Journal of Business, Economics & Finance ISSN: 2146-7943

Journal of Business, Economics & Finance
Year: 2013 Volume: 2 Issue: 3

MIXTURE DISTRIBUTION APPROACH IN FINANCIAL RISK ANALYSIS

Keziban Kocak[1], Nazif Calis[2], Deniz Unal[1]*

[1]Department of Statistics, Faculty of Arts and Sciences, Çukurova University, Adana, Turkey.
Email: kkocak@student.cu.edu.tr, dunal@cu.edu.tr.
[2]Department of Management, Faculty of Economic and Administrative Sciences, Adıyaman University, Adıyaman Turkey. Email: ncalis@adiyaman.edu.tr.

KEYWORDS

Financial risk, value-at-risk (VaR), normal mixture distribution.

ABSTRACT

In recent years, major changes occurred in the prices of stock exchange appeared the necessity of measuring the financial risk. Nowadays, Value-at-Risk (VaR) is often used to calculate the financial risk. Parametric methods which need normality are mostly used in the calculation of VaR.If the financial data does not fit the normal distribution, mixture of normal distribution models can be fitted to this data. In this study, the financial risk is calculated by using normal mixture distribution models as a new approach to parametric method.

1. INTRODUCTION

In daily life almost everything involves a risk that is in all areas of life lots of people and company exposed to risk. For the finance and economy, uncertainty of future returns cause a risk and this risk has an increasing importance.Financial Risks can be defined as incidents that affect the strategies and goals of an organization negatively. This risk also causes it to gain less than expected and may damage the organization.The financial risks which investors and operations come across are categorized as systematic and unsystematic risks.Systematic risks occur as a result of unexpected economical events and effect the whole organization negatively.Unsystematic risks do not occur in the whole economy but they are the risks which occur in according to the change of the organization's situation due to the fluctuations on the cost of instruments such as stock certificates, bond and treasury notes (Aven, T. 2008).The risks that might occur in banking do not only affect that field but also affects the whole economical system,however it can be reduced by a successful management.The major changes occurred in the prices of stock exchange revealed the necessity of measuring the financial risk. The firms' works about the financial risk measurement started in 1970's. VaR which was developed by Morgan (1994) is used to measure the highest loss in stock exchange in a certain confidence level and in a certain time period.

The basic presentation of VaRin mathematical form is;

$$\mathrm{Var} = \left(\mu + z_{1-\alpha} \times \sigma \times \sqrt{t}\right) \times A \quad (1)$$

whereμ is average return, $1-\alpha$is confidence level, σ is standard deviation, $t$is given timeperiod and$A$ is the amount of investment.

VaRcan be calculated by using different methodsbut the simplest way of VaR calculation is the Parametric Method.However there is normality assumption in the parametric method and also based on the variation of portfolio. In this method for the portfolio incomes, each portfolio's variation should be computed. When there is a tailed normal structure for financial data or this data has not a normal distribution, calculated VaR value will not reflect the real risk level.

If the financial data has a heterogeneous structure, the normal distribution doesn't fit the data, properly. In this case financial risk can be calculated by using a mixture of normal distribution model approach in the Parametric Method (Alexander, 2008).Alexander (2008) used EM (Expectation-Maximization) algorithm with the maximum likelihood method to estimate the unknown parameters of mixture of normal distribution model. In addition to Alexander (2008), in case of moderate and volatile money policy within a certain time, Dardac and Grigore (2011) showed that using mixture of normal distribution models in Parametric method for estimating portfolio returns will give more accurate results than Historical Simulation and Monte-Carlo Simulation.

In this paper, the daily changes of the shares of Tofas, Turkcell, Vestel, Ulker, Eczacıbaşı from IMKB (İstanbul Stock Exchange) are taken and using them, a portfolio created. Also four currency units (Euro/TL, Dollar/TL, Pound/TL, Franc/TL) and a portfolio created from these units are used as a new financial data sets. For these data sets normality tests are applied and the VaR is calculated by using normal and mixture of normal distributions.

2. MIXTURE OF NORMAL DISTRIBUTION MODELS

Mixture distribution models are more appropriate distribution models for the heterogeneous data structures in many areas.In mixture distribution models, the most appropriate method to estimate the unknown parameters is EM algorithm with maximum likelihood estimation method (Dempster et al., 1977).

In univariate case, the probability density function of X is as follows (Çalış, 2005);

$$p(x) = \pi_1 f_1(x) + \cdots + \pi_g f_g \qquad (x \in C) \quad (2)$$

where C is the sample space and X's are random variables. In this case X has finite mixture distribution. Where $\pi_1, \pi_2, \dots, \pi_g$are the mixture weights and $f_1(.), f_2(.), \dots f_g(.)$ are functions providing the following properties:

$$0 \leq \pi_i \leq 1 \qquad (i = 1, \dots, g) \quad (3)$$

$$\pi_1 + \pi_2 + \cdots + \pi_g = 1 \quad (4)$$

$$f_i(.) \geq 0 \quad (5)$$

$$\int_c f_i\,(.)\,dx = 1 \qquad (i = 1, \dots, g) \quad (6)$$

Here the mixture of normal distribution model with two components may be as follows,

$$p(x|\Psi) = \pi\Phi(x|\mu_1, \sigma) + (1-\pi)\Phi(x|\mu_2, \sigma) \quad (7)$$

where$\Phi(x|\mu_1, \sigma); (i = 1,2)$ is normally distributed with meanμ_i and varianceσ^2 for the univariate case. Also $\pi_1 = \pi,\ \pi_2 = 1 - \pi$ are mixture weights of mixture of normal distribution model with two components.

3. DATA AND METHODOLOGY

In this section, two different portfolios created, one is from the daily changes of the shares of Tofas, Turkcell, Vestel, Ulker, Eczacıbaşı from IMKB (İstanbul Stock Exchange), and the other is from four currency units (Euro/TL, Dollar/TL, Pound/TL, Franc/TL). For these data sets normality tests are applied and the VaR is calculated by using normal and mixture of normal distributions.

3.1. Stock Certificate Data

In this section, normal distribution and mixture of normal distribution approaches are used for the Parametric method and also VaR values are calculated for the ISE-30 Index (Istanbul Stock Exchange National 30 Index). The daily changes of the shares of Tofas, Turkcell, Vestel, Ulker, Eczacıbaşı and the portfolio of these stocks from December 2, 2008 to May 14, 2012 are analyzed.

According to the normal and the mixture of normal distribution approach, VaR values are calculated for each of the Tofas, Turkcell, Vestel, Ulker, Eczacıbaşı stocks and also the portfolio of them.

For this purpose normality tests are performed and descriptive statistics that belong to portfolio data and the stocks are examined.

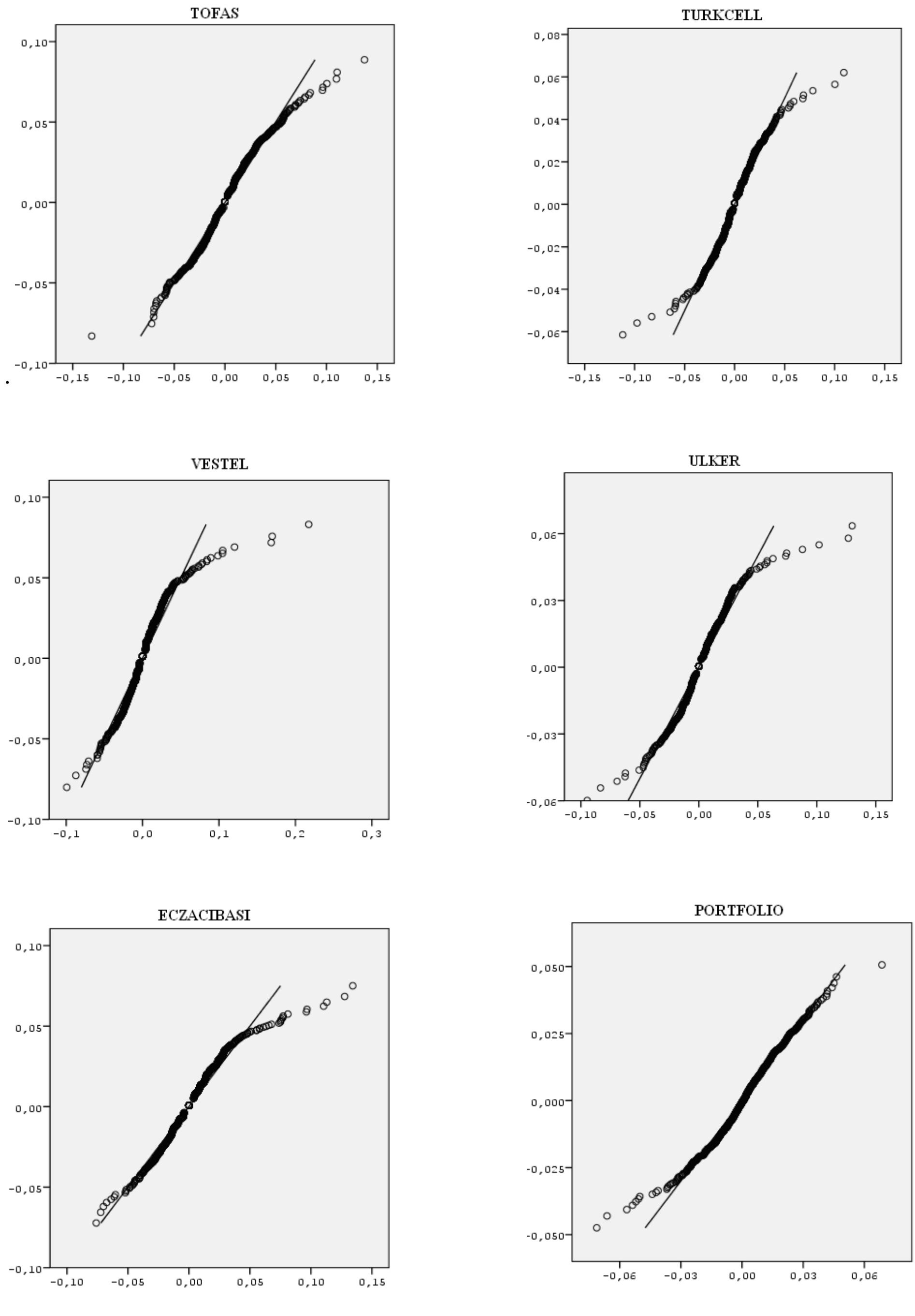
Table 1: Descriptive statistics that belong to portfolio data and stocks and Normality Tests

	Mean	**Minimum**	**Maximum**	**Standart Dev.**	**Skewness**	**Kurtosis**	**K-S test p**
PORTFOLIO	0,00158	-0,07129	0,06865	0,01539	-0,324400	1,906296	**<,001**
TOFAS	0,00280	-0,13121	0,13736	0,02695	0,339280	2,350493	**<,001**
TURKCELL	0,00029	-0,11186	0,10918	0,01938	0,023342	4,668816	**<,001**
VESTEL	0,00153	-0,09914	0,21739	0,02561	1,657750	11,35580	**<,001**
ULKER	0,00186	-0,09462	0,12998	0,01935	0,649661	6,669241	**<,001**
ECZACIBASI	0,00141	-0,07595	0,13426	0,02311	0,905236	4,334453	**<,001**

In Table 1, kurtosis, skewness and the results of the Kolmogrov-Smirnov test statistics show that the data are not normally distributed.

In order to see graphically whether the stocks and the portfolio have normal distribution or not, the Q-Q plots are constructed to see graphically.

Figure 1: Q-Q plots of stocks and the return series of the portfolio



In Q-Q plots, the distribution line of the return series follows a different pattern from the standard normal line. As a result, it can be said that return series don't correspond with the normal pattern.

Table 2: The MSE and KS (Kolmogorov-Smirnov) test statistics values which belong to normal and mixture of normal distribution models for the portfolio

