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Some Weaker Forms of Continuous and Irresolute Mappings in Nano Ideal Topological Spaces

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Abstract - The aim of this paper is to define and study certain new classes of continuous and irresolute mappings namely nano α-I-continuous, nano semi I-continuous, nano pre I-continuous, nano α-I-irresolute, nano semi I-irresolute and nano pre I-irresolute mappings in nano ideal topological spaces and we discuss some of its properties. All these concepts will be helpful for further generalizations of nano continuous mappings in nano ideal topological spaces. Finally we obtain a decomposition of nano α-I-continuous mapping.

Keywords *nano pre I-continuous map, nano semi I-continuous map, nano α-I-continuous map, nano I-irresolute map, nano ideal topological spaces.*
2010 Subject classification: *54A05, 54A10, 54B05*

1 Introduction

The concept of continuity is fundamental in large part of contemporary mathematics. In the nineteenth century, definitions of continuity were formulated for functions of real or complex variables. In the early of twentieth century, the concept of continuity was generalized so as to applicable to functions between topological spaces. Over the year, many generalizations of continuous mapping have been introduced and studied. Levine (1963), Mashhour et al.(1982) and Mashhour et al.(1983) have introduced the notion of semi-continuity, pre-continuity and α-continuity respectively.

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In 1972, the concept of irresolute functions were introduced and studied in Crossely and Hildebrand (1972). He proved that irresolute functions are stronger than semi continuous but are independent of continuous functions. In Hamlett and Jankovic (1990), the notion of ideal topological spaces was introduced . Weak forms of continuity via idealization have been introduced and studied in the papers of Dontchev (1999), Hatir and Noiri(2002). Study of irresolute functions in ideal topological spaces was extended by several authors in Acikgoz et al.(2005), Hatir and Noiri (2009).

Recently, Thivagar and Richard (2013a) established the field of nano topological spaces. In 2016, Thivagar and Devi (2016) introduced the notion of nano local functions and explore the field of nano topological spaces. Quite recently, Parimala and Jafari (2018) have introduced the notion of nano I-continuous mapping in nano ideal topological spaces.

In this paper we introduce and study the new classes of continuous and irresolute mappings namely nano α-I-continuous, nano semi I-continuous, nano pre I-continuous, nano α-I-irresolute, nano semi I-irresolute and nano pre I-irresolute mappings in nano ideal topological spaces and we discuss some of its properties. Finally we obtain a decomposition of nano α-I-continuity using nano pre I-continuity and nano semi I-continuity.

2 Preliminaries

Definition 2.1. *(Thivagar and Richard, 2013a) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let* $X \subseteq U$.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by* $L_R(X)$. *That is,* $L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$, *where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by* $U_R(X)$. *That is,* $U_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}$

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by* $B_R(X)$. *That is,* $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. *(Thivagar and Richard, 2013a) Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then* $\tau_R(X)$ *satisfies the following axioms:*

1. ϕ *and U are in* $\tau_R(X)$.

2. *The union of the elements of any subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

3. *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets.

Definition 2.3. *(Dontchev, 1999) Let $(U, \tau_R(X))$ be a nano topological space, the set $B = \{\phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.4. *(Thivagar and Richard, 2013a) If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then*
(1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A).
(2) The nano closure of the set A is defined as the intersection of all nano closed subsets containing A and is denoted by NCl(A).

Definition 2.5. *(Thivagar and Richard, 2013a) Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*
(i) nano α-open if $A \subseteq NInt(NCl(NInt(A)))$
(ii) nano pre open if $A \subseteq NInt(NCl(A))$
(iii) nano semi open if $A \subseteq NCl(NInt(A))$

Definition 2.6. *(Thivagar and Richard, 2013a) A subset A of a nano topological space $(U, \tau_R(X))$ is said to be nano α-closed (resp. nano semi closed, nano pre closed), if its complement is nano α-open (resp. nano semi open,nano pre open).*

Definition 2.7. *(Thivagar and Richard, 2013b) Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. A mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano continuous if the inverse image of every nano open set in $(V, \tau_{R'}(Y))$ is nano open set in $(U, \tau_R(X))$.*

Definition 2.8. *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. A mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be*
(i). nano α-continuous (Thivagar et al., 2017) if $f^{-1}(B)$ is nano α-open set in $(U, \tau_R(X))$ for every nano open set B in $(V, \tau_{R'}(Y))$.
(ii). nano semi continuous (Sathishmohan, 2017) if $f^{-1}(B)$ is nano semi open set in $(U, \tau_R(X))$ for every nano open set B in $(V, \tau_{R'}(Y))$.
(iii). nano pre continuous (Thivagar et al., 2017) if $f^{-1}(B)$ is nano pre open set in $(U, \tau_R(X))$ for every nano open set B in $(V, \tau_{R'}(Y))$.

