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Numerical Methods for FGM Composites Shells and Plates

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Abstract

Main formulations for free vibration analysis of functionally graded composite shells have been given in numerical concept. Equations of motions for conical shells are listed in differential form. First-order shear deformation (FSDT) shell theory is used for obtaining the equations. Then two methods have been applied for solution. These methods are differential quadrature (DQ) and discrete singular convolution (DSC). The discrete forms of these equations have been given.

Keywords: Functionally graded composites, frequency, conical shells, annular plates, sector plates, DSC, HDQ.

1. Introduction

Functionally graded materials (FGM) are greatly used in different applications in engineering. Thus, many papers have been published for beams, plate and shell problems in order to obtain reasonable accurate results for design via different numerical methods [1-44]. By using the FSDT, the related governing equation for free vibration of conical shell can be written as

$$L_{11} + L_{12} + L_{13} + L_{14} + L_{15} - \rho h \cdot \omega^2 = 0 \tag{1}$$

$$L_{21} + L_{22} + L_{23} + L_{24} + L_{25} - \rho h \cdot \omega^2 = 0 \tag{2}$$

$$L_{31} \cdot U + L_{32} \cdot V + L_{33} \cdot W + L_{34} \cdot \Phi_x + L_{35} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \tag{3}$$

$$L_{41} \cdot U + L_{42} \cdot V + L_{43} \cdot W + L_{44} \cdot \Phi_x + L_{45} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \tag{4}$$

$$L_{51} \cdot U + L_{52} \cdot V + L_{53} \cdot W + L_{54} \cdot \Phi_x + L_{55} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \tag{5}$$

2. Solution by DSC method

By DSC method, governing differential equation of motion of truncated conical panel, Eqs. (1-5), can be discrete

$$^{DSC}L_{11}\cdot U+{}^{DSC}L_{12}\cdot V+{}^{DSC}L_{13}\cdot W+{}^{DSC}L_{14}\cdot\Phi_x+{}^{DSC}L_{15}\cdot\Phi_y-\rho h\cdot\omega^2=0 \quad (6)$$

$$^{DSC}L_{21}\cdot U+{}^{DSC}L_{22}\cdot V+{}^{DSC}L_{23}\cdot W+{}^{DSC}L_{24}\cdot\Phi_x+{}^{DSC}L_{25}\cdot\Phi_y-\rho h\cdot\omega^2=0 \quad (7)$$

$$^{DSC}L_{31}\cdot U+{}^{DSC}L_{32}\cdot V+{}^{DSC}L_{33}\cdot W+{}^{DSC}L_{34}\cdot\Phi_x+{}^{DSC}L_{35}\cdot\Phi_y-\rho h\cdot\omega^2=0 \quad (8)$$

$$^{DSC}L_{41}\cdot U+{}^{DSC}L_{42}\cdot V+{}^{DSC}L_{43}\cdot W+{}^{DSC}L_{44}\cdot\Phi_x+{}^{DSC}L_{45}\cdot\Phi_y-\rho h^3\cdot\omega^2/12=0 \quad (9)$$

$$^{DSC}L_{51}\cdot U+{}^{DSC}L_{52}\cdot V+{}^{DSC}L_{53}\cdot W+{}^{DSC}L_{54}\cdot\Phi_x+{}^{DSC}L_{55}\cdot\Phi_y-\rho h^3\cdot\omega^2/12=0 \quad (10)$$

The coefficients of L_{ij} are:

$$^{DSC}L_{11}=A_{11}\cdot\Xi_x^{(2)}+\frac{A_{11}}{R(x)}\sin\alpha\cdot\Xi_x^{(1)}-\frac{A_{22}}{R^2(x)}\cdot U(i)\cdot\sin^2\alpha+\frac{A_{33}}{R^2(x)}\cdot\Xi_s^{(2)} \quad (11)$$

$$^{DSC}L_{12}=\frac{(A_{12}+A_{33})}{R(x)}\sin\alpha\cdot\Xi_{xs}^{(2)}\frac{\partial^2 V}{\partial x\partial s}-\frac{(A_{22}+A_{33})}{R^2(x)}\sin\alpha\cdot\Xi_s^{(1)} \quad (12)$$

$$^{DSC}L_{13}=\frac{A_{12}}{R(x)}\cos\alpha\cdot\Xi_x^{(1)}-\frac{A_{22}}{R^2(x)}\cdot W(i)\cdot\sin\alpha\cdot\cos\alpha \quad (13)$$

$$^{DSC}L_{14}=B_{11}\cdot\Xi_x^{(2)}+\frac{B_{11}}{R(x)}\sin\alpha\cdot\Xi_x^{(1)}-\frac{B_{22}}{R^2(x)}\cdot\Psi_x(i)\sin^2\alpha+\frac{B_{33}}{R^2(x)}\cdot\Xi_s^{(2)} \quad (14)$$

$$^{DSC}L_{15}=\frac{(B_{12}+B_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}-\frac{(B_{22}+B_{33})}{R^2(x)}\cdot\Xi_s^{(1)}\cdot\sin\alpha \quad (15)$$

$$^{DSC}L_{21}=\frac{(A_{12}+A_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}+\frac{(A_{22}+A_{33})}{R^2(x)}\sin\alpha\cdot\Xi_s^{(1)} \quad (16)$$

$$^{DSC}L_{22}=A_{33}\Xi_x^{(2)}+A_{33}\frac{\sin\alpha}{R(x)}\Xi_s^{(1)}$$
$$-\frac{A_{33}}{R^2(x)}\cdot V(i)\cdot\sin^2\alpha+\frac{A_{22}}{R^2(x)}\Xi_s^{(2)}-\frac{A_{44}}{R^2(x)}\cdot V(i)\cdot\cos^2\alpha \quad (17)$$

$$^{DSC}L_{23}=\frac{(A_{22}+A_{44})}{R^2(x)}\cdot\cos\alpha\cdot\Xi_s^{(1)} \quad (18)$$

$$^{DSC}L_{24}=\frac{(B_{12}+B_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}+\frac{(B_{22}+B_{33})}{R^2(x)}\sin\alpha\cdot\Xi_s^{(1)} \tag{19}$$

$$^{DSC}L_{25}=B_{33}\cdot\Xi_x^{(2)}+B_{33}\frac{\sin\alpha}{R(x)}\cdot\Xi_x^{(1)}$$

$$-\frac{B_{33}}{R^2(x)}\cdot\Psi_s(i)\cdot\sin^2\alpha+\frac{B_{22}}{R^2(x)}\cdot\Xi_s^{(2)}+A_{44}\cdot\frac{\cos\alpha}{R(x)}\cdot\Psi_s(i) \tag{20}$$

$$^{DSC}L_{31}=-\frac{A_{12}}{R(x)}\cos\alpha\cdot\Xi_x^{(1)}-\frac{A_{22}}{R^2(x)}\cdot U(i)\cdot\sin\alpha\cdot\cos\alpha \tag{21}$$

$$^{DSC}L_{32}=-\frac{(A_{22}+A_{44})}{R^2(x)}\cos\alpha\cdot\Xi_s^{(1)} \tag{22}$$

$$^{DSC}L_{33}=A_{55}\cdot\Xi_x^{(2)}+\frac{A_{55}}{R(x)}\sin\alpha\cdot\Xi_s^{(1)}+\frac{A_{44}}{R^2(x)}\cdot\Xi_s^{(2)}-\frac{A_{22}}{R^2(x)}\cdot W(i)\cdot\cos^2\alpha \tag{23}$$

$$^{DSC}L_{34}=A_{55}\cdot\Xi_x^{(1)}-\frac{B_{12}}{R(x)}\cos\alpha\cdot\Xi_x^{(1)}$$

$$+\frac{A_{55}}{R(x)}\cdot\Psi_x(i)\cdot\sin\alpha-\frac{B_{22}}{R^2(x)}\cdot\Psi_x(i)\cdot\sin\alpha\cdot\cos\alpha \tag{24}$$

$$^{DSC}L_{35}=\frac{A_{44}}{R(x)}\cdot\Xi_s^{(1)}-\frac{B_{22}}{R^2(x)}\cdot\cos\alpha\cdot\Xi_s^{(1)} \tag{25}$$

$$^{DSC}L_{41}=B_{11}\cdot\Xi_x^{(2)}+\frac{B_{11}}{R(x)}\sin\alpha\cdot\Xi_x^{(1)}-\frac{B_{22}}{R^2(x)}\cdot U(i)\cdot\sin^2\alpha+\frac{B_{33}}{R^2(x)}\cdot\Xi_s^{(2)} \tag{26}$$

$$^{DSC}L_{42}=\frac{(B_{12}+B_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}-\frac{(B_{22}+B_{33})}{R^2(x)}\sin\alpha\cdot\Xi_s^{(1)} \tag{27}$$

$$^{DSC}L_{43}=-A_{55}\cdot\Xi_x^{(1)}+B_{12}\frac{\cos\alpha}{R(x)}\cdot\Xi_x^{(1)}-\frac{B_{22}}{R^2(x)}\cdot W(i)\cdot\sin\alpha\cos\alpha \tag{28}$$

$$^{DSC}L_{44}=D_{11}\cdot\Xi_x^{(2)}+D_{11}\frac{\sin\alpha}{R(x)}\cdot\Xi_x^{(1)}-\frac{D_{22}}{R^2(x)}\sin^2\alpha+\frac{D_{33}}{R^2(x)}\cdot\Xi_s^{(2)}-A_{55}\cdot\Psi_x(i) \tag{29}$$

$$^{DSC}L_{45}=\frac{(D_{12}+D_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}-\frac{(D_{22}+D_{33})}{R^2(x)}\cdot\Xi_s^{(1)}\sin\alpha \tag{30}$$

$$^{DSC}L_{51}=\frac{(B_{12}+B_{33})}{R(x)}\cdot\Xi_x^{(1)}\cdot\Xi_s^{(1)}+\frac{(B_{22}+B_{33})}{R^2(x)}\cdot\Xi_s^{(1)}\sin\alpha \tag{31}$$

$$ {}^{DSC}L_{52} = B_{33} \cdot \Xi_x^{(2)} + B_{33} \frac{\sin\alpha}{R(x)} \cdot \Xi_x^{(1)} $$

$$ - B_{33} \cdot \frac{\sin^2\alpha}{R^2(x)} \cdot V(i) + \frac{B_{22}}{R^2(x)} \cdot \Xi_s^{(2)} + \frac{A_{44}}{R(x)} \cdot \cdot V(i) \cdot \cos\alpha \tag{32} $$

$$ {}^{DSC}L_{53} = -\frac{A_{44}}{R(x)} \cdot \Xi_s^{(1)} + \frac{B_{22}}{R^2(x)} \cos\alpha \cdot \Xi_s^{(1)} \tag{33} $$

$$ {}^{DSC}L_{54} = \frac{(D_{12} + D_{33})}{R(x)} \cdot \Xi_x^{(1)} \cdot \Xi_s^{(1)} + \frac{(D_{22} + D_{33})}{R^2(x)} \sin\alpha \cdot \Xi_s^{(1)} \tag{34} $$

$$ {}^{DSC}L_{55} = D_{33} \cdot \Xi_x^{(1)} + D_{33} \frac{\sin\alpha}{R(x)} \cdot \Xi_x^{(1)} $$

$$ - \frac{D_{33}}{R^2(x)} \cdot \Psi_s(i) \cdot \sin^2\alpha + \frac{D_{22}}{R^2(x)} \cdot \Xi_s^{(2)} - A_{44} \cdot \Psi_s(i) \tag{35} $$

DSC derivation is given as

$$ \Xi_x^n(\,) = \frac{\partial^{(n)}(\,)}{\partial x^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k \cdot \Delta x)(\,)_{i+k,j} \tag{36} $$

$$ \Xi_s^n(\,) = \frac{\partial^{(n)}(\,)}{\partial s^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k \cdot \Delta s)(\,)_{i,j+k} \tag{37} $$

$$ \Xi_x^1 \Xi_s^{(n-1)}(\,) = \frac{\partial^{(n)}(\,)}{\partial x \cdot \partial s^{(n-1)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k \cdot \Delta x)(\,)_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k \cdot \Delta s)(\,)_{i,k+j} \tag{38} $$

$$ \Xi_x^{(n-1)} \Xi_s^1(\,) = \frac{\partial^{(n)}(\,)}{\partial x^{(n-1)} \partial s} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k \cdot \Delta x)(\,)_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k \cdot \Delta s)(\,)_{i,k+j} \tag{39} $$

3. Solution by DQ method

If DQ used above derivation can be define as

$$ \Xi_x^n(*) = \frac{\partial^{(n)}(*)}{\partial x^{(n)}} = \sum_{k=1}^{N} C_{i+k,j}^{(n)}(i)(*)_{i+k,j} \tag{40} $$

$$ \Xi_s^n(*) = \frac{\partial^{(n)}(*)}{\partial s^{(n)}} = \sum_{k=1}^{N} C_{i,j+k}^{(n)}(j)(*)_{i,j+k} \tag{41} $$

$$\Xi_x^1 \Xi_s^{(n-1)}(*) = \frac{\partial^{(n)}(*)}{\partial x \cdot \partial s^{(n-1)}} = \sum_{k=1}^{N} C_{i+k,j}(i) \sum_{k=1}^{N} C_{i,k+j}^{(n-1)}(j)(*)_{i,k+j} \tag{42}$$

$$\Xi_x^{(n-1)} \Xi_s^1(*) = \frac{\partial^{(n)}(*)}{\partial x^{(n-1)} \partial s} = \sum_{k=1}^{N} C_{i,k+j}^{(n-1)}(j) \sum_{k=1}^{N} C_{i+k,j}^{(1)}(i)(*)_{i,k+j} \tag{43}$$

C_{ijk} are weighting coefficients. The equations of motion are:

$${}^{DQ}L_{11} \cdot U + {}^{DQ}L_{12} \cdot V + {}^{DQ}L_{13} \cdot W + {}^{DQ}L_{14} \cdot \Phi_x + {}^{DQ}L_{15} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \tag{44}$$

$${}^{DQ}L_{21} \cdot U + {}^{DQ}L_{22} \cdot V + {}^{DQ}L_{23} \cdot W + {}^{DQ}L_{24} \cdot \Phi_x + {}^{DQ}L_{25} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \tag{45}$$

$${}^{DQ}L_{31} \cdot U + {}^{DQ}L_{32} \cdot V + {}^{DQ}L_{33} \cdot W + {}^{DQ}L_{34} \cdot \Phi_x + {}^{DQ}L_{35} \cdot \Phi_y - \rho h \cdot \omega^2 = 0 \tag{46}$$

$${}^{DQ}L_{41} \cdot U + {}^{DQ}L_{42} \cdot V + {}^{DQ}L_{43} \cdot W + {}^{DQ}L_{44} \cdot \Phi_x + {}^{DQ}L_{45} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \tag{47}$$

$${}^{DQ}L_{51} \cdot U + {}^{DQ}L_{52} \cdot V + {}^{DQ}L_{53} \cdot W + {}^{DQ}L_{54} \cdot \Phi_x + {}^{DQ}L_{55} \cdot \Phi_y - \rho h^3 \cdot \omega^2 / 12 = 0 \tag{48}$$

In FGM material some properties are not constant:

$$E(z) = (E_c - E_m)V_c + E_m \tag{49}$$

$$\upsilon(z) = (\upsilon_c - \upsilon_m)V_c + \upsilon_m \tag{50}$$

For example if four-parameter power law is used then volume fractions are given for two cases.

$$\text{Case-1} \quad V_f = \left[1 - a\left(\frac{z}{h} + \frac{1}{2}\right) + b\left(\frac{z}{h} + \frac{1}{2}\right)^c\right]^p \tag{51}$$

$$\text{Case-2:} \quad V_f = \left[1 - a\left(-\frac{z}{h} + \frac{1}{2}\right) + b\left(-\frac{z}{h} + \frac{1}{2}\right)^c\right]^p \tag{52}$$

4. Conclusion

These equations can also be used for circular cylindrical shell and panel, annular, circular plates, sector and annular sector plates. Each methods have own advantages. But for higher modes, the method of DSC is more effective.

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