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Comparison of Recent Meta-Heuristic Optimization Algorithms Using Different Benchmark Functions

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Abstract

Meta-heuristic optimization algorithms are used in many application areas to solve optimization problems. In recent years, meta-heuristic optimization algorithms have gained importance over deterministic search algorithms in solving optimization problems. However, none of the techniques are equally effective in solving all optimization problems. Therefore, researchers have focused on either improving current meta-heuristic optimization techniques or developing new ones. Many alternative meta-heuristic algorithms inspired by nature have been developed to solve complex optimization problems. It is important to compare the performances of the developed algorithms through statistical analysis and determine the better algorithm. This paper compares the performances of sixteen meta-heuristic optimization algorithms (AWDA, MAO, TSA, TSO, ESMA, DOA, LHHO, DSSA, LSMA, AOSMA, AGWOCS, CDDO, GEO, BES, LFD, HHO) presented in the literature between 2021 and 2022. In this context, various test functions, including single-mode, multi-mode, and fixed-size multi-mode benchmark functions, were used to evaluate the efficiency of the algorithms used.

1. Introduction

Optimization has received more attention in recent years, and various new optimization methods have been developed [1–15].These newly discovered techniques are applied to real-world challenges. An optimization problem is about finding the optimal answer from a collection of possible solutions. The main goal of optimization is to find selection variables that lead to the minimization or maximization of an objective function. These problems are classified as constrained or unconstrained, discrete or continuous, static or dynamic, and single-or multi-objective. Most real-world problems are nonlinear, incur significant computational costs, and have many complicated solution spaces. For this reason, several researchers have proposed optimization techniques to solve these problems, often referred to as mathematical programming approaches or meta-heuristic methods. Therefore, solving problems with a large number of variables and constraints is very challenging. Since most traditional optimization techniques are based on classical mathematical and probabilistic assumptions, they are not able to provide useful answers to the increasingly complicated optimization problems of recent years. Often, basic optimization problems can be effectively solved using traditional optimization approaches such as mathematical programming. However, solving real-world engineering optimization problems using classical optimization methods is very difficult. Therefore, several researchers [16–24] have proposed novel solution strategies, called meta-heuristic algorithms, to solve difficult optimization problems within reasonable time and cost. Most conventional optimization approaches are based on classical mathematics and probabilistic assumptions that cannot provide useful answers to emerging, complicated optimization problems. Meta-heuristics, which have gained popularity among researchers due to their numerous advantages over conventional optimization strategies, have a number of advantages over conventional optimization strategies, including their simplicity, non-differentiation, adaptability, and avoidance of local optima [25]. The main advantage of these techniques over conventional optimization methods is that they are able to solve optimization problems without requiring gradient information. Moreover, they can be adapted to a variety of working situations. The efficiency and effectiveness of meta-heuristic optimization algorithms in addressing known constrained mathematical and engineering design problems is one of their main advantages. Evolutionary algorithms, physics-based algorithms, swarm intelligence algorithms, and human-based algorithms are the four types of meta-heuristic algorithms [26,27]. Table 1.1 shows some of the optimization algorithms presented in the literature between 2021 and 2022 that were investigated in this study.

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Algorithms	Year
Artificial Water Drop Algorithm (AWDA) [28]	2022
Mexican Axolotl Optimization (MAO) [3]	2022
Tunicate Swarm Algorithm (TSA) [2]	2022
Tuna Swarm Optimization (TSO) [1, 8]	2022
Equilibrium Slime Mould Algorithm (ESMA) [8, 26]	2021
Dingo Optimization Algorithm (DOA) [29]	2021
Leader Harris hawks optimization (LHHO) [5, 30]	2021
Differential Squirrel Search Algorithm (DSSA) [9, 10]	2021
Leader Slime Mould Algorithm (LSMA) [31]	2021
Adaptive Opposition Slime Mould Algorithm (AOSMA) [32]	2021
Hybrid Augmented Grey Wolf Optimizer & Cuckoo Search (AGWOCS) [33, 34]	2021
Child Drawing Development Optimization Algorithm (CDDO) [34]	2021
Golden Eagle Optimizer (GEO) [35]	2021
Bald eagle search Optimization algorithm (BES) [14]	2021
Lévy Flight Distribution (LFD) [6]	2021
Harris hawks optimization (HHO) [36]	2021

Table 1.1: Optimization algorithms

In this study, the performance of some meta-heuristic algorithms, listed in Table 1.1, was evaluated using a series of test functions. These are meta-heuristic algorithms inspired by the behavior of natural organisms. Meta-optimization is the process of optimizing the performance of an algorithm by changing its parameters. This strategy not only increases the efficiency of the algorithm, but also allows us to better understand how the algorithm responds to different types of challenges. These techniques fall into two broad categories: offline and online. Offline techniques specify the parameter settings of the algorithm before execution and work with a training set as an example. Offline approaches work well when the selected examples have the same structure as the other examples in the training set. However, these approaches may fail if the class of instances is heterogeneous. This is because finding the appropriate parameter settings for each class of instances takes a lot of time in this case. Online approaches, on the other hand, collect feedback and try to determine the optimal parameter values while the algorithm is solving a problem scenario. These approaches reduce computation time by trying to find the parameter settings while the algorithm is running. Although several optimization methods have been proposed in the literature, no algorithm is able to provide the optimal answer to all optimization questions [37]. As a result, the established optimization techniques and the field of new meta-heuristic optimization algorithms are constantly being improved through innovations and further developments. By evaluating the success of newly developed meta-heuristic optimization algorithms and comparing them with previously published algorithms, new studies on improving existing optimization algorithms or developing new optimization algorithms based on successful algorithms are added to the literature on a daily basis. In this context, Artificial Water Drop Algorithm [28], Mexican Axolotl Optimization: a novel bio-inspired heuristic [3], Tunicate Swarm Algorithm [2], Tuna Swarm Optimization [1, 8], Equilibrium Slime Mould Algorithm [8, 26], Dingo Optimization Algorithm [29], Leader Harris hawks optimization [5], Differential Squirrel Search Algorithm [9, 10], Leader Slime Mould Algorithm [31], Adaptive Opposition Slime Mould Algorithm [32], CLA- New Meta-Heuristic Algorithm [38], Hybrid Augmented Grey Wolf Optimizer and Cuckoo Search [33, 34], Child Drawing Development Optimization Algorithm [34], Golden Eagle Optimizer [35], Bald eagle search Optimization algorithm [14], Chimp Optimization Algorithm [39], Lévy Flight Distribution [6] and Harris hawks optimization [36] are some of them. This paper compares the performances of sixteen meta-heuristic optimization algorithms presented in the literature between 2021 and 2022. Various test functions, including single-modal, multi-modal and fixed-size multi-modal comparison functions, have been used to evaluate the effectiveness of the algorithms used in this context.
The rest of the article is structured as follows: Section 2 describes the methodology and mathematical framework of the benchmark functions; Section 3 presents the experimental results. Finally, section 4 presents the conclusion.

2. Methodology

An important aspect of testing and validating a new algorithm is comparing it to existing algorithms that use benchmark functions. This type of bench-marking is also crucial to better understand the advantages and weaknesses of the algorithm. Typically, the new technique is evaluated against a set of test functions that ideally have different properties such as mode shapes. However, these bench-marking methods suffer from a crucial weakness. Although it is a comparative function, it is rarely used in practice. There are several reasons for this. While real-world scenarios are much more complex and different than these test items, an explanation is usually well thought out and concise. Another problem is that these test functions often use unconstrained or regular fields, whereas in the real world, non-linear, complex constraints often apply, and the field may contain multiple isolated partitions or islands. The functions used to compare the performances of the algorithms discussed here are detailed below.

2.1. Mathematical framework of benchmark functions

The following functions are among the most used for evaluating optimization strategies. They are categorized based on their basic physical properties and shapes (see Table 1.1). Uni-modal, Multi-modal and Fixed-dimension multi-modal benchmark test functions were used in this study. In Table 2.1, D indicates the size of the function, Range is the variation range of the optimization variable, and F_{min} is the minimum. Twenty-three test functions were used to evaluate the performance of the sixteen algorithms considered in this study. Figure 2.1, Figure 2.2, and Figure 2.3 show two-dimensional (2D) views of the different functions.

	Function	D	Range	F_{min}
Uni-modal	$F_1(X)=\sum_{i=1}^{n} x_i^2$	30	[-100, 100]	0
	$F_2(X)=\sum_{i=1}^{n}\lvert X_i\rvert+\prod_{i=1}^{n}\lvert X_i\rvert$	30	[-10, 10]	0
	$F_3(X)=\sum_{i=1}^{n}\left(\sum_{j-1}^{1} x_i\right)^2$	30	[-100, 100]	0
	$F_4(X)=\max_i\left(\lvert x_i\rvert .1\leq i\leq n\right\rvert$	30	[-30, 30]	0
	$F_5(X)=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_i^2\right)^2+\left(x_i-1\right)^2\right]$	30	[-100, 100]	0
	$F_6(X)=\sum_{i=1}^{n}\left([x_i+0.5]\right)^2$	30	[-100, 100]	0
	$F_7(X)=\sum_{i=1}^{n} ix_i^4+\text{random}[0.1)$	30	[-1.28, 1.28]	0
Multi-modal	$\mathrm{F}_8(\mathrm{x})=\sum_{i=1}^{n}-\mathrm{x}_i\sin\left(\sqrt{\lvert\mathrm{x}_i\rvert}\right)-418.9829\times d$	30	[-500, 500]	-418.9829xd
	$F_9(x)=\sum_{i=1}^{n}\left[x_i^2-10\cos\left(2\pi x_i\right)+10\right]$	30	[-5.12, 5.12]	0
	$F_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos\left(2\pi x_i\right)\right)+20+e$	30	[-32, 32]	0
	$F_{11}(x)-\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	30	[-600, 600]	0
	$F_{12}(x)=\frac{\pi}{n}\left\{10\sin\left(\pi y_1\right)+\sum_{i=1}^{n-1}\left(y_i-1\right)^2\left[1+10\sin 2\left(\pi y_{i+1}\right)\right]+\left(y_n-1\right)^2\right\}$ $+\sum_{i=1}^{n}u\left(x_i,10,100,4\right)$	30	[-50, 50]	0
	$y_i=1+\frac{x_i+1}{4}\times u(x_i,a,k,m)=\begin{cases}k\left(x_i-a\right)^m x_i>a\\ 0-a<x_i<a\\ k(-x_i-a)^m x_i<-a\end{cases}$			
	$F_{13}(x)=0.1\mid\sin 2\left(3\pi x_1\right)+\sum_{i=1}^{n}\left(x_i-1\right)^2\left[1+\sin 2\left(3\pi x_i+1\right)\right]$ $+\left(x_n-1\right)^2\left[1+\sin 2\left(2\pi x_n\right)\mid l+\sum_{i=1}^{n}u\left(x_1,5,100,4\right)\right.$	30	[-50, 50]	0
Fixed-dimension multi-modal	$F_{14}(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum\left(x_i-a_j\right)^6}\right)^{-1}$	2	[-65, 65]	0.998
	$F_{15}(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1\left(b_1^2+b_1x_2\right)}{b_i^2+b_1x_3+x_4}\right]^2$	4	[-5,5]	0.00030
	$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	[-5, 5]	-1.0316
	$F_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^3+\frac{5}{\pi}x_1-6\right)^3+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	2	[-5,5]	0.398
	$F_{18}(x)=\left[1+\left(x_1+x_2+1\right)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]$ $\times\left[30+\left(2x_1-3x_2\right)^2\times\left(18-32x_1+12x_1^2+48x_2+36x_1x_2+27x_2^2\right)\right]$	2	[-2, 2]	3
	$\mathrm{F}_{19}(\mathrm{x})=-\sum_{i=1}^{4}\mathrm{c}_i\exp\left(-\sum_{i=1}^{3}\mathrm{a}_{ij}\left(\mathrm{x}_j-\mathrm{p}_{ij}\right)^2\right)$	3	[1,3]	-3.86
	$\mathrm{F}_{20}(\mathrm{x})=-\sum_{i=1}^{4}\mathrm{c}_i\exp\left(-\sum_{i=1}^{5}\mathrm{a}_{ij}\left(x_j-\mathrm{p}_{ij}\right)^2\right)$	6	[0,1]	-3.32
	$\mathrm{F}_{21}(\mathrm{x})=-\sum_{i=1}^{5}\left[\left(\mathrm{X}-\mathrm{a}_i\right)\left(\mathrm{X}-\mathrm{a}_i\right)^{\mathrm{T}}+\mathrm{c}_i\right]^{-1}$	4	[0,10]	-10.1532
	$\mathrm{F}_{22}(\mathrm{x})=-\sum_{i=1}^{7}\left[\left(\mathrm{X}-\mathrm{a}_i\right)\left(\mathrm{X}-\mathrm{a}_i\right)^{\mathrm{T}}+\mathrm{c}_i\right]^{-1}$	4	[0,10]	-10.4028
	$\mathrm{F}_{23}(\mathrm{x})=-\sum_{i=1}^{10}\left[\left(\mathrm{X}-\mathrm{a}_i\right)\left(\mathrm{X}-\mathrm{a}_i\right)^{\mathrm{T}}+\mathrm{c}_i\right]^{-1}$	4	[0,10]	-10.5363

Table 2.1: A mixture of uni-modal, multi-modal, and fixed-dimension multi-modal benchmark functions.

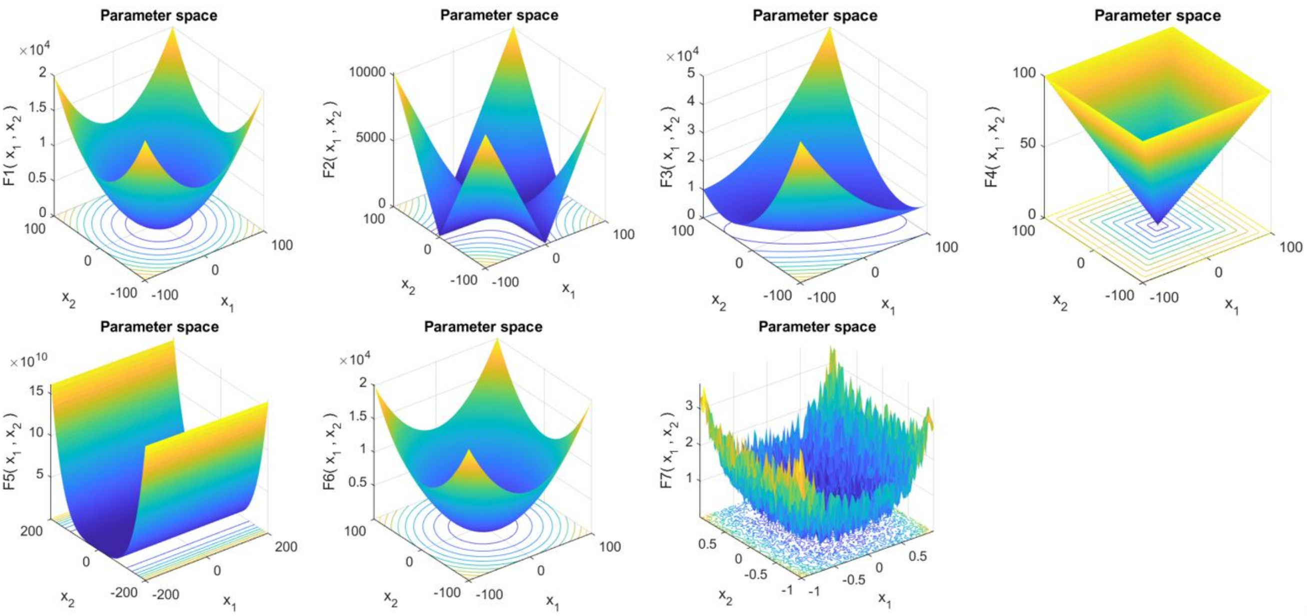

Figure 2.1: 2-D version of uni-modal benchmark function.

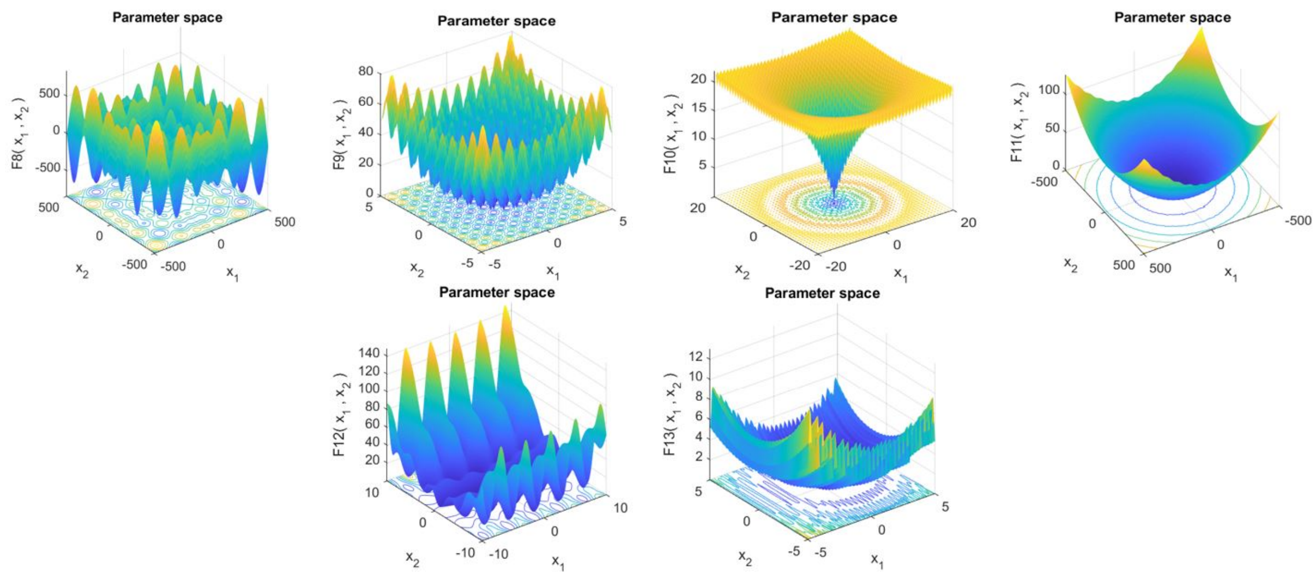

Figure 2.2: 2-D version of multi-modal benchmark function

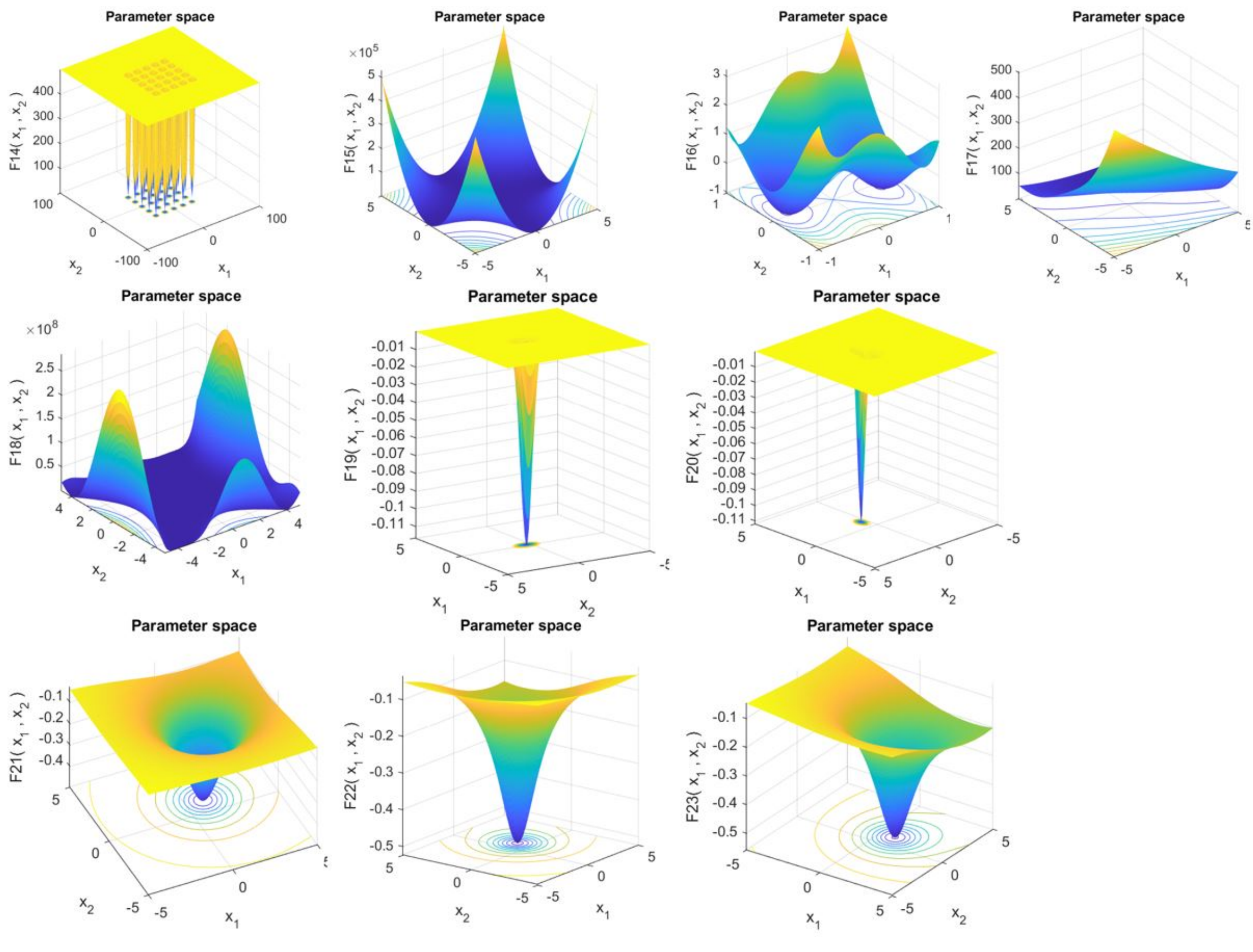

Figure 2.3: 2-D version of fixed-dimension multi-modal benchmark function

3. Experimental Results

In this section, we demonstrate the effectiveness of the algorithm used for 23 commonly used uni-modal, multi-modal and fixed-dimensional multi-modal bench-marking functions using qualitative metrics such as best, worst, mean, standard deviation and median scores. Table 2.1 illustrates these functions with category representations of three mathematical functions. Figure 2.1, Figure 2.2, and Figure 2.3 also represent two-dimensional shapes. The first group includes functions with a single solution path (F1-F7), which have a single ideal solution and are intentionally difficult to use. The second group includes functions (F8-F13) that have many optimal solutions. While the local optimal solutions are used in these functions to evaluate the algorithm's exploration performance, an algorithm must be able to search the space globally to find the global optimum and avoid being trapped in the local optimum. The third group contains multi-modal functions with fixed dimensions (F14-F23), which are similar to multi-modal functions but have fixed dimensions. The dimensions of these functions, as well as the constant coefficients used in this work, are accessible in [40, 41]. For each test category, 30 particles with 500 iterations were used. Each function was run 30 times and its average values were used for a fair evaluation. Table 3.1 shows how the parameters of each algorithm are configured.

Algorithms	Parameter	Value
For all algorithms	Population	30
	Maximum Iterations	500
AGWOCS	Control Parameter (a)	[2 1]
		[1 3]
AOSMA	Control Parameter (a, b)	
		[1 0]
	diffusion factor(pp)	0.1
AWDA	upper limit	5
	lower limit	2
BES	Control parameter (a)	[1.5 2]
	Control parameter (r)	[0 1]
	child level rate(LR)	0.01
CDDO	Child skill rate(SR)	0.9
	Creativity Rate (CR)	0.1
	alpha	0.85
CLA	zeta	0.6
	pConf	0.25
	mu	0.05
DOA	Hunting or Scavenger rate (p)	0.5
	Group attack or persecution (Q)	0.7
	Gliding constant (Gc)	1.9;
DSSA	Crossover rate (Cr)	Cr=0.5;
	Random gliding distance (dg)	dg=0.8;
	Predator presence probability(Pdp)	Pdp=0.1;
ESMA	adjustable param (q)	0.2
	vectors of random numbers in the range (r, λ)	[0 1]
GEO	Propensity to attack (pa)	[0.5 2]
	Propensity to cruise (pc)	[1 0.5]
	escaping energy (E0)	[-1 1]
	are random number (q)	[0.5 0.5]
HHO	Harris Hawks Number	30
	β	1.5
	E0 variable	ε [-1, 1]
	Search agents	30
	Threshold	2
	CSV	0.5
	β	1.5
LFD	$\alpha 1$	10
	$\alpha 2$	0.00005
	$\alpha 3$	0.005
	$\partial 1$	0.9
	$\partial 2$	0.1
	Harris Hawks Number	30
LHHO	β	1.5
	E0 variable	ε [-1, 1]
	Entropic parameter (r)	0.5
LSMA	N	20
	z	0.03
	CrossOver Probability (cop)	0.5
	damage probability(dp)	0.5
MAO	regeneration probability (rp)	0.1
	tournament size (k)	3
	differentiation constant (λ)	0.5
	Search agents	30
TSA	Parameter P_{min}	1
	Parameter P_{max}	4
TSO	a	0.7
	z	0.05

Table 3.1: Algorithm parameter settings

The results of the performance comparison are shown in Table 3.2 for uni-modal, Table 3.3 for multi-modal and Table 3.4 for multi-modal fixed dimensions. Due to the stochastic nature of meta-heuristic algorithms, the results of two consecutive runs often do not match. Since we performed many independent experiments with each method, the average values of the results for each function are tabulated. The experiments were conducted in MATLAB 2021a on an Intel Core i7, with 16 GB RAM and in a Windows 10 environment. The algorithms of each benchmark function were run 30 times under the same conditions. The tables contain statistical data in the form of best value, worst value, median value, mean value and corresponding standard deviation.

Algorithms		F1	F2	F3	F4	F5	F6	F7
	Best	3.3506e-49	1.6539e-30	1.2297e-12	3.6704e-15	0.029243	0.0021407	1.7915e-06
AGWOCS	Mean	0.25209	0.0016347	2.0228	0.0031702	130.791	0.25954	7.0832e-05
	Std	1.14	0.0060045	5.5347	0.0081379	735.3995	1.1485	0.0002045
	Best	0	0	0	0	0.0051288	3.6623e-06	2.7551e-06
AOSMA	Mean	5.5695	52.0231	75.962	0.045495	17156.5852	4.7483	0.013632
	Std	92.779	45.0205	349.9254	0.21304	354706.9755	103.5421	0.24843
	Best	0.11767	0.00095195	1.0586	0.015044	6.0371	0.031703	7.2871e-05
AWDA	Mean	1.8643	0.016292	3.8985	0.028343	1045.6344	1.9835	0.0006933
	Std	2.8366	0.032433	2.1564	0.010301	2800.0147	2.8459	0.0014098
	Best	0	0	0	0	0.65699	3.9129e-21	2.3361e-06
BES	Mean	1.1926e-15	6.1888e-10	2.9025e-08	2.5315e-09	0.79875	0.0076214	8.3538e-06
	Std	2.6668e-14	1.3839e-08	6.4902e-07	5.6606e-08	0.087603	0.029749	2.9198e-05
	Best	2.1644e-60	1.4406e-171	0	1.7792e-173	0.9624	0.052593	2.653e-05
CDDO	Mean	1.2876	0.0033599	8.7454	0.0058665	2015.8209	1.406	0.00036195
	Std	9.7277	0.042286	143.1281	0.075178	44221.5524	26.4666	0.0038432
	Best	1.1469e-148	4.477e-124	8.6369e-219	4.6275e-164	0.96158	0.18751	3.7285e-05
DOA	Mean	7.157	16322776.9037	21.7977	0.013491	20147.9618	5.993	0.010845
	Std	114.5315	365353138.5064	347.2062	0.16287	448149.7826	96.0457	0.17488
	Best	0	2.5337e-184	2.4815e-233	1.2598e-173	0	0	3.8027e-05
DSSA	Mean	12.7504	4122824.3096	29.7994	0.025507	37561.7976	16.4352	0.020802
	Std	167.4216	65122224.9887	382.15	0.26481	483778.2616	180.258	0.25229
	Best	0	5.2536e-256	0	3.6113e-247	0.0081807	3.7081e-05	6.4925e-06
ESMA	Mean	13.8772	228481.2873	53.2054	0.0089188	15700.6462	5.6668	0.010327
	Std	176.0067	5108996.7524	486.9561	0.12994	350975.4737	102.2555	0.20037
	Best	5.6973e-27	4.9817e-32	2.5913e-24	2.115e-20	3.6978e-33	1.0374e-32	1.2008e-3
GEO	Mean	0.000979	0.0002412	4.9683e-05	0.00011026	0.00020344	0.00072641	0.00031885
	Std	0.0069597	0.00077319	0.00013601	0.00060643	0.0016597	0.0046885	0.0022391
	Best	3.2657e-114	1.3239e-56	9.4819e-94	2.6659e-52	0.00029177	2.1529e-05	1.4017e-05
HHO	Mean	7.1772	517294.4506	27.5972	0.015173	19452.9728	6.4293	0.01276
	Std	121.1378	11567055.3729	329.9414	0.1707	386839.017	116.7763	0.20452
	Best	1.0547e-08	0.00052114	2.5953e-07	2.2107e-05	1.6065	0.081866	0.00016718
LFD	Mean	25.1795	2.4779605994158	213.2922	0.041412	86637.4102	23.6023	0.071877
	Std	295.4489	5.5404824897375	1880.1345	0.31196	1142071.398	252.4117	0.73438
	Best	4.0603e-161	4.1437e-80	1.0195e-112	1.6601e-80	4.0084e-06	4.8004e-08	1.0863e-06
LHHO	Mean	2.4816	219.2769	5.6402	0.0065608	668.1418	0.86073	0.00065924
	Std	52.2959	4903.0323	73.9713	0.094363	14709.5343	16.1954	0.012678
	Best	0	0	0	6.0899e-320	0.0056562	5.9639e-05	1.9479e-06
LSMA	Mean	8.1105	74.5910	384.4104	0.018095	18847.2574	6.626	0.0067008
	Std	125.5814	16.6790	1203.5107	0.17735	358882.8341	112.3465	0.12339
	Best	13.7968	0.15669	31.1561	0.55386	96.4628	9.8023	0.0017507
MAO	Mean	87.4892	416.7457	154.6912	1.0213	119163.5785	72.2031	0.024319
	Std	162.0843	5648.4623	306.2596	0.64014	448476.2389	157.1719	0.09077
	Best	1.301e-202	1.8046e-103	1.5165e-185	1.8193e-92	0.955	0.20092	1.1951e-05
TSA	Mean	7.3101	88640985.7881	9.4265	0.018317	21884.5316	8.1195	0.013698
	Std	114.2715	1982072697.8507	163.5931	0.19395	418587.6844	117.503	0.25689
	Best	2.1705e-257	5.4605e-128	3.9708e-224	9.279e-118	0.03069	1.9959e-05	2.737e-05
TSO	Mean	19.8437	4187.2582	82.5075	0.018975	26214.3009	14.8457	0.014592
	Std	151.2259	93628.9913	490.0847	0.18541	489293.5139	145.0812	0.19667

Table 3.2: The results of benchmark functions with uni-modality, (D= 30, Max it=500)

Table 3.2 shows the convergence of the algorithms used. In this step, the performance of the algorithms was evaluated against the benchmark functions in which they were run. In this evaluation step, the initial population number was assumed to be 30 and the iteration number was assumed to be 500. Figure3.1 shows convergence plots of uni-modal benchmark functions. In the evaluation algorithm, the solutions tend to search extensively for promising regions of the search spaces and exploit the optimal point. In these uni-modal model functions, it is observed that there is an effective balance between exploration and exploitation so that the solutions move toward the optimal point. In the initial steps, a repetition of sudden changes can be observed, which gradually decreases as the iteration progresses. The convergence behavior of an algorithm at a point in the search space leads to solution fitness. The convergence diagram of solution fitness is shown in Figure 3.1. The graphs show decreasing behavior across all test functions. They show that the approximate optimum significantly improves the point at all iterations. Figure 3.1−Figure3.4 shows the convergence plot of 23 functions compared to different algorithms (16 algorithms). Those that can reach the point of global optimum (0) with high performance in functions F1-F7 of the algorithms.

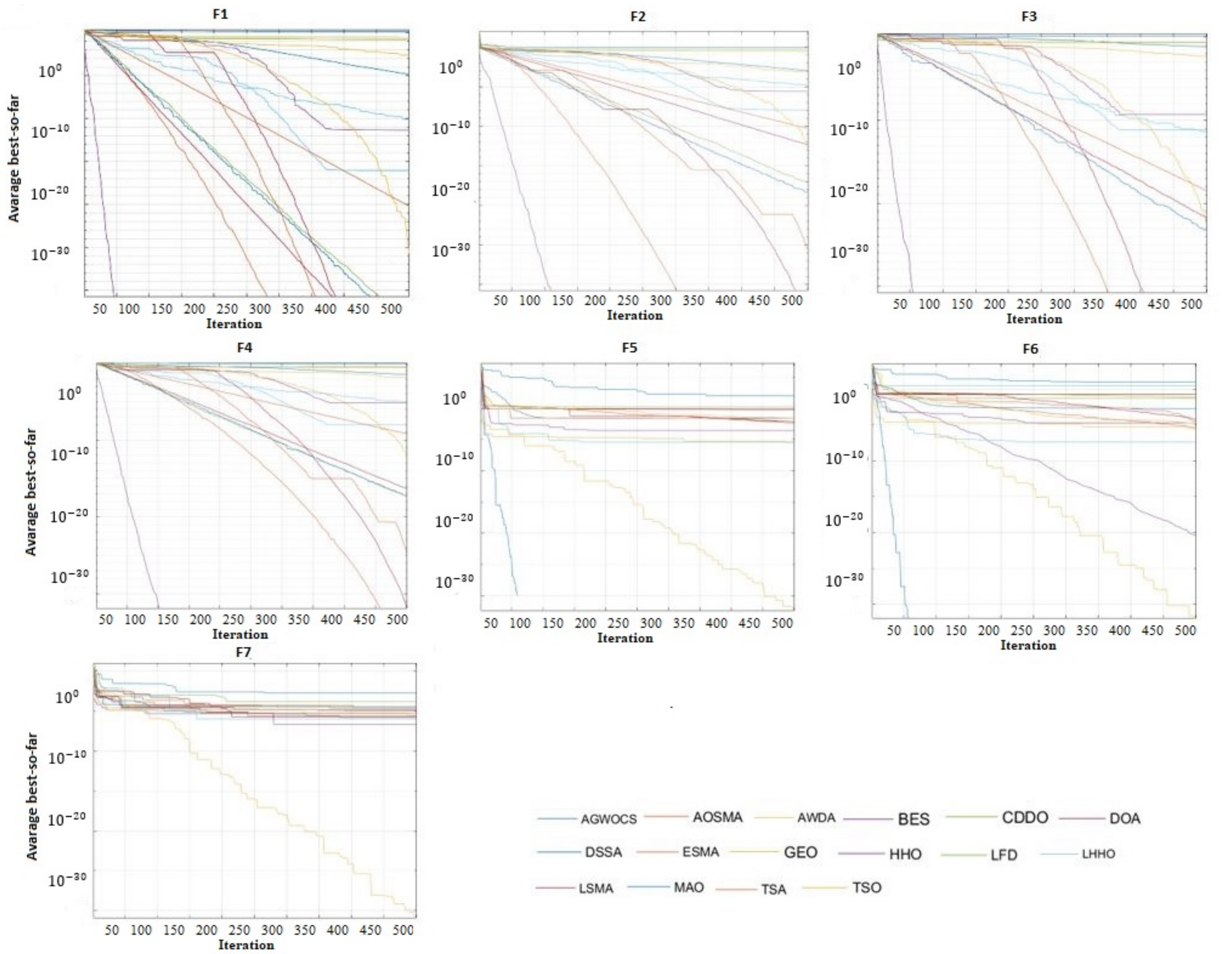

Figure 3.1: Convergence curves of the algorithms on F1–F7

Table 3.3 evaluates the convergence of the algorithms. The performance of the algorithms is tested at this stage using the multi-modal (F8-F13) benchmark functions. The initial population is 30 and the number of iterations in this evaluation stage is 500. The consistency diagrams for multi-modal benchmark functions are shown in Figure 3.2. It was found that there is an efficient balance between search and utilization for these multi-modal model functions, which ensures that the solutions approach the optimal point. The initial phases show a pattern of dramatic shifts that diminishes as the iteration progresses. At a certain point in the search space, the convergence behavior of an algorithm leads to solution fitness. For all test functions, the graphs show decreasing convergence.

Table 3.4 shows the convergence performance of the fixed-dimension multi-modal algorithms (F14-F23) in this phase. In this evaluation phase, the initial population is 30 and the number of iterations is 500. The consistency diagrams for fixed-dimension multi-modal benchmark functions are shown in Figure3.3. The early phases show a pattern of dramatic shifts that decrease as the iteration progresses. The diagrams show decreasing convergence for all test functions. The convergence plots for functions F14-F23 are shown in Figure 3.3.

		Multimodal benchmark functions					
Algorithms		F8	F9	F10	F11	F12	F13
	Best	-7.8365	0	8.8818e-18	0	0.00010519	0.00092445
AGWOCS	Mean	-7.3794	0.041007	0.00097152	0.0027997	1.3831	220.9848
	Std	0.24301	0.082966	0.0027633	0.012436	17.098	1239.4441
	Best	-418.9827	0	2.9606e-17	0	9.9386e-07	6.7812e-06
AOSMA	Mean	-415.9582	0.075541	0.0047513	0.044797	52393.0677	55823.3721
	Std	20.5681	0.95516	0.046634	0.99497	1171544.4371	1248248.3047
	Best	-2.5589	0.039849	0.0060703	0.0018924	0.0041695	0.0029728
AWDA	Mean	-2.0609	0.076849	0.012512	0.015766	3153.6857	10144.2236
	Std	0.39046	0.021872	0.0044522	0.015993	5547.4062	16607.6865
	Best	-192.849	0	2.9606e-17	0	3.9419e-25	0.098869
BES	Mean	-159.6664	3.0316e-17	1.6406e-12	1.1145e-16	0.00047515	0.098944
	Std	21.8772	6.779e-16	3.6684e-11	2.4921e-15	0.0033075	0.00019872
	Best	-414.9186	4.1436	1.4803e-16	0	1.9783e-06	0.010553
CDDO	Mean	-409.2277	4.1992	0.0029311	0.005143	143.3606	802.3993
	Std	30.587	0.43545	0.030446	0.1003	3205.583	17941.9473
	Best	-174.9058	0	2.9606e-17	0	0.024691	0.078147
DOA	Mean	-159.6674	0.67532	0.0060553	0.10222	35187.4366	96073.2885
	Std	18.7455	2.6085	0.052274	1.2424	785684.0574	2032984.8721
	Best	-2.718491268946872e+68	0	2.9606e-17	0	5.2351e-34	4.4993e-34
DSSA	Mean	-1.90597575847278e+66	0.12554	0.0058799	0.12345	72488.5008	202488.5563
	Std	1.717311818082357e+67	1.2337	0.060015	1.5571	1072057.3871	2600745.5342
	Best	-418.9721	0	2.9606e-17	0	2.4456e-05	1.7282e-06
ESMA	Mean	-410.9383	0.10006	0.0038052	0.058521	43636.2202	69284.3225
	Std	31.5805	0.99944	0.039319	0.83932	975735.3523	1549156.0305
	Best	5.3515e-32	4.1497e-32	4.1512e-16	5.7436e-21	2.0543e-33	1.1894e-28
GEO	Mean	0.00039421	0.00013089	0.00047771	0.00013806	0.00015149	0.0012698
	Std	0.004072	0.00049639	0.0047613	0.0023189	0.00084138	0.0046525
	Best	-418.9774	0	2.9606e-17	0	2.801e-06	7.0288e-07
HHO	Mean	-414.5949	0.11334	0.0051101	0.054826	41053.6651	113010.0108
	Std	29.3184	1.0612	0.048748	0.99619	902002.1664	2150141.3942
	Best	-937.7924	7.4903e-06	2.6392e-05	2.5532e-09	0.00049674	0.16477
LFD	Mean	-413.8811	3.3044	0.0091952	0.28953	300136.7078	233181.9774
	Std	235.8705	5.0429	0.065979	2.6183	3366941.541	3359582.3175
	Best	-418.9829	0	2.9606e-17	0	1.0112e-08	4.471e-07
LHHO	Mean	-416.4481	0.091388	0.0023909	0.017484	673.9148	9749.6928
	Std	21.7364	0.79404	0.027909	0.37956	15066.1738	216313.5404
	Best	-418.9775	0	2.9606e-17	0	6.7502e-06	9.812e-05
LSMA	Mean	-407.2941	0.084856	0.0047234	0.052499	49885.3932	99542.8596
	Std	36.8422	0.86632	0.048209	0.98755	1115471.0593	1892313.5595
	Best	-92.5983	0.9515	0.27227	0.22361	0.13535	12.4004
MAO	Mean	-68.2304	1.9137	0.37329	1.1615	459699.3387	792254.9447
	Std	17.0243	0.90728	0.10648	1.8415	1991167.4424	2327180.6163
	Best	-105.0841	0.06663	1.4803e-16	0	0.022997	0.075074
TSA	Mean	-98.7788	0.64792	0.0067426	0.06771	43727.5153	53020.4358
	Std	5.8538	2.1449	0.054204	0.99494	949251.061	1106040.1279
	Best	-418.9829	0	2.9606e-17	0	1.3514e-06	2.7818e-06
TSO	Mean	-412.5186	0.21564	0.01235	0.097162	41742.4897	98319.3265
	Std	30.9786	0.90544	0.05926	1.0356	933389.2961	1697227.7022

Table 3.3: The results of the benchmark functions with multi modality, with 30 dimensions

Although the comparison has a slower convergence rate at the beginning of the search for most functions, after a few iterations it shows good convergence performance and gives a better answer for most functions, especially for fixed multi-modal functions. The frequency diagram can be seen in Figure3.4. In this way, the performance of all algorithms in all functions is shown together. The frequency by best case indicates the number of algorithms that can reach the optimal point in the functions. According to this scheme, the algorithm GEO has the highest frequency, while the algorithms AWDA and LFD have the lowest frequency. To allow a fair comparison, the necessary conditions for the algorithms have remained the same. It is worth noting that due to the meta-heuristic nature of the algorithms, the comparisons made here are not constant and do not always give the same result.

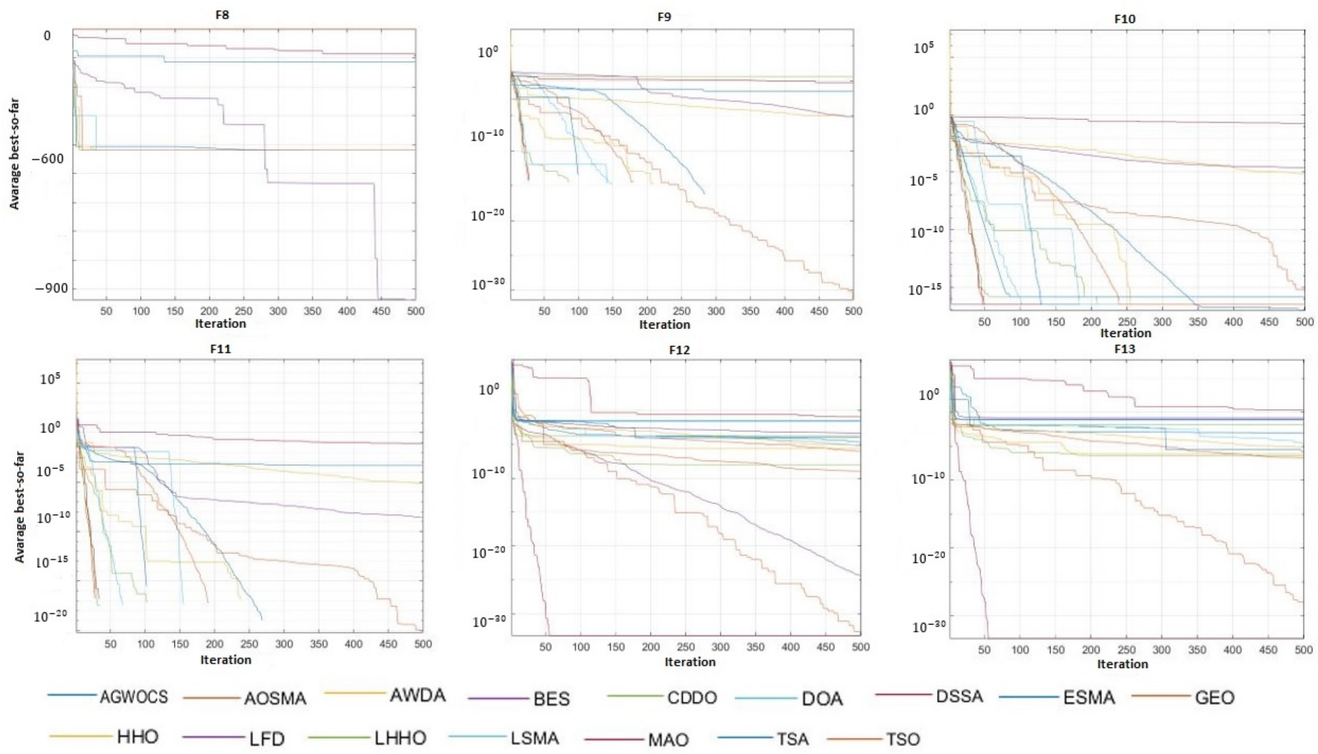

Figure 3.2: Convergence curves of the algorithms on F8–F13

		Fixed-dimension multi-modal benchmark functions									
Alg.		F14	F15	F16	F17	F18	F19	F20	F21	F22	F23
	Best	0.0033135	3.5141e-07	-0.0011462	0.00044281	0.0033333	-0.0042913	-0.0036521	-0.0074093	-0.0079195	-0.0038682
AGWOCS	Mean	0.0036284	1.0034e-06	-0.0011444	0.00046888	0.0035728	-0.0042828	-0.0036105	-0.0057	-0.005937	-0.0030972
	Std	0.0014804	5.3683e-06	3.0457e-05	0.00047603	0.0028832	7.3445e-05	0.00015844	0.0015926	0.0019506	0.00048692
	Best	0.033267	1.4088e-05	-0.034388	0.013263	0.1	-0.12876	-0.10658	-0.33844	-0.34676	-0.35121
AOSMA	Mean	0.05901	7.2452e-05	-0.034346	0.013281	0.13901	-0.12868	-0.10537	-0.32794	-0.33328	-0.34115
	Std	0.16589	0.00044571	0.00053593	0.00016387	0.47073	0.00030876	0.0068892	0.049831	0.047258	0.030526
	Best	0.0033135	1.0092e-06	-0.0011463	0.0033335	0.0033335	-0.004292	-0.0036895	-0.0029809	-0.011482	-0.0030016
AWDA	Mean	0.0034069	4.743e-05	-0.0011413	0.00617	0.0042119	-0.0041809	-0.0033073	-0.0025423	-0.0065387	-0.0026121
	Std	0.0003169	0.00015492	1.0756e-05	0.0035232	0.0025558	0.00021722	0.00048191	0.00075494	0.0038831	0.00032105
	Best	0.033267	1.025e-05	-0.034388	0.013263	0.1	-0.12876	-0.11073	-0.16851	-0.34676	-0.35121
BES	Mean	0.03489	1.6113e-05	-0.034383	0.013293	0.10002	-0.12868	-0.11038	-0.16805	-0.34264	-0.34873
	Std	0.010095	0.00010823	7.938e-05	0.00020801	0.00049561	0.0013077	0.0035659	0.0059867	0.026726	0.019388
	Best	0.033267	8.2041e-05	-0.034346	0.013268	0.10234	-0.12618	-0.094687	-0.3371	-0.33279	-0.34979
CDDO	Mean	0.035301	0.00013376	-0.034288	0.013495	0.34636	-0.12422	-0.094687	-0.32306	-0.28744	-0.34583
	Std	0.045474	9.0075e-05	0.00083044	0.0014674	0.40121	0.00189	2.9173e-16	0.041391	0.05882	0.013276
	Best	0.033267	0.00067878	-0.034388	0.013263	0.1	-0.12876	-0.11055	-0.33844	-0.34676	-0.35121
DOA	Mean	0.03453	0.00073193	-0.034305	0.013619	0.10346	-0.12807	-0.11018	-0.33236	-0.34238	-0.34561
	Std	0.0080879	0.00067655	0.0012122	0.0030308	0.046907	0.0026997	0.0027794	0.036855	0.031086	0.03435
	Best	0.033267	2.0486e-05	-0.034012	0.013335	0.38346	-0.12736	-0.095157	-0.17003	-0.34676	-0.35121
DSSA	Mean	0.12317	4.6844e-05	-0.033502	0.013935	0.45034	-0.12705	-0.089494	-0.16933	-0.3419	-0.3495
	Std	0.84121	0.00024589	0.0016329	0.0021464	0.46138	0.00024224	0.0057824	0.0096559	0.032963	0.021966
	Best	0.033267	1.0342e-05	-0.034388	0.013263	0.1	-0.12876	-0.10677	-0.33844	-0.34676	-0.35118
ESMA	Mean	0.047522	5.1199e-05	-0.034055	0.013476	0.22587	-0.12864	-0.10438	-0.33534	-0.32294	-0.34799
	Std	0.14829	0.00041763	0.003931	0.002481	0.95846	0.00059844	0.0049511	0.019334	0.060308	0.027668
	Best	1.5268e-30	1.915e-30	6.6766e-33	0	4.2114e-33	1.5216e-15	4.1227e-23	8.7103e-32	1.7117e-30	6.5173e-31
GEO	Mean	6.3114e-05	0.00040485	0.00068608	2.2865e-05	0.00043754	0.00011214	0.00016022	0.00067317	0.00064407	0.0001355
	Std	0.0004438	0.0027602	0.0031767	8.8517e-05	0.0035416	0.0010354	0.00066701	0.005151	0.0055138	0.0009007
	Best	0.033267	1.0422e-05	-0.034388	0.013263	0.1	-0.12856	-0.098654	-0.16851	-0.16946	-0.17081
HHO	Mean	0.072033	2.4821e-05	-0.034353	0.013447	0.25284	-0.12721	-0.090329	-0.16665	-0.16705	-0.16894
	Std	0.57258	0.00010526	0.00032258	0.0031932	1.0818	0.0012712	0.0044844	0.01272	0.013189	0.012809
	Best	0.033267	6.1611e-05	-0.034388	0.10129	0.10834	-0.12869	-0.10531	-0.33844	-0.12414	-0.17095
LFD	Mean	0.10675	0.00024484	-0.034195	0.10975	0.11857	-0.12842	-0.10396	-0.30615	-0.10692	-0.15115
	Std	0.16519	0.0015185	0.0011812	0.017921	0.032809	0.0019299	0.006337	0.083296	0.030128	0.039268
	Best	0.033267	1.0269e-05	-0.034388	0.013263	0.1	-0.12876	-0.10361	-0.33833	-0.16959	-0.3512
LHHO	Mean	0.036795	2.604e-05	-0.034358	0.013475	0.10315	-0.12868	-0.099576	-0.30315	-0.16949	-0.28795
	Std	0.028064	0.00017386	0.00041314	0.0027182	0.069287	0.00075748	0.0020618	0.070785	0.00068774	0.088846
	Best	0.033267	1.0297e-05	-0.034388	0.013263	0.1	-0.12876	-0.11073	-0.33843	-0.34676	-0.35121
LSMA	Mean	0.043432	3.426e-05	-0.034116	0.014432	0.48988	-0.12824	-0.10629	-0.32716	-0.32563	-0.30189
	Std	0.033784	9.4922e-05	0.00088212	0.005674	3.3495	0.002802	0.010143	0.029421	0.065147	0.081489
	Best	0.066401	0.00036673	-0.034387	0.78119	0.82362	-0.12841	-0.10869	-0.27841	-0.080423	-0.086101
MAO	Mean	1.3265	3.2519	0.13343	3.636	2.2516	-0.12241	-0.094856	-0.15349	-0.059535	-0.066526
	Std	4.0638	74.4946	3.3885	7.9148	8.2122	0.016864	0.024606	0.10096	0.021443	0.01934
	Best	0.45395	0.00028049	-0.034388	0.01334	0.1	-0.12843	-0.10819	-0.20731	-0.056239	-0.33368
TSA	Mean	0.47901	0.00086876	-0.032249	0.014729	0.49815	-0.12746	-0.10047	-0.17529	-0.050901	-0.23924
	Std	0.56031	0.0045565	0.0072198	0.0051053	1.0072	0.0047008	0.01357	0.043324	0.0075194	0.055414
	Best	0.033267	4.0772e-05	-0.034388	0.013263	0.1	-0.12876	-0.11073	-0.33844	-0.34676	-0.35121
TSO	Mean	0.038004	0.0001935	-0.034156	0.013405	0.11071	-0.12866	-0.10999	-0.3307	-0.34009	-0.33744
	Std	0.017447	0.0016314	0.0016097	0.0016331	0.1609	0.00091173	0.0059657	0.042262	0.034612	0.048519