Definition 2.9. *(Hamlett and Jankovic 1990) An ideal I on a topological space is a non-empty collection of subsets of X which satisfies*
1. $A \in I$ and $B \subseteq A$ implies $B \in I$.
2. $A \in I$ and $B \in I$ implies $A \cup B \in I$.

Definition 2.10. *(Thivagar and Devi, 2016) A nano topological space $(U, \tau_R(X))$ with an ideal I on U is called a nano ideal topological space and denoted as $(U, \tau_R(X), I)$.*

Definition 2.11. *(Thivagar and Devi, 2016) Let $(U, \tau_R(X), I)$ be a nano ideal topological space. A set operator $(A)^{*N} : P(U) \to P(U)$ is called the nano local function of I on U with respect to I on $\tau_R(X)$ is defined as $(A)^{*N} = \{x \in U : U \cap A \notin I \ ; \text{for every } U \in \tau_R(X)\}$ and is denoted by $(A)^{*N}$, where nano closure operator is defined as $NCl^*(A) = A \cup (A)^{*N}$.*

Definition 2.12. *(Parimala and Jafari, 2018) A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be nano I-open if $A \subseteq NInt((A)^{*N})$.*

Definition 2.13. *(Parimala and Jafari, 2018) A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be nano I-closed if its complement is nano I-open.*

Definition 2.14. *A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be*
(i) nano α-I - open (Thivagar and Devi, 2016) if $A \subseteq NInt[NCl^*[NInt(A)]]$.
(ii) nano semi I - open (Thivagar and Devi, 2016) if $A \subseteq NCl^*[NInt(A)]$.
(iii) nano pre I-open (Inthumathi et al. 2018) if $A \subseteq NInt[NCl^*(A)]$.
The family of all nano α-I-open (resp., nano semi I-open, nano pre I-open) sets of a nano ideal topological space is denoted by $N\alpha IO(U, X)$ (resp., $NSIO(U, X), NPIO(U, X)$.
A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be nano α-I-closed (resp.nano semi I-closed, nano pre I-closed), if its complement is nano α-I-open (resp. nano semi I-open).

Definition 2.15. *(Parimala and Jafari, 2018) A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called nano I-continuous if $f^{-1}(V)$ is nano I-open set in $(U, \tau_R(X), I)$ for every nano open set V in $(V, \tau_{R'}(Y))$.*

3 Some weaker forms of nano I-continuity

Definition 3.1. *A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is said to be nano α-I-continuous (resp. nano semi-I-continuous and nano pre-I-continuous) if $f^{-1}(B)$ is nano α-I-open (resp. nano semi I-open and nano pre I-open) set in $(U, \tau_R(X), I)$ for every nano open set B in $(V, \tau_{R'}(Y))$.*

Theorem 3.2. *Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ be a nano continuous mapping. Then the following statements are holds.*

1. *f is nano α-I-continuous*
2. *f is nano semi I-continuous*
3. *f is nano pre I-continuous.*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ be a nano continuous mapping and B be a nano open set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(B)$ is nano open set in $(U, \tau_R(X), I)$.
(1). Since every nano open set is nano α-I-open, $f^{-1}(B)$ is nano α-I-open in $(U, \tau_R(X), I)$. Thus f is nano α-I-continuous.
(2). Since every nano open set is nano semi-I-open, $f^{-1}(B)$ is nano semi-I-open in $(U, \tau_R(X), I)$. Thus f is nano semi I-continuous.
(3). Since every nano open set is nano pre-I-open, $f^{-1}(B)$ is nano pre-I-open in $(U, \tau_R(X), I)$. Thus f is nano pre I-continuous. □

Theorem 3.3. *In a nano ideal topological space $(U, \tau_R(X), I)$, the following are holds.*
(i). Every nano α-I-continuous mapping is nano semi I-continuous.
(ii). every nano α-I-continuous mapping is nano pre I-continuous

Proof. The proof follows from the fact that every nano α-I-open set is nano semi I-open and nano pre I-open. □

Remark 3.4. *The converse of Theorems 3.2 and 3.3 are need not be true as shown in the following examples.*

Example 3.5. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{a, d\} \subseteq U$, $U/R = \{\{a, d\}\,\{b\}\,\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a, d\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c\}$, $Y = \{a, c, d\} \subseteq V$, $V/R' = \{\{b\}, \{a, c\}, \{d\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a, c, d\}\}$. *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = a,\ f(b) = b,\ f(c) = c,\ f(d) = d$.
Then, $N\alpha IO(U, X) = \{\phi, \{a, d\}, \{a, b, d\}, \{a, c, d\}, U\}$. *We note that,* $f^{-1}(\{a, c, d\}) = \{a, c, d\} \in N\alpha IO(U, X)$ *but* $\{a, c, d\} \notin \tau_R(X)$. *Hence,* f *is nano* α*-I-continuous but not nano continuous.*

Example 3.6. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{a, b\} \subseteq U$, $U/R = \{\{a\}\,\{b, d\}\,\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c\}$, $Y = \{a\} \subseteq V$, $V/R' = \{\{a\}, \{b, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a\}\}$. *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = f(b) = f(c) = a,\ f(d) = c$. *Then* f *is nano pre I-continuous but neither nano* α*-I-continuous nor nano continuous.*

Example 3.7. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{a, b\} \subseteq U$, $U/R = \{\{a, d\}\,\{b\}\,\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c\}$, $Y = \{a\} \subseteq V$, $V/R' = \{\{a\}, \{b, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a\}\}$. *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = c,\ f(b) = f(c) = f(d) = a$. *Then* f *is nano semi I-continuous but neither nano* α*-I-continuous nor nano continuous.*

Theorem 3.8. *Let* f *be a nano* α*-I-continuous mapping in a nano ideal topological space* $(U, \tau_R(X), I)$ *. Then the following are holds.*

1. f *is nano* α*-continuous.*
2. f *is nano semi-continuous.*
3. f *is nano pre-continuous.*

Proof. The proof is immediately follows from the fact that every nano α-I-open is nano α-open, nano semi open and nano pre open. □

However the converse not true as shown in the following examples.

Example 3.9. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{a\} \subseteq U$, $U/R = \{\{a\}\,\{b, d\}\,\{c\}\}$, *and* $\tau_R(X) = \{\phi, U, \{a\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c\}$, $Y = \{a\} \subseteq V$, $V/R' = \{\{a\}, \{b, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a\}\}$. *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *as* $f(a) = f(b) = a,\ f(c) = c,\ f(d) = b$. *Then* f *is nano* α*-continuous but not nano* α*-I-continuous.*

Example 3.10. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{a, b\} \subseteq U$, $U/R = \{\{a\}\,\{b, d\}\,\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c\}$, $Y = \{a\} \subseteq V$, $V/R' = \{\{a\}, \{b, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a\}\}$. *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = f(b) = f(c) = a,\ f(d) = b$. *Then* f *is nano pre continuous but not nano* α*-I-continuous.*

Example 3.11. *Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subseteq U$, $U/R = \{\{a\}\{b, d\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ and the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c\}$, $Y = \{a\} \subseteq V$, $V/R' = \{\{a\}, \{b, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a\}\}$. Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ by $f(a) = b$, $f(b) = f(c) = f(d) = a$. Then f is nano semi continuous but not nano α-I-continuous.*

Theorem 3.12. *Let $(U, \tau_R(X), I)$ be a nano ideal topological spaces. Then every nano I-continuous mapping is nano pre I-continuous.*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ be a nano I-continuous mapping and B be a nano open set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(B)$ is nano I-open set in $(U, \tau_R(X), I)$ for every nano open set B in $(V, \tau_{R'}(Y))$. Since every nano I-open set is nano pre-I-open, $f^{-1}(B)$ is nano pre-I-open in $(U, \tau_R(X), I)$. Thus f is nano pre I-continuous. □

The converse need not be true as shown in the following example.

Example 3.13. *Let $U = \{a, b, c, d\}$ be the universe, $X = \{b, d\} \subseteq U$, $U/R = \{\{a, d\}\{b\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a, d\}, \{b\}, \{a, b, d\}\}$ and the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c\}$, $Y = \{b\} \subseteq V$, $V/R' = \{\{b\}, \{a, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{b\}\}$. Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ by $f(a) = a$, $f(b) = b$, $f(c) = f(d) = c$. Then f is nano pre I-continuous but not nano I-continuous.*

Theorem 3.14. *In a nano ideal topological space $(U, \tau_R(X), I)$, the following are holds.*
(i). Every nano pre I-continuous mapping is nano pre-continuous
(ii). Every nano semi I-continuous mapping is nano semi-continuous

Proof. The proof follows from the respective facts
(i). Every nano pre-I-open set is nano pre open.
(ii). Every nano semi I-open set is nano semi open. □

The converse need not be true as shown in the following examples.

Example 3.15. *Let $U = \{a, b, c, d\}$ be the universe, $X = \{b, d\} \subseteq U$, $U/R = \{\{a, d\}\{b\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a, d\}, \{b\}, \{a, b, d\}\}$ and the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c, d\}$, $Y = \{a, b\} \subseteq V$, $V/R' = \{\{a\}, \{b\}, \{c, d\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a, b\}\}$. Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. Then f is nano pre-continuous but not nano pre I-continuous.*

Example 3.16. *Let $U = \{a, b, c\}$ be the universe, $X = \{a, b\} \subseteq U$, $U/R = \{\{a\}\{b, c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c\}$, $Y = \{a, c\} \subseteq V$, $V/R' = \{\{b\}, \{a, c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a, c\}\}$. Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ by $f(a) = a$, $f(b) = b$ $f(c) = c$. Then f is nano semi-continuous but not nano semi I-continuous.*

Corollary 3.17. *Let $(U, \tau_R(X), I)$ be a nano ideal topological space. Then every nano I-continuous mapping is nano pre-continuous.*

Theorem 3.18. *In a nano ideal topological space, the composition of nano α-I-continuous and nano continuous mapping is nano α-I-continuous.*

Proof. Assume that $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano α-I-continuous mapping and $g : (V, \tau_{R'}(Y), J) \to (W, \tau_{R''}(Z))$ is a nano continuous mapping. Let B be a nano open set in $(W, \tau_{R''}(Z))$. Then $g^{-1}(B)$ is a nano open set in $(V, \tau_{R'}(Y), J)$ for every nano open set B in $(W, \tau_{R''}(Z))$ Since f is nano α-I-continuous, $f^{-1}(g^{-1}(B))$ is nano α-I-open in $(U, \tau_R(X), I)$. i.e., $(g \circ f)^{-1}(B)$ is nano α-I-open set. Hence $(g \circ f)$ is a nano α-I-continuous mapping. □

Theorem 3.19. *Let $(U, \tau_R(X), I)$ be a nano ideal topological space. Then the composition of nano semi I-continuous and nano continuous mapping is nano semi I-continuous.*

Proof. The proof is similar to the proof of Theorem 3.18. □

Theorem 3.20. *Let $(U, \tau_R(X), I)$ be a nano ideal topological space. Then the composition of nano pre I-continuous and nano continuous mapping is nano pre I-continuous.*

Proof. The proof is similar to the proof of Theorem 3.18. □

Theorem 3.21. *If $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is both nano continuous and nano pre I-continuous mapping then f is also nano α-I-continuous mapping.*

Proof. Assume that $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is both nano continuous and nano pre I-continuous mapping. Let A be a nano open set in $(V, \tau_{R'}(Y))$. Now, f is continuous then $f^{-1}(A)$ is nano open in $(U, \tau_R(X), I)$.
Also, f is nano pre I-continuous, then $f^{-1}(A)$ is nano pre I-open in $(U, \tau_R(X), I)$. Since $f^{-1}(A)$ is both nano open and nano pre I-open, $f^{-1}(A)$ is nano α-I-open. Thus $f^{-1}(A)$ is nano α-I-open in $(U, \tau_R(X), I)$ for every nano open set A in $(V, \tau_{R'}(Y))$. Hence f is nano α-I-continuous. □

Theorem 3.22. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is nano α-I-continuous if and only if the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is nano α-I-closed set in $(U, \tau_R(X), I)$.*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ be a nano α-I-continuous mapping and let A be a nano closed set in $(V, \tau_{R'}(Y))$. Then $V - A$ is nano open set in $(V, \tau_{R'}(Y))$. Since f is nano α-I-continuous, $f^{-1}(V - A)$ is nano α-I-open in $(U, \tau_R(X), I)$. i.e., $U - f^{-1}(A)$ is nano α-I-open in $(U, \tau_R(X), I)$. Therefore $f^{-1}(A)$ is nano α-I-closed in $(U, \tau_R(X), I)$.
Conversely, suppose that the inverse image of every nano closed set in $(V, \tau_{R'}(Y), J)$ is nano α- I-closed set in $(U, \tau_R(X), I)$. Let B be a nano open set in $(V, \tau_{R'}(Y))$. Then $V - B$ is nano closed set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(V - B)$ is nano α-I-closed in $(U, \tau_R(X), I)$. i.e., $U - f^{-1}(B)$ is nano α-I-closed in $(U, \tau_R(X), I)$. Therefore $f^{-1}(B)$ is nano α-I-open in $(U, \tau_R(X), I)$. Hence f is nano α-I-continuous. □

Theorem 3.23. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is nano semi I-continuous if and only if the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is nano semi I-closed set in $(U, \tau_R(X), I)$.*

Proof. The Proof is similar to the proof of Theorem 3.22. □

Theorem 3.24. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is nano pre I-continuous if and only if the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is nano pre I-closed set in $(U, \tau_R(X), I)$.*

Proof. The Proof is similar to the proof of Theorem 3.22. □

Remark 3.25. *In a nano ideal topological space, the notion of nano semi I-continuous and nano pre I-continuous are independent.*

Example 3.26. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{b, d\} \subseteq U$, $U/R = \{\{a, d\}\{b\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a, d\}, \{b\}, \{a, b, d\}\}$ *and the ideal* $I = \{\phi, \{a\}\}$ *and let* $V = \{a, b, c, d\}$, $Y = \{a, b, d\} \subseteq V$, $V/R' = \{\{a\}, \{b, d\}, \{c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a, b, d\}\}$.
(1) *Define* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = a$, $f(b) = c$, $f(c) = b$, $f(d) = d$. *Then* f *is nano semi I-continuous but not nano pre I-continuous.*
(2) *Define* $g : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $g(a) = c$, $g(b) = b$, $g(c) = a$, $g(d) = d$. *Then* f *is nano pre I-continuous but not nano semi I-continuous.*

Remark 3.27. *In a nano ideal topological space* $(U, \tau_R(X), I)$, *the notion of nano semi continuous and nano pre continuous are independent.*

Example 3.28. *Let* $U = \{a, b, c, d\}$ *be the universe,* $X = \{b, d\} \subseteq U$, $U/R = \{\{a, d\}\{b\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a, d\}, \{b\}, \{a, b, d\}\}$ *and let V={a,b,c,d} be the universe,* $Y = \{c, d\}$, $V/R' = \{\{a\}, \{b\}, \{c, d\}\}$, $\tau_{R'}(Y) = \{\phi, U, \{c, d\}\}$.
(i). Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *by* $f(a) = d$, $f(b) = f(c) = c$, $f(d) = b$. *Then* f *is nano pre continuous but not nano semi continuous.*
(ii). Define $g : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ *by* $g(a) = g(c) = c$, $g(b) = b$, $g(d) = d$. *Then* g *is nano semi continuous but not nano pre continuous.*

Remark 3.29. *From the above results and from the Theorem 4.7 in Thivagar et al. (2015),we have the implications.*

Theorem 3.30. *Let* A *be a subset of a nano ideal topological space. Then* f *is nano* α*-I-continuous if and only if it is both nano semi I-continuous and nano pre I-continuous.*

Proof. Necessary part is evident from the fact that, every nano α-I-continuous is nano semi I-continuous and nano pre I-continuous.
Conversely, assume $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is both nano semi I-continuous and nano pre I-continuous. Let A be a nano open set in $(V, \tau_{R'}(Y))$. Then $f^{-1}(A)$ is both nano semi I-open and nano pre I-open set in $(U, \tau_R(X), I)$, $f^{-1}(A)$ is nano α-I-open set in $(U, \tau_R(X), I)$. Hence f is nano α-I-continuous. □

4 Nano I-Irresolute Mappings

Definition 4.1. *A mapping* $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ *is said to be nano* α*-I-irresolute (resp. nano semi I-irresolute and nano pre I -irresolute) if* $f^{-1}(B)$ *is nano* α*-I-open (resp. nano semi I-open and nano pre I-open) set in* $(U, \tau_R(X), I)$ *for every nano* α*-I-open (resp. nano semi I-open and nano pre I-open) set* B *in* $(V, \tau_{R'}(Y), J)$.