Distribution	**MSE**	**KS**
Normal	18,99	0,0526
Normal Mixture	4,15*	0,0147*

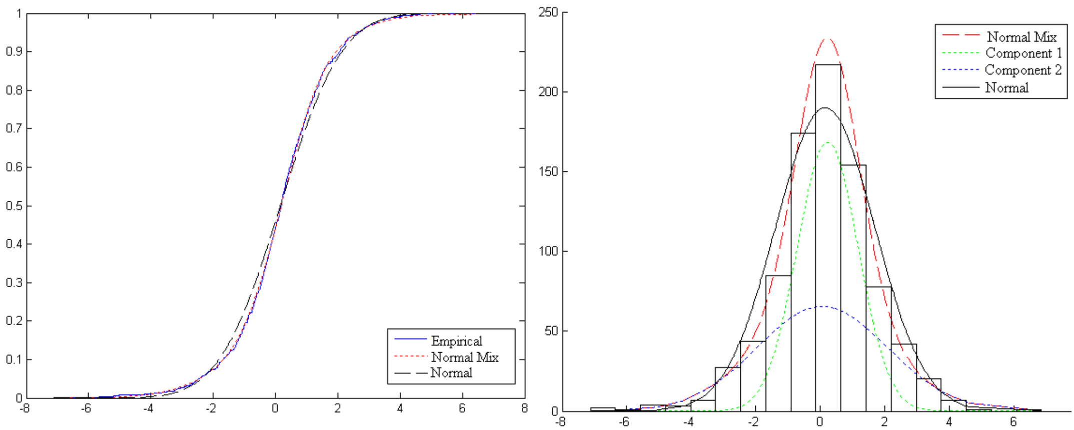
From Table 2 it can be seen that the mixture of normal distribution is more convenient for the portfolio data.

Table 3: The parameters regarding the mixture of normal distribution developed for the portfolio data (Here, π_1 and π_2 areselectedusing EM algorithm)

Component	**Mixture Weight**	**Mean**	**Covariance Matrices**
Component 1	$\pi_1 = 0{,}8303$	$\mu_1 = \begin{bmatrix} 0{,}0021 \\ 0{,}0008 \\ -0{,}0003 \\ 0{,}0007 \\ 0{,}0005 \end{bmatrix}$	$\Sigma_1 = \begin{bmatrix} 0{,}00056 & 0{,}00009 & 0{,}00019 & 0{,}00015 & 0{,}00017 \\ 0{,}00009 & 0{,}00025 & 0{,}00006 & 0{,}00005 & 0{,}00006 \\ 0{,}00019 & 0{,}00006 & 0{,}00030 & 0{,}00013 & 0{,}00018 \\ 0{,}00015 & 0{,}00005 & 0{,}00013 & 0{,}00020 & 0{,}00013 \\ 0{,}00017 & 0{,}00006 & 0{,}00018 & 0{,}00013 & 0{,}00032 \end{bmatrix}$
Component 2	$\pi_2 = 0{,}1697$	$\mu_2 = \begin{bmatrix} 0{,}0021 \\ -0{,}0020 \\ 0{,}0106 \\ 0{,}0075 \\ 0{,}0060 \end{bmatrix}$	$\Sigma_2 = \begin{bmatrix} 0{,}00156 & 0{,}00007 & 0{,}00061 & 0{,}00031 & 0{,}00047 \\ 0{,}00007 & 0{,}00099 & 0{,}00016 & 0{,}00022 & 0{,}00023 \\ 0{,}00061 & 0{,}00016 & 0{,}00229 & 0{,}00051 & 0{,}00057 \\ 0{,}00031 & 0{,}00022 & 0{,}00051 & 0{,}00117 & 0{,}00039 \\ 0{,}00047 & 0{,}00023 & 0{,}00057 & 0{,}00039 & 0{,}00157 \end{bmatrix}$

Figure 2: (a) Normal, mixture of normal and empirical cumulative distribution functions for the stocks portfolio

(b) The mixture of normal distribution and the approximate distribution of the returns



(a) **(b)**

The functions of the normal, mixture of normal and empirical cumulative distributions of the stocks portfolio are shown in Figure 2 (a). For the stocks portfolio data, the mixture of normal distribution function is fitted to achieved the empirical distribution function in a more adequate level according to the normal distribution

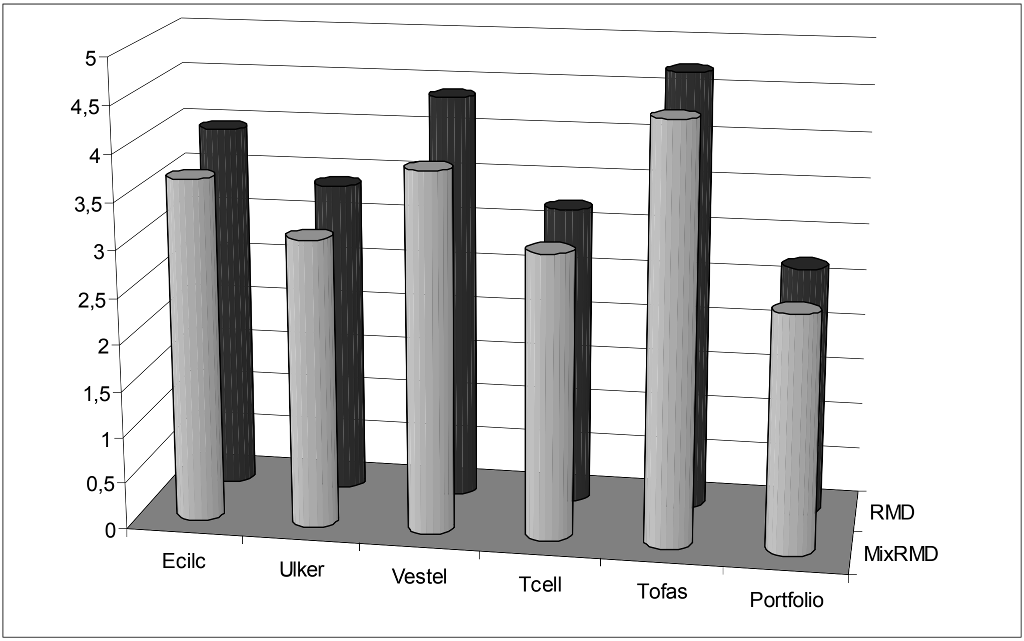
In Figure 2 (b), the approximate distribution of the returns is shown by using the mixture of normal distribution approach for the portfolio. This distribution combines 2 different normal distribution. One of them has a slighter standard deviation and this is related with a moderate market regime. The other one has a larger standard deviation that is it has a more volatile market regime.

In Table 4,VaR and Mixture VaR for the stocks and the stock portfolio are given.

Table 4: VaR and Mixture VaR values calculated for stocks and the portfolio

α=0,05	VAR	Mix VAR	W
PORTFOLIO	2,6974	2,5551	[0,2 0,2 0,2 0,2 0,2]
TOFAS	4,7274	4,4817	[1 0 0 0 0]
TURKCELL	3,2271	3,0678	[0 1 0 0 0]
VESTEL	4,3795	3,8691	[0 0 1 0 0]
ULKER	3,3797	3,1013	[0 0 0 1 0]
ECZACIBASI	3,9546	3,6855	[0 0 0 0 1]

Figure 3: The bar charts of the VaR and Mixture VaR values of Table 4



3.2. Currency Units Data

In this section, the normal distribution and the mixture of normal distribution approach are used for the Parametric method.And VaR values are calculated for the data (four currency units (Euro, Dollar, Pound, Franc) and a portfolio of them)from December 2, 2008 to May 14, 2012.

According to normal and mixture of normal approach, VaR values are calculated for each of the Euro, Dollar, Pound, Franc currencies and also the portfolio created from these currencies.

For this purpose, descriptive and normality tests are examined.

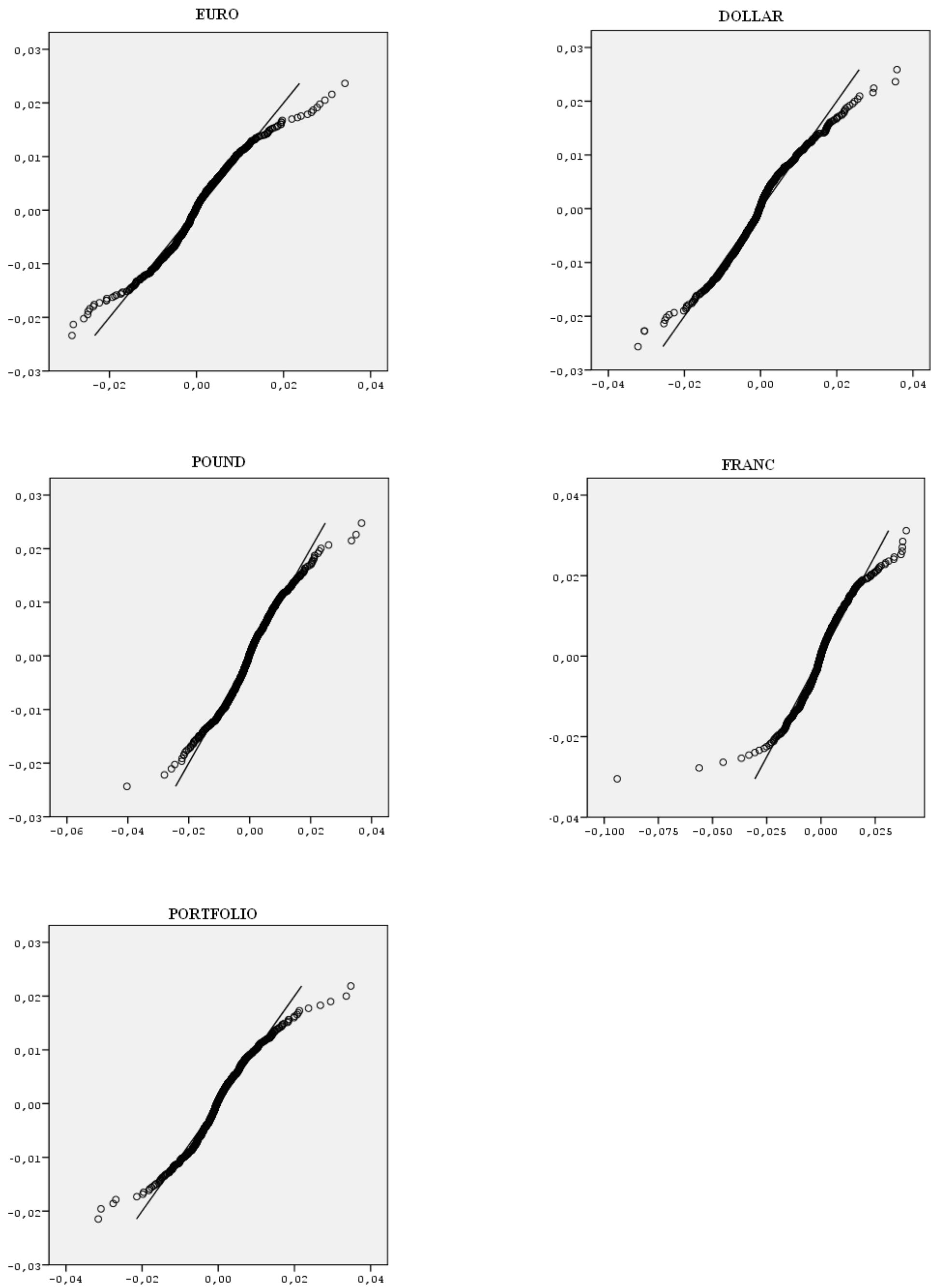
Table 5: Descriptive statistics that belong to portfolio data and currencies and the Normality Tests

	Mean	**Minimum**	**Maximum**	**Standart Dev.**	**Skewness**	**Kurtosis**	**K-S test p**
PORTFOLIO	0,00020	-0,0315	0,0348	0,00668193	0,227144	3,342304	**<,001**
EURO	0,00014	-0,0286	0,0341	0,00724964	0,218171	2,680215	**<,001**
DOLLAR	0,00011	-0,0322	0,0358	0,00794295	0,329346	2,043935	**<,001**
POUND	0,00022	-0,0403	0,0367	0,00756838	0,092997	2,618297	**<,001**
FRANC	0,00035	-0,0940	0,0393	0,00949999	-0,85117	12,07023	**<,001**

In Table 5, kurtosis, skewness and the results of the Kolmogrov-Smirnov test statistics show that the data is not normally distributed.

In order to see graphically whether the currencies and the portfolio have normal distribution, the Q-Q plots can be given.

Figure 4: Q-Q plots of currencies and the return series of the portfolio



In Q-Q plots, the distribution line of the return series follows a different pattern from the standard normal line. As a result, it can be said that return series doesn't correspond with the normal pattern.

Table 6: The MSE and KS (Kolmogorov-Smirnov) test statistics

Distribution	MSE	KS
Normal	28,138	0,0696
Normal Mixture	6,570*	0,0264*

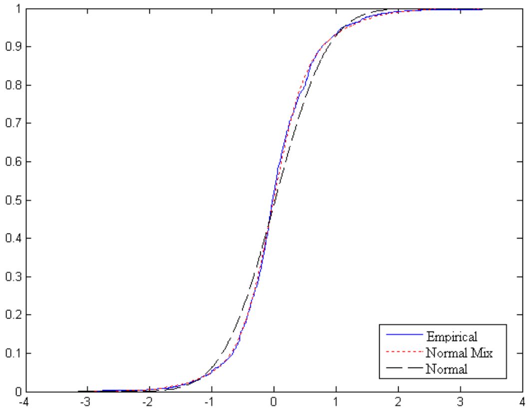
From Table 6, it can be seen that the mixture of normal distribution is more suitable for the portfolio.

Table 7: The parameters of the mixture of normal distribution for the portfolio.

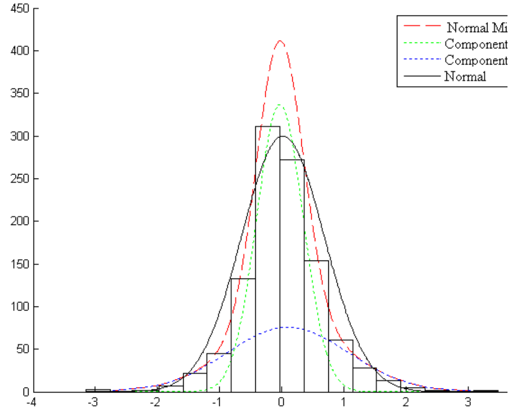
Component	Mixture weight	Mean	Covariance Matrices
Component 1	$\pi_1 = 0{,}4384$	$\mu_1 = \begin{bmatrix} 0{,}000495 \\ 0{,}001018 \\ 0{,}000773 \\ 0{,}000773 \end{bmatrix}$	$\Sigma_1 = \begin{bmatrix} 0{,}000095 & 0{,}000052 & 0{,}000060 & 0{,}00009 \\ 0{,}000052 & 0{,}000112 & 0{,}000055 & 0{,}00008 \\ 0{,}000060 & 0{,}000055 & 0{,}000101 & 0{,}00006 \\ 0{,}000093 & 0{,}000080 & 0{,}000069 & 0{,}00018 \end{bmatrix}$
Component 2	$\pi_2 = 0{,}5616$	$\mu_2 = \begin{bmatrix} -0{,}000143 \\ -0{,}000595 \\ -0{,}000219 \\ 0{,}000016 \end{bmatrix}$	$\Sigma_2 = \begin{bmatrix} 0{,}000019 & 0{,}000010 & 0{,}000014 & 0{,}00001 \\ 0{,}000010 & 0{,}000024 & 0{,}000014 & 0{,}00001 \\ 0{,}000014 & 0{,}000014 & 0{,}000023 & 0{,}00001 \\ 0{,}000017 & 0{,}000010 & 0{,}000013 & 0{,}00002 \end{bmatrix}$

Figure 5: (a) Normal, mixture of normal and empirical cumulative distribution functions for the currencies portfolio

(b) The mixture of normal distribution and the approximate distribution of the returns



(a)



(b)

The functions of the normal, mixture of normal and empirical cumulative distributions of the portfolio of currencies are shown in Figure 5 (a). For this data, the mixture of normal distribution function is fitted to the empirical distribution function in a more adequate level then the normal distribution.

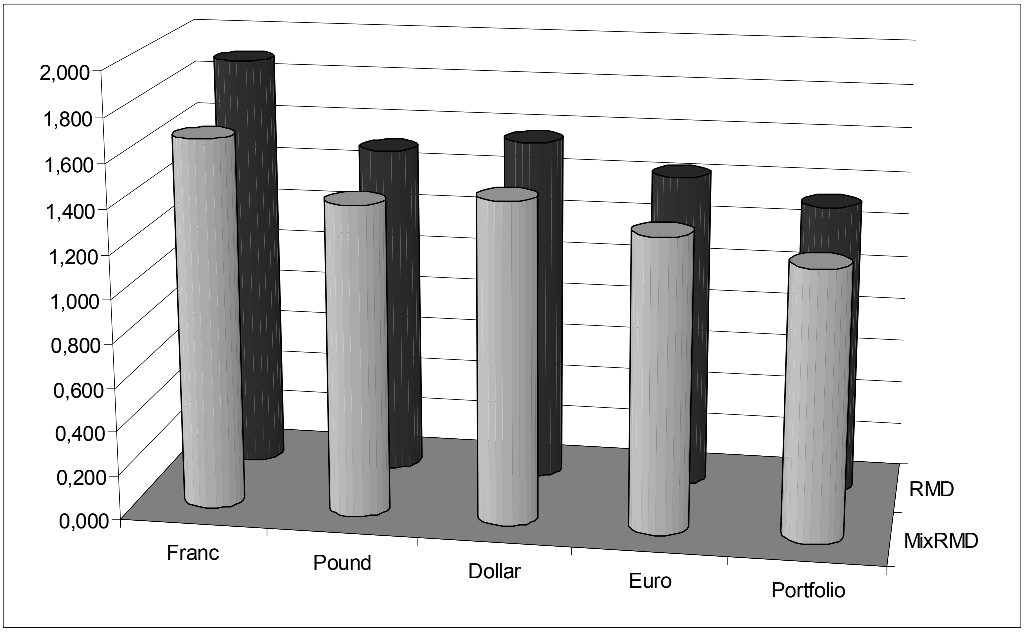
In Figure 5 (b) the approximate distribution of the returns is shown by using mixture of normal distribution.This mixture distribution combines two different Normal Distributions. One of them has a slighter standard deviation i.e. this has a moderate market regime. The other distribution has a larger standard deviation that is a more volatile market regime is available here.

In Table 8, VaR and Mixture VaR for currencies and currency portfolio are given.

Table 8: VaR and Mixture VaR values calculated for currencies and the portfolio

α**=0,05**	**VAR**	**Mix VAR**	**W**
PORTFOLIO	1.1212	1.0334	[0,25 0,25 0,25 0,25]
EURO	1.2099	1.1232	[1 0 0 0]
DOLLAR	1.3218	1.2273	[0 1 0 0]
POUND	1.2704	1.1898	[0 0 1 0]
FRANC	1.6023	1.4160	[0 0 0 1]

Figure6: The bar charts of the VaR and Mixture VaR values of Table 8.



4. CONCLUSION

In this study, the normal distribution and the mixture of normal distribution models are compared in the Parametric method to analyse the financial risks. For this purpose, VaR values are calculated for the Stock Certificate Data and Currency Units Data.

The results show that mixture of normal distribution models is more appropriate according to normal distribution for the data. And also it is seen that Mixture VaR values are less then VaR values for the normal distributions. So it can be said that Mixture VaR values are more realistic. In financial risk analysis, the Mixture VaR calculation in parametric method is a new and an alternative approach to VaR based on normal distribution. Consequently it would be more convenient to calculate VaR or Mixture VaRvalues, according to distribution of the data, to compareand to comment the results.In future studies, other methods for calculating VaR (such as Historical Simulation Method and Monte-Carlo Simulation Method) can be used and this methods can be compared totheMixturedistributionapproach.

REFERENCES

Aven, T., 2008. Risk Analysis: AssessingUncertainties Beyond ExpectedValuesandProbabilities, John Wiley & Sons.

Alexander, C., 2008. Market Risk Analysis, Value-at-Risk Models, Volume IV, John Wiley & Sons, Chichester.

Calış, N., 2005. The methods for estimating the number of components in mixture distribution models.ÇukurovaÜniversitesi, Fen BilimleriEnstitüsü, YüksekLisansTezi - Adana.

Dardac, N., Grigore, A., 2011. Modeling the Market Risk in the Context of the Basel III Acord. Theoretical and Applied Economics Volume XVIII(2011), No. 11(564), pp.5-20.

Dempster, A.P., Laird, N.M., and Rubin, D.M., 1977. Maximum likelihood form incomplete data via the EM algorithm, with discussion. Journal of the Royal Statistical Society B 39, 1-38.

Morgan, JP., 1994. "RiskMetrics".Second Edition, JP Morgan.

www.forexprostr.com.