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MODAL ANALYSIS OF TAPERED BEAM-COLUMN EMBEDDED IN WINKLER ELASTIC FOUNDATION

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Abstract

Modal analysis of tapered piles embedded in elastic foundations is investigated. The pile is modeled via Bernoulli-Euler beam theory and discrete singular convolution is used for modeling. Some parametric results have been presented for tapered pile in elastic foundation.

Keywords: Beam-column; elastic foundation; tapered piles; discrete singular convolution.

1. Introduction

There are different type problems related to soil-structure interaction can be modeled by means of a beam or a beam-column on an elastic foundation. Winkler foundation model is extensively used by engineers and researchers because of its simplicity. The analysis of beam-columns on elastic foundations have been carried out in the literature, namely by Zhaohua and Cook [1], Yankelevsky and Eisenberger [2], Doyle and Pavlovic [3], Yokoyama [4], Valsangkar and Pradhanang [5], De Rosa and Maurizi [6], Halabe and Jain [7], West and Mