

Fibonacci and Lucas numbers of some taxa naturally distributed in North East Anatolia

Kuzeydoğu Anadolu'da doğal olarak yayılış gösteren bazı taksonların Fibonacci ve Lucas sayıları

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Abstract

In this study, general information about obtaining Fibonacci and Lucas number sequences from the rabbit problem is given. Obtaining the golden ratio and phyllotaxis ratio with Fibonacci numbers is examined. The Fibonacci and Lucas number characteristics of plants belonging to the same species are the same all over the world. The phyllotaxis ratio of 47 plant taxa and the number of cone spirals of two pines belonging to Gymnospermae distributed in North East Anatolia were determined. 34 of 47 taxa which phyllotaxis ratio investigated are herbs and 13 ones are woody/scrubs. Three of the herbaceous plants are endemic (*Psephellus straminecephalus*(Hub.-Mor.) Wagenitz, *Tripleurospermum fissurale*(Sosn.) E.Hossainand *Chesneya elegans*Fomin) and two are rare species (*Callicephalus nitens*(M.Bieb. ex Willd.) C.A.Mey.and *Cirsium rigidum*DC.). With these studies, a literature contribution was provided to the studies in the field.

Özet

Bu çalışmada tavşan probleminden yola çıkarak Fibonacci ve Lucas sayı dizilerinin elde edilmesi hakkında genel bilgiler verilmiştir. Fibonacci sayılarıyla altın oran ve yaprak diziliş oranının elde edilmesi incelendi. Aynı türe ait bitkilerin Fibonacci ve Lucas sayı özellikleri dünyanın her yerinde aynıdır. Kuzeydoğu Anadolu florasında yer alan 47 adet taksonun yaprak diziliş oranları ve iki açık tohumlu çam türünün kozalak sarmalları belirlendi. Yaprak dizilişleri incelenen 47 taksonun 34'ü otsu ve 13'ü odunsu/ çalı formundadır. Bu taksonlardan üçü endemik (*Psephellus straminecephalus*(Hub.-Mor.) Wagenitz, *Tripleurospermum fissurale*(Sosn.) E.Hossainve *Chesneya elegans*Fomin) ve iki tanesi nadir takson (*Callicephalus nitens*(M.Bieb. ex Willd.) C.A.Mey. ve *Cirsium rigidum*DC.)'dur. Bu incelemelerde alanda yapılan çalışmalara literatür katkısı sağlanmıştır.

INTRODUCTION

Early on, mathematics was seen as a tool helping to understand the facts in some branches of science by putting them into numbers and it has developed as a science in the historical process. Many scientific events have been tried to be associated with mathematics and in this way, scientificness has been tried to be added to the events. Mathematical facts and ratios have begun to be sought in many events and phenomena that approach perfection. The sequence obtained as a result of the examination of the rabbit question in the book Liber Abaci published by Leonardo Fibonacci in 1202 and the golden ratio obtained from this sequence can be given as examples. Fibonacci numbers and ratios began to be studied intensively, especially in the 1900s. From architecture to art, from mathematics to human structure, the golden ratio has been observed in many

phenomena and events approaching perfection. Examples of these are the calculation of the equatorial circumference, the array of flowers and leaves on a branch, the array of branches in trees, and the spirals in sunflowers. The Fibonacci and Lucas number properties of objects are the same all over the world in the same species. (Alfred 1965, Verner E. Hoggatt 1969, Koshy 2001, Yentür and Cevahir Öz 2013, Şahin 2021).

Turkey is very rich country from plant diversity by means of three phytogeographical regions having different climate types, altitudes from sea level to 5100 m and various soil structures, and new taxa have been determined day by day for the Turkish flora. According to recent reports, number plant taxa was reached to approximately 12400 (Güner et al. 2012, Özhatay et al. 2019). Most of them are herbs, and woody plants are represented by more or less 650 taxa (Akkemik 2018).

34.5 % out of the total numbers are endemic to our country. In the present investigation, we aimed to determine Fibonacci and Lucas number sequences of 47 plant taxa including 32 herbs (three of them are endemic and two ones are rare species) and 13 scrubs / woody plants and evaluate the importance of these characters. In one previous study, we examined Fibonacci and Lucas number samples, leaf and branch arrangement ratios in plants growing in Artvin flora (Gökcan and Özcan 2020). This study is a continuation of our previous investigation about Fibonacci and Lucas numbers of the plants naturally growing in North East Anatolia.

MATERIAL AND METHODS

Rabbit Problem, Fibonacci and Lucas Number Sequences

Leonardo Fibonacci is an Italian mathematician. He took lessons from Muslim mathematicians in North Africa, where he went on the occasion with his father, and learned the Arabic number system. He saw the beauty of the Arabic number system, in contrast to the Roman number system used in Italy. For this reason, he wrote the book Liber Abaci, which includes information on arithmetic, algebra and the Arabic number system. The most important effect of this book was the transfer of the

Arabic number system to the western world. As a result of the re-examination of the book in the 19th century, a problem in the book gave rise to a truth that will be studied a lot in mathematics and science and will be associated with many scientific facts. This problem was the rabbit problem. The rabbit question in Leonardo Fibonacci's book published in 1202 can be summarized as follows:

On the first day of January, there is a pair of rabbits in the cage, and these rabbits will give birth to a pair of rabbits on the first day of each subsequent month. These pairs of baby rabbits will become adults in a month, and adult couples will be able to give birth to a new pair of rabbits a month later. How many pairs of rabbits will there be after a year if reproduction continues in this way, so that the rabbits in the cage do not die?

In the solution of the problem, the adult rabbit pairs in the cage should be represented by X , and the baby rabbit pairs by Y . In this case, the table below showing the adult and baby pairs according to the months can be given. In addition, let the number of adult rabbit pairs be denoted by X_n , the number of baby rabbit pairs by Y_n , and the total number of rabbit pairs in the relevant month by Z_n , where n is the number of the month.

Table 1.Numbers and distribution of couples according to months

Date	Couples	X_n	Y_n	$Z_n = X_n + Y_n$
1 January	X	1	0	1
1 February	XY	1	1	2
1 March	XYX	2	1	3
1 April	$XYXXY$	3	2	5
1 May	$XYXXYXYX$	5	3	8
1 June	$XYXXYXYXXYXY$	8	5	13
1 July	$XYXXYXYXXYXYXXYXYXXYXY$	13	8	21
1 August	$XYXXYXYXXYXYXXYXYXXYXYXXYXYXXYXY$	21	13	34

Some conclusions are obtained from Table 1:

Each of the columns X_n, Y_n and Z_n in Table 1 creates a sequence. If the terms of the sequence are examined, it is seen that the sum of the consecutive terms gives the next term. Recurrence relation $X_n = X_{n-1} + X_{n-2}$

defined to identify other terms of sequence for $n \geq 3$, with initial conditions $X_1 = 1$ and $X_2 = 1$. Value of X_n for n^{th} month is equal to value of Y_n for $(n + 1)^{th}$ month. In other words, $X_n = Y_{n+1}$. The sum of X_n and Y_n for n^{th} month is equal to X_n for $(n + 1)^{th}$ month. Then, $X_n +$

$Y_n = X_{n+1}$. $X_{n-1} = Y_n$ is obtained from $X_n = Y_{n+1}$. So, recurrence relation $X_{n-1} + X_n = X_{n+1}$ is obtained.

Let n^{th} term of Fibonacci sequence denoted by F_n for $n \geq 1$ with initial conditions $F_1 = 1$ and $F_2 = 1$. This sequence corresponds to sequence X_n in Table 1. Therefore, recurrence relation $F_{n-1} + F_n = F_{n+1}$ can be defined. Some terms of Fibonacci sequences can be given as follows.

$$F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8,$$

$$F_7 = 13, F_8 = 21, F_9 = 34, F_{10} = 55$$

The Fibonacci name for the sequence was given by the French mathematician Lucas because the terms of the sequence were derived from a problem in the book Liber Abaci (Verner E Hoggatt 1969, Koshy 2001).

For more information, (Alfred 1965) can be given as references.

Recurrence relation $F_{n-1} + F_n = F_{n+1}$ can be written in the quadratic form as $\alpha^{n+1} = \alpha^n + \alpha^{n-1}$. Then, $\alpha^2 - \alpha - 1 = 0$ for $n = 1$. The roots of quadratic equation are found as $\alpha_1 = \frac{1+\sqrt{5}}{2}, \alpha_2 = \frac{1-\sqrt{5}}{2}$.

Lucas number sequence is defined by using $L_n = F_{n+1} + F_{n-1}$ by Lucas. Some terms of Lucas number sequence can be obtained as follows:

$$L_1 = F_2 + F_0 = 1 + 0 = 1, L_2 = F_3 + F_1 = 2 + 1 = 3,$$

$$L_3 = F_4 + F_2 = 3 + 1 = 4, L_4 = F_5 + F_3 = 5 + 2 = 7,$$

$$L_5 = F_6 + F_4 = 8 + 3 = 11$$

In other words, Lucas number sequence can be obtained by using recurrence relation $L_{n+1} = L_n + L_{n-1}$ with initial conditions $L_1 = 1$ and $L_2 = 3$.

Golden Ratio

After the discovery of the Fibonacci sequence, many scientific facts were associated with this sequence and its terms. One of the terms obtained with the terms of this sequence is the golden ratio. There is a golden ratio in

nature and in the construction of many structures that have fascinated humanity from past to present.

There are many methods for obtaining the golden ratio. One of them is the obtained with the terms of the Fibonacci sequence. The golden ratio is the ratio of the elements of the consecutive Fibonacci sequence from the largest to the smallest at infinity.

$$\frac{F_2}{F_1} = 1, \frac{F_3}{F_2} = 2, \frac{F_4}{F_3} = \frac{3}{2} = 1.5, \frac{F_5}{F_4} = \frac{5}{3} = 1.666 \dots,$$

$$\frac{F_6}{F_5} = \frac{8}{5} = 1.6, \frac{F_7}{F_6} = \frac{13}{8} = 1.625, \dots$$

For $n \rightarrow \infty, \lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} \cong 1.618 \dots$ golden ratio is obtained.

Fibonacci and Lucas Numbers in Plants

How many branches will be on a tree, which branch will vegetate from where, how many leaves will be on the branch and the order of these leaves on the branch, the positions and shapes of the flowers blooming on the branch are formed according to a rule determined by miraculous measures.

Each plant has a unique branching and leaf array. Plants of the same species in different parts of the world show similar characteristics. Plants can be classified based on these characteristics.

Array shapes differ according to the plant species, either circular or spiral. In these arrays, one leaf is positioned in such a way that it does not shade the sunlight of the other, and there is a certain mathematical ratio in the array of the leaves on the branch.

The first two leaves on a tree branch vegetate at an angle of 180° to each other. The third leaf vegetates from the right of the first two leaves by making an angle. From here, four leaves vegetate at an angle of 90° to each other.

The number of turns and leaf numbers in plants, starting from one leaf and reaching the same directional leaf, gives the Fibonacci numbers. For example, there are 2 leaves in one turn in blackwood and linden ($F_2 =$

1, $F_3 = 2$), 3 leaves in one turn on beech ($F_2 = 1, F_4 = 3$), 5 leaves in 2 turns on an apple tree ($F_3 = 2, F_5 = 5$) and 13 leaves in 5 turns in larch ($F_5 = 5, F_7 = 13$). If the ratio in the leaf array is taken as $\frac{\text{Number of Turns}}{\text{Number of Leaves}}$, it becomes $\frac{F_2}{F_3} = \frac{1}{2}$ for blackwood and linden, $\frac{F_2}{F_4} = \frac{1}{3}$ for beech, $\frac{F_3}{F_5} = \frac{2}{5}$ for apple tree and $\frac{F_5}{F_7} = \frac{5}{13}$ for larch.

Plant Collection

Plant samples used in this investigation were obtained from naturally distributed habitats in North East Anatolia (Figures 1-3). The taxa are arranged in alphabetical order according to family names and their collections data are listed in Table 2 and 3. Local names of plants were obtained from Güner et al. (2012). Specimens were identified by Prof. Dr. Melahat ÖZCAN according to Davis (1965-1982), dried according to standard herbarium

techniques and deposited in Artvin Coruh University Herbarium (ARTH).

RESULTS AND DISCUSSION

This research includes the findings obtained from the studies carried out by expanding the sample of our previous research (Gökcan and Özcan 2020) to the flora of North East Anatolia. These findings provided the opportunity to extend our previous results. The study is a continuation of the previous research on the compatibility with Fibonacci sequences and ratios of endemic plants in the Artvin flora. Other plant specimens in the North East flora were included in the research, and it was determined that these plants were formed in accordance with the Fibonacci sequence and ratios.

Table 2. Phyllotaxis ratios in some North East Anatolian taxa. + rare species, *: endemic species

Family	Taxon	Local name	Circle number/ leaf number	
Astreaceae	<i>Achillea biserrata</i> M. Bieb.	Aksırıkotu	$\frac{F_5}{F_6} = \frac{5}{8}$	
	<i>Achillea arabica</i> Kotschy	Hanzabel	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_4}{F_5} = \frac{3}{5}$	
	<i>Callicephalus nitens</i> (M.Bieb. ex Willd.) C.A.Mey.+	Parlak düğme	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_2}{F_3} = \frac{1}{2}$	
	<i>Carlina biebersteinii</i> Hornem.	Kaya diken	$\frac{F_4}{F_5} = \frac{3}{5}$	
	<i>Carlina oligocephala</i> Boiss. & Kotschy	Domuzdiken	$\frac{F_4}{F_5} = \frac{3}{5}, \frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Centaurea macrocephala</i> Puschk. ex Willd.	Sarıbaş	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_4}{F_5} = \frac{3}{5}, \frac{F_2}{F_3} = \frac{1}{2}, \frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Centaurea salicifolia</i> M.Bieb. subsp. <i>abbreviata</i> K.Koch.	Ordu serçebaşı	$\frac{F_2}{F_3} = \frac{1}{2}, \frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Cirsium adjaricum</i> Sommier & Levier	Kızıl Kobuk	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_2}{F_3} = \frac{1}{2}$	
	<i>Cirsium aggregatum</i> Ledeb.	Top kangal	$\frac{F_4}{F_5} = \frac{3}{5}, \frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Cirsium hypoleucum</i> DC.	Vişne kangalı	$\frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Cirsium obvallatum</i> (M.Bieb.) Fisch.	Dağ kangalı	$\frac{F_3}{F_4} = \frac{2}{3}$	
	<i>Cirsium pubigerum</i> DC. var. <i>glomeratum</i> (Frey & Sint.) P.H.Davis & Parris	Dere kangalı	$\frac{F_3}{F_4} = \frac{2}{3}$	
	Asteraceae	<i>Cirsium rigidum</i> DC.*	Erken kangal	$\frac{F_3}{F_4} = \frac{2}{3}$
		<i>Cirsium simplex</i> C.A.Mey. subsp. <i>armenum</i> (DC.) Petr.	Posof kangalı	$\frac{F_2}{F_3} = \frac{1}{2}, \frac{F_3}{F_4} = \frac{2}{3}, \frac{F_4}{F_5} = \frac{3}{5}$

Table 2 (Continued). Phyllotaxis ratios in some North East Anatolian taxa. + rare species, *: endemic species

Family	Taxon	Local name	Circle number/ leaf number
Asteraceae	<i>Cyanus cheiranthifolius</i> (Willd.) Wagenitz var. <i>purpurescens</i> (DC.) Wagenitz	Gökbaş, perpatikan	$\frac{F_3}{F_4} = \frac{2}{3}$
	<i>Leucanthemum vulgare</i> Lam.	Ay papatyä	$\frac{L_2}{L_3} = \frac{3}{4}, \frac{L_1}{L_2} = \frac{1}{3}$
	<i>Matricaria matricarioides</i> (Less.) Porter ex Britton	Kelkız çiçeği	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}, \frac{F_4}{F_5} = \frac{3}{5}$
	<i>Psephellus pulcherrimus</i> (Willd.) Wagenitz	Zarif tülübaş	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_2}{F_3} = \frac{1}{2}$
	<i>Psephellus straminicephalus</i> (Hub.- Mor.) Wagenitz*	Tortum tülübaşı	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{L_1}{L_2} = \frac{1}{3}$
	<i>Tanacetum balsamitoides</i> Sch.Bip.	Marsuvanotu	$\frac{F_3}{F_4} = \frac{2}{3}$
	<i>Tanacetum coccineum</i> (Willd.) Grierson subsp. <i>chamaemelifolium</i> (Sommier & Levier) Grierson	Pire otu	$\frac{F_4}{F_5} = \frac{3}{5}$
	<i>Telekia speciosa</i> (Schreb.) Baumg.	Puğre	$\frac{L_2}{L_3} = \frac{3}{4}, \frac{F_3}{F_4} = \frac{2}{3}$
	<i>Tripleurospermum fissurale</i> (Sosn.) E.Hossain*	Seki papatyası	$\frac{F_3}{F_4} = \frac{2}{3}$
	Brassicaceae	<i>Cardamine bulbifera</i> (L.) Crantz	Dişlikök
Campanulaceae	<i>Campanula alliarifolia</i> Willd.	Akçan	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{L_2}{L_3} = \frac{3}{4}$
Fabaceae	<i>Chesneya elegans</i> Fomin*	Hoş çesneya	$\frac{F_2}{F_3} = \frac{1}{2}$
	<i>Hedysarum huetii</i> Boiss.	Acı mercimek	$\frac{F_2}{F_3} = \frac{1}{2}$
	<i>Lathyrus aphaca</i> L.var. <i>biflorus</i> Post.	Sarı burçak	$\frac{L_1}{L_3} = \frac{1}{4}$
	<i>Lathyrus cicera</i> L.	Çimen burçak/colban	$\frac{F_2}{F_3} = \frac{1}{2}$
Fabaceae	<i>Lathyrus roseus</i> Steven	Gül mürdümüğü	$\frac{L_1}{L_3} = \frac{1}{4}, \frac{F_2}{F_3} = \frac{1}{2}$
	<i>Lathyrus vernus</i> (L.) Bernh.	Bahar külürü	$\frac{F_2}{F_3} = \frac{1}{2}$
Lamiaceae	<i>Origanum rotundifolium</i> Boiss.	Yuvarlak mercan	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}, \frac{F_2}{F_3} = \frac{1}{2} = \frac{3}{6}$
	<i>Salvia glutinosa</i> L.	Oklu şalba	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}$
Rosaceae	<i>Rubus saxatilis</i> L.	Köslek	$\frac{F_2}{F_3} = \frac{1}{2}$

Table 3. Phyllotaxis ratio (branching ratio) in some scrubs or woody plants

Family	Scrubs/woody plants	Local name	Circle number/ leaf number per branch
Anacardiaceae	<i>Cotinus coggygria</i> Scop.	Boyacı sumağı, Peruka çalısı	$\frac{F_4}{F_5} = \frac{3}{5}$
	<i>Rhus coriaria</i> L.	Sumak	$\frac{F_4}{F_5} = \frac{3}{5}$
Cistaceae	<i>Cistus salviifolius</i> L.	Kartli	$\frac{L_1}{L_3} = \frac{1}{4}, \frac{L_1}{L_2} = \frac{1}{3} = \frac{2}{6}$ $\frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}$

Table 3 (Continued). Phyllotaxis ratio (branching ratio) in some scrubs or woody plants

Family	Scrubs/woody plants	Local name	Circle number/ leaf number per branch
Ericaceae	<i>Rhododendron caucasicum</i> Pall.	Dağ kumarı	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{3}{6}, \frac{F_4}{F_5} = \frac{3}{5}$ $\frac{L_2}{L_3} = \frac{3}{4}$
Fagaceae	<i>Quercus petraea</i> (Matt.) Liebl. Subsp. <i>iberica</i> (Steven ex M. Bieb.) Krassiln.	Ballık meşesi	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}$
Oleaceae	<i>Jasminum fruticans</i> L.	Yasemin, boruk	$\frac{F_3}{F_4} = \frac{2}{3}, \frac{L_2}{L_3} = \frac{3}{4}$
Rosaceae	<i>Crataegus monogyna</i> Jacq.	Yemişen	$\frac{F_3}{F_5} = \frac{2}{5}, \frac{F_4}{F_5} = \frac{3}{5}$
	<i>Rubus idaeus</i> L.	Ahududu	$\frac{F_4}{F_5} = \frac{3}{5}, \frac{F_3}{F_4} = \frac{2}{3}$
Rhamnaceae	<i>Rhamnus microcarpa</i> Boiss.	Kaya çehrisi	$\frac{F_2}{F_3} = \frac{1}{2}$
Thymelaeaceae	<i>Daphne pontica</i> L.	Sırmağuşu	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{2}{4}, \frac{F_3}{F_5} = \frac{2}{5}$
	<i>Daphne glomerata</i> Lam.	Ezentere	$\frac{F_2}{F_3} = \frac{1}{2} = \frac{3}{6}$
	<i>Daphne oleoides</i> Schreb.	Gövçek	$\frac{F_4}{F_5} = \frac{3}{5}, \frac{L_2}{L_4} = \frac{3}{7}$
Ulmaceae	<i>Celtis plachoniana</i> L.	Çitlenbik	$\frac{F_2}{F_3} = \frac{1}{2}$
Pinaceae ((Gymnospermae)	<i>Pinus pinea</i> L.	Fıstık çamı	$F_6 = 8$ spirals from center to right $F_7 = 13$ spirals from center to left
	<i>Pinus sylvestris</i> L. var. <i>hamata</i> Steven	Sarıçam	$F_6 = 8$ spirals from center to right $F_7 = 13$ spirals from center to left



Figure 1. Natural habitus of plants. a: *Achillea biserrata*, b: *Achillea arabica*, c: *Carlina oligocephala*, d: *Centaurea macrocephala*, e: *Cirsium adjaricum*, f: *Cirsium aggregatum*, g: *Cirsium hypoleucum*, h: *Cirsium obvallatum*, i: *Cirsium pubigerum* var. *glomeratum*, j: *Cirsium rigidum*, k: *Cirsium simplex* subsp. *armenum*, l: *Cyanus cheiranthifolius* var. *purpurescens*, m: *Leucanthemum vulgare*, n: *Matricaria matricarioides*, o: *Psephellus pulcherrimus*, p: *Tanacetum coccineum* subsp. *chamaemelifolium*.

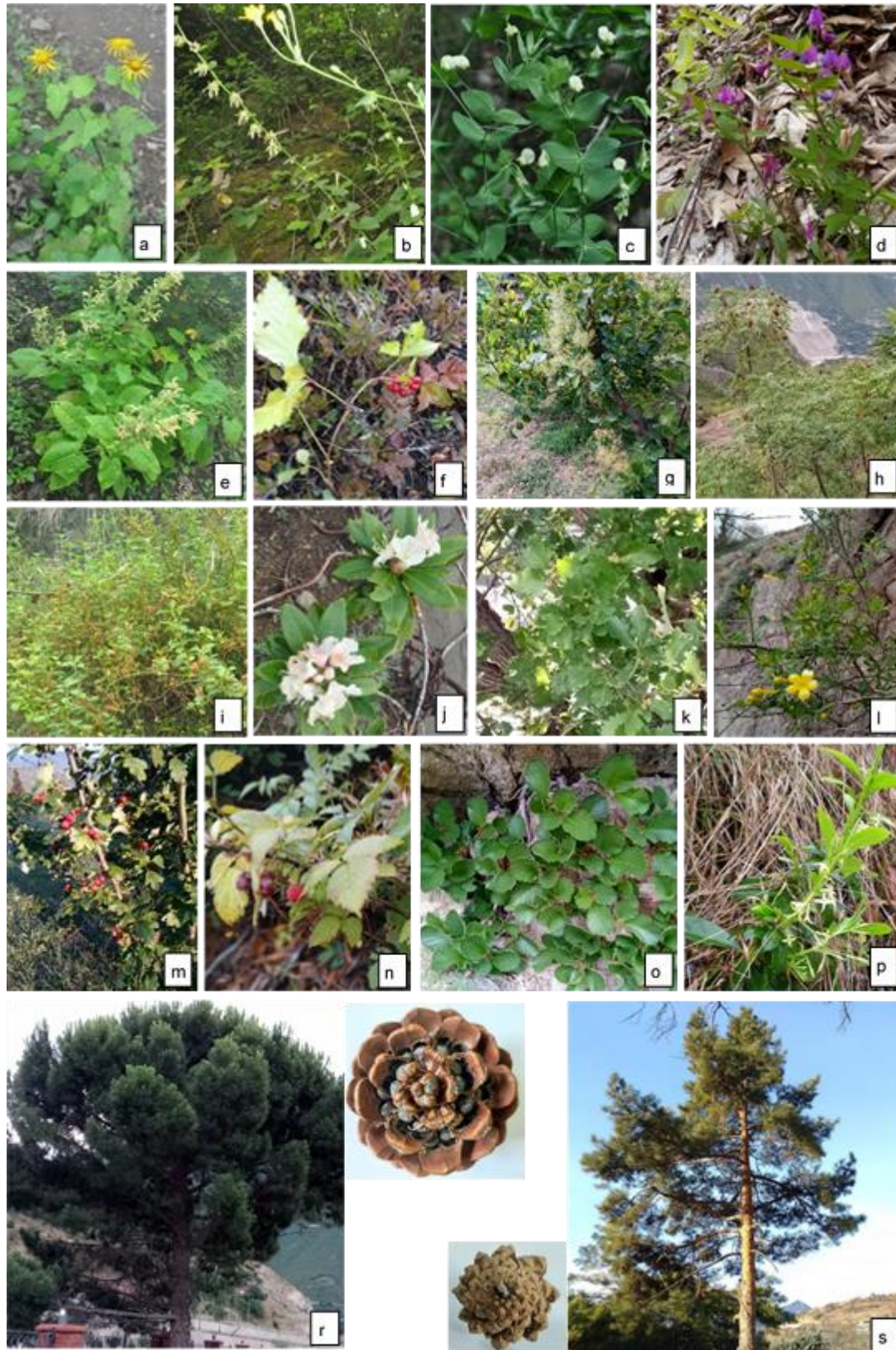


Figure 2. Natural habitus of plants. a: *Telekia speciosa*, b: *Campanula alliarifolia*, c: *Lathyrus aphacavar. biflorus*, d: *Lathyrus vernus*, e: *Salvia glutinosa*, f: *Rubus saxatilis*, g: *Cotinus coggygria*, h: *Rhus coriaria*, i: *Cistus salviifolius*, j: *Rhododendron caucasicum*, k: *Quercus petraea* subsp. *iberica*, l: *Jasminum fruticans*, m: *Crataegus monogyna*, n: *Rubus idaeus*, o: *Rhamnus microcarpa*, p: *Daphne pontica*, r: *Pinus pinea* with cone(on the right), s: *Pinus sylvestris* var. *hamata* with cone(on the left).

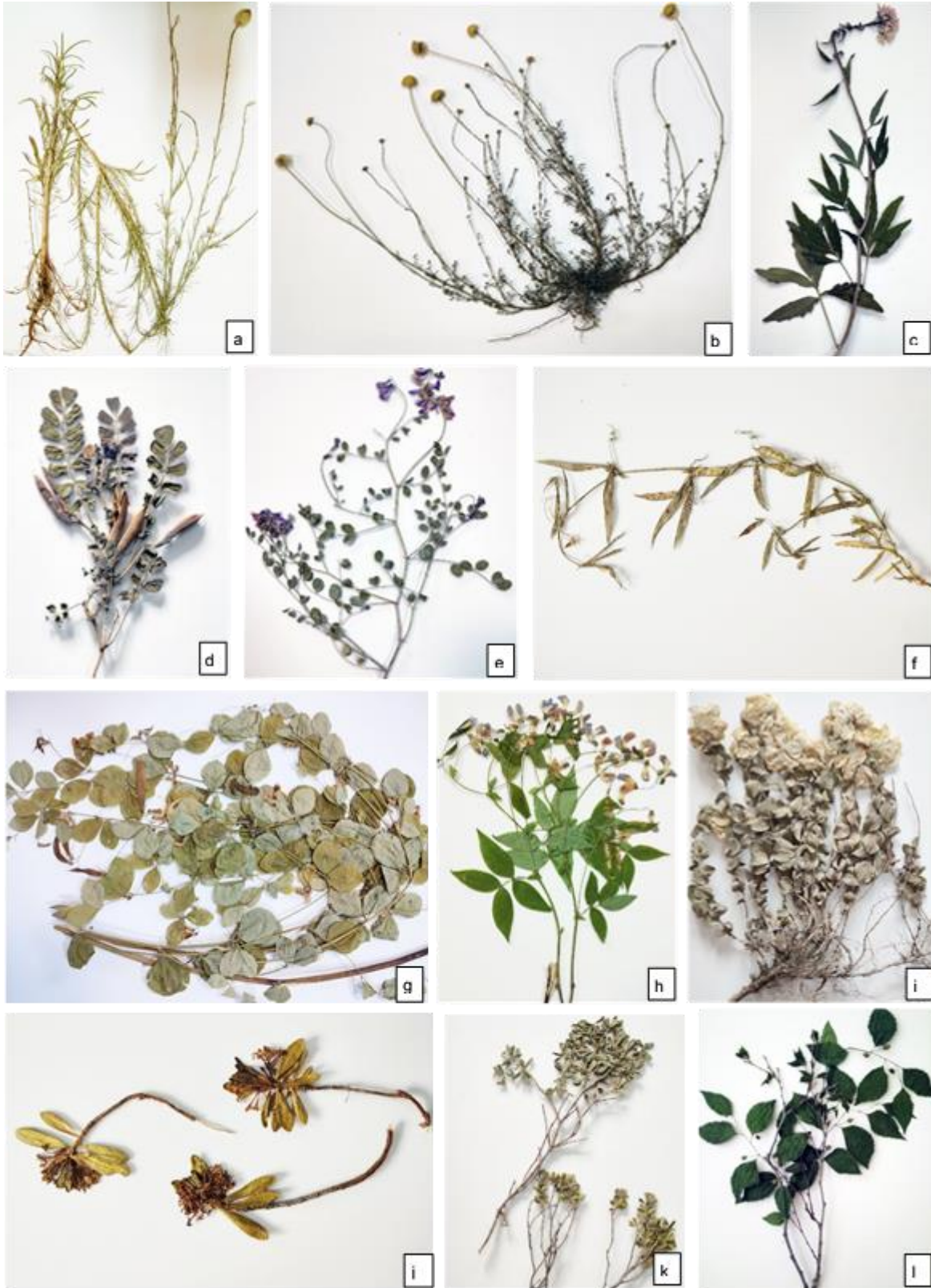


Figure 3. Herbarium samples of plants. a: *Callicephalus nitens*, b: *Tripleurospermum fissurale*, c: *Cardamine bulbifera*, d: *Chesneya elegans*, e: *Hedysarum huetii*, f: *Lathyrus cicera*, g: *Lathyrus roseus*, h: *Lathyrus vernus*, i: *Origanum rotundifolium*, j: *Daphne glomerata*, k: *Daphne oleoides*, l: *Celtis plachoniana*.

In conclusion, with this study, which is the continuation of the study in (Gökcan and Özcan 2020), the Fibonacci and

Lucas number characteristics of 47 taxa in the North East Anatolian flora were examined. Since the Fibonacci and

Lucas number characteristics of the plants are universal in plants belonging to the same species, the universal values of these species were recorded.

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