Table 3.4: The results of the benchmark functions with fixed-dimension multi modality, with 30 dimensions

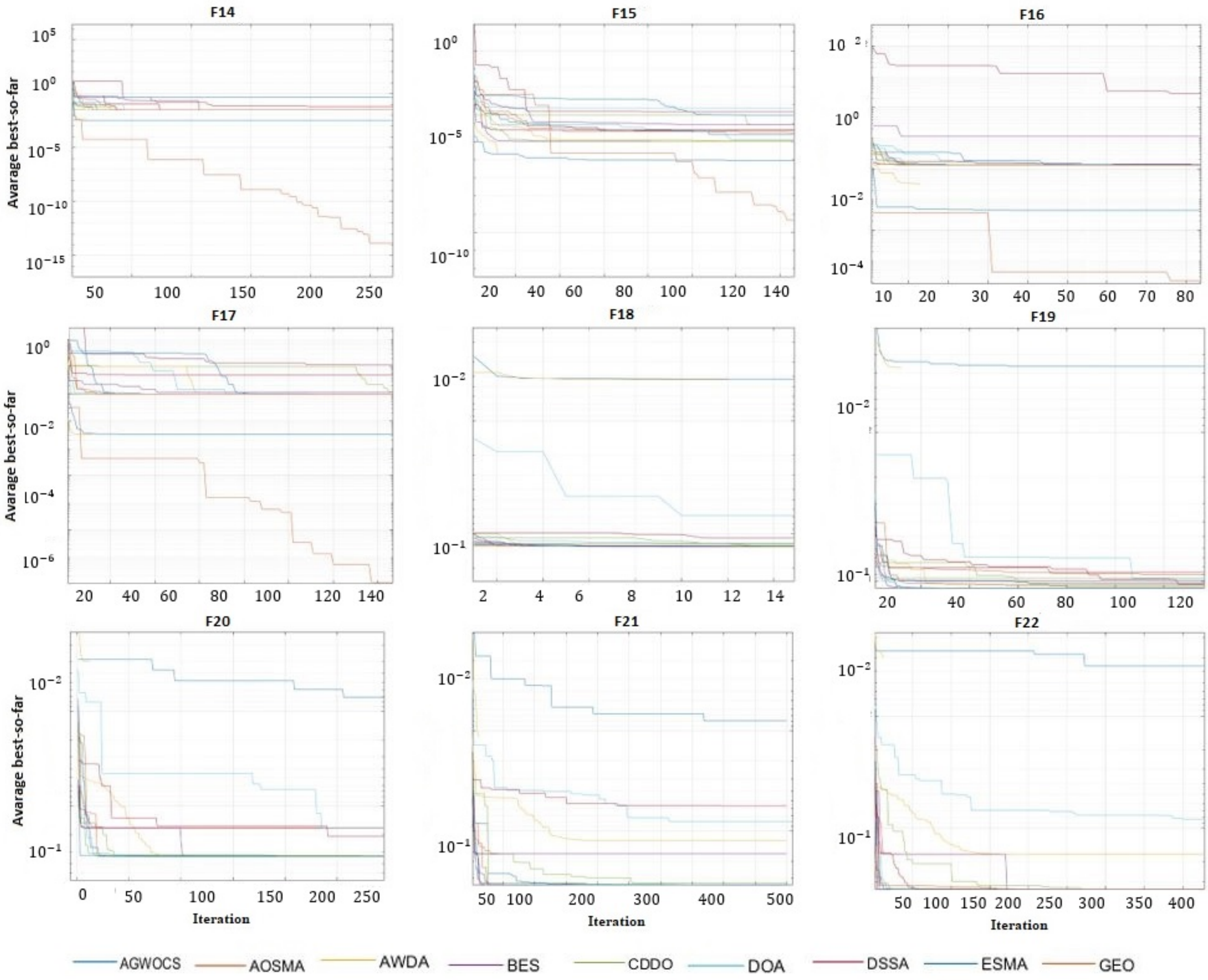

Figure 3.3: Convergence curves of the algorithms on F14–F23

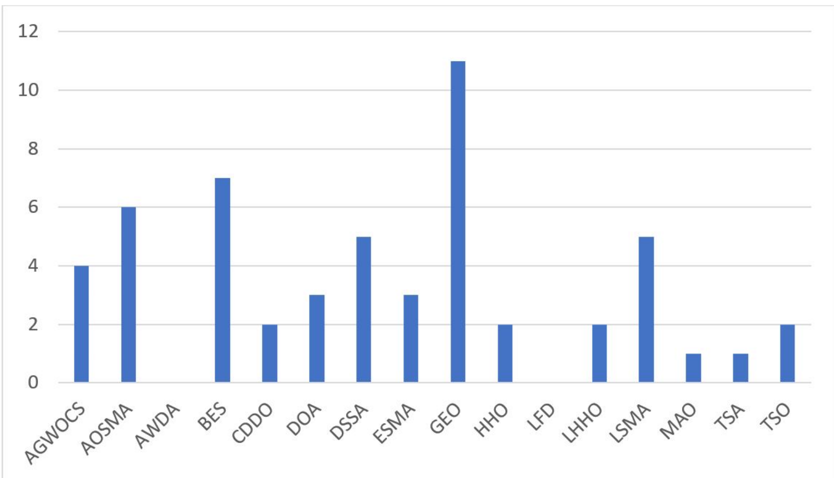

Figure 3.4: Performance histogram of the algorithms depending on the benchmark functions used

4. Conclusion

This paper compares the performance of sixteen meta-heuristic algorithms inspired by natural events. In this work, uni-modal, multi-modal, and fixed-dimension multi-modal benchmark functions were utilized to evaluate the efficiency of the optimization algorithms (AWDA, MAO, TSA, TSO, ESMA, DOA, LHHO, DSSA, LSMA, AOSMA, AGWOCS, CDDO, GEO, BES, LFD, HHO). The functions used contain sixteen test functions of three different types to test their performance in terms of usage, avoidance of local optimum, and convergence. The results are presented in the form of tables and diagrams. For future work, the use of different types of functions with a greater variety of curvatures, slopes and intercepts in the optimization of real problems is considered.

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Competing interests

The author declare that they have no competing interests.

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