Theorem 4.2. *In a nano ideal topological space* $(U, \tau_R(X), I)$, *the followings are holds.*

1. *Every nano* α*-I-irresolute mapping is nano* α*-I-continuous.*

2. *Every nano semi-I-irresolute mapping is nano semi-I-continuous.*

3. *Every nano pre-I-irresolute mapping is nano pre-I-continuous.*

Proof. (1). Let$f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ be a nano α-I-irresolute mapping and let B be a nano open set in $(V, \tau_{R'}(Y), J)$.
Since every nano open set is nano α-I-open, we have B is nano α-I-open in $(V, \tau_{R'}(Y), J)$. By our assumption $f^{-1}(B)$ is nano α-I-open in $(U, \tau_R(X), I)$. Thus f is nano α-I-continuous.
The proof of (2) and (3) are similar to proof of (1). □

Example 4.3. *Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subseteq U$, $U/R = \{\{a\}\{b, d\}\{c\}\}$, $\tau_R(X) = \{\phi, U, \{a\}, \{b, d\}, \{a, b, d\}\}$ and the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c, d\}$, $Y = \{a, d\} \subseteq V$, $V/R' = \{\{a, d\}, \{b\}, \{c\}\}$, $\tau_{R'}(Y) = \{\phi, V, \{a, d\}\}$. Define $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$. Then f is nano α-I-continuous, nano semi I-continuous and nano pre I-continuous but not nano α-I-irresolute, nano semi I-irresolute and nano pre I-irresolute.*

Theorem 4.4. *In a nano ideal topological space, the composition of nano α-I-irresolute and nano α-I-continuous mapping is nano α-I-continuous.*

Proof. Assume that $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is a nano α-I-irresolute mapping and $g : (V, \tau_{R'}(Y), J) \to (W, \tau_{R''}(Z))$ is a nano α-I- continuous mapping. Let B be a nano open set in $(W, \tau_{R''}(Z))$. Then $g^{-1}(B)$ is a nano α-I- open set in $(V, \tau_{R'}(Y), J)$. Since f is nano α-I-irresolute, $f^{-1}(g^{-1}(B))$ is nano α-I-open in $(U, \tau_R(X), I)$. i.e., $(g \circ f)^{-1}(B)$ is a nano α-I-open set.Hence $(g \circ f)$ is a nano α-I-continuous mapping. □

Theorem 4.5. *In a nano ideal topological spaces, the composition of nano semi I-irresolute and nano semi I- continuous mapping is also nano semi I-continuous.*

Proof. The proof is similar to the proof of the Theorem 4.4. □

Theorem 4.6. *In a nano ideal topological spaces, the composition of nano pre I-irresolute and nano pre I-continuous mapping is also nano pre I-continuous.*

Proof. The proof is similar to the proof of the Theorem 4.4. □

Theorem 4.7. *A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano α-I-irresolute if and only if $N\alpha ICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ be a nano α-I-irresolute mapping and $B \subseteq V$, then $N\alpha ICl(B)$ is a nano α-I-closed set in $(V, \tau_{R'}(Y), J)$ and hence $f^{-1}(N\alpha ICl(B))$ is nano α-I-closed in $(U, \tau_R(X), I)$. Therefore, $N\alpha ICl(f^{-1}(N\alpha ICl(B))) = f^{-1}(N\alpha ICl(B))$. Since $B \subseteq (N\alpha ICl(B))$, $f^{-1}(B) \subseteq f^{-1}(N\alpha ICl(B))$.
Therefore $N\alpha ICl(f^{-1}(B)) \subseteq N\alpha ICl(f^{-1}(N\alpha ICl(B))) = f^{-1}(N\alpha ICl(B))$.
Conversely, $N\alpha ICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$ for every subset $B \subseteq V$.
Let B be a nano α-I-closed set in $(V, \tau_{R'}(Y), J)$. Then $N\alpha ICl(B) = B$. By assumption, $N\alpha ICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B)) = f^{-1}(B)$. i.e.,$N\alpha IClf^{-1}(B) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq N\alpha ICl(f^{-1}(B))$ hence $N\alpha ICl(f^{-1}(B)) = f^{-1}(B)$, $f^{-1}(B)$ is nano α-I-closed in $(U, \tau_R(X), I)$ for every nano α-I-closed set B in V. Thus f is nano α-I-irresolute on $(U, \tau_R(X), I)$. □

Theorem 4.8. *A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano semi I-irresolute if and only if $NSICl(f^{-1}(B)) \subseteq f^{-1}(NSICl(B))$.*

Proof. The proof is similar to the proof of the Theorem 4.7. □

Theorem 4.9. *A mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano pre I-irresolute if and only if $NPICl(f^{-1}(B)) \subseteq f^{-1}(NPICl(B))$.*

Proof. The proof is similar to the proof of the Theorem 4.7. □

Theorem 4.10. *If the mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano semi I-irresolute then $NSICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$.*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano semi I-irresolute mapping and let B be any set in $(V, \tau_{R'}(Y), J)$. Now, $N\alpha ICl(B)$ is a nano α-I-closed set in $(V, \tau_{R'}(Y), J)$. Since every nano α-I-closed set is nano semi I-closed in $(V, \tau_{R'}(Y), J)$ and f is nano semi I-irresolute we have $f^{-1}(N\alpha ICl(B))$ is nano semi I-closed in $(U, \tau_R(X), I)$. *i.e.,* $NSICl[f^{-1}(N\alpha ICl(B))] = f^{-1}(N\alpha ICl(B))$ we have, $NSICl(f^{-1}(B)) \subseteq NSICl[f^{-1}(N\alpha ICl(B))] = f^{-1}(N\alpha ICl(B))$. Thus $NSICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$. □

Theorem 4.11. *If the mapping $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano pre I-irresolute then $NPICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano pre I-irresolute mapping and let B be any set in $(V, \tau_{R'}(Y), J)$. Now , $N\alpha ICl(B)$ is a nano α-I-closed set in$(V, \tau_{R'}(Y), J)$. Since every nano α-I-closed set is pre I-closed in $(V, \tau_{R'}(Y), J)$ and f is nano pre I-irresolute then $f^{-1}(N\alpha ICl(B))$ is nano pre I-closed in $(U, \tau_R(X), I)$. *i.e.,* $NPICl[f^{-1}(N\alpha ICl(B))] = f^{-1}(N\alpha ICl(B))$. we have, $NPICl(f^{-1}(B)) \subseteq NPICl[f^{-1}(N\alpha ICl(B))] = f^{-1}(N\alpha ICl(B))$. Thus $NPICl(f^{-1}(B)) \subseteq f^{-1}(N\alpha ICl(B))$. □

Theorem 4.12. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano α-I-irresolute if and only if the inverse image of every nano α-I-closed set in $(V, \tau_{R'}(Y), J)$ is nano α-I-closed set in $(U, \tau_R(X), I)$.*

Proof. Let $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ be a nano α-I-irresolute mapping and let A be a nano α-I-closed set in $(V, \tau_{R'}(Y), J)$. Then $V - A$ is nano α-I-open set in $(V, \tau_{R'}(Y), J)$. Since f is nano α-I-irresolute, $f^{-1}(V - A)$ is nano α-I-open in $(U, \tau_R(X), I)$. i.e., $U - f^{-1}(A)$ is nano α-I-open in $(U, \tau_R(X), I)$. Therefore $f^{-1}(A)$ is nano α-I-closed in $(U, \tau_R(X), I)$. Conversely, suppose that the inverse image of every nano closed set in $(V, \tau_{R'}(Y), J)$ is nano α-I-closed set in $(U, \tau_R(X), I)$. Let B be a nano open set in $(V, \tau_{R'}(Y), J)$. Then $V - B$ is nano closed set in $(V, \tau_{R'}(Y), J)$. Then $f^{-1}(V - B)$ is nano α-I-closed in $(U, \tau_R(X), I)$.
i.e., $U - f^{-1}(B)$ is nano α-I-closed in $(U, \tau_R(X), I)$. Therefore $f^{-1}(B)$ is nano α-I-open in $(U, \tau_R(X), I)$. Hence f is nano α-I-irresolute. □

Theorem 4.13. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano semi I-irresolute if and only if the inverse image of every nano semi I-closed set in $(V, \tau_{R'}(Y), J)$ is nano semi I-closed set in $(U, \tau_R(X), I)$.*

Proof. Proof is similar to proof of the Theorem 4.12 □

Theorem 4.14. *A function $f : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is nano pre I-irresolute if and only if the inverse image of every nano pre I-closed set in $(V, \tau_{R'}(Y), J)$ is nano pre I-closed set in $(U, \tau_R(X), I)$.*

Proof. Proof is similar to proof of the Theorem 4.12. □

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