable” with respect to variations of the given signal, $j(t)$, which participates in the formulas for A_n and B_n in (24).

There are three important cases in choice of the format of the cavity surface S: see (4). 1) If $S_1 = 0$, all the cavity surface S is perfectly electric conducting where boundary conditions (6) hold. 2) If $S_2 = 0$, all the cavity surface S is lossy, over which Leontovich boundary condition (5) holds. The third case, when $S_1 \neq 0$ and $S_2 \neq 0$, but $S_1 + S_2 = S$, is considered herein.

8. DISCUSSION

In the novel simple SI format of Maxwell’s equations, and also in the novel format of Leontovich boundary condition, the new electric, $\mathbb{E}(\mathbf{r},t)$, and magnetic, $\mathbb{H}(\mathbf{r},t)$, field vectors; have *a common physical dimension*, as opposed to the standard electric, $\mathcal{E}(\mathbf{r},t)$, and magnetic field, $\mathcal{H}(\mathbf{r},t)$, which have the distinct ones. *Just this property of the new fields* permits one to denote the mechanical equivalents (mass and mechanical momentum) of the energetic field characteristics of the local fields in free space, cavities, and in waveguides. This result may be useful for study of the unsolved as yet problems (Erden et al., 2018) in *radio frequency resonant cavity thruster*, i.e., EmDrive.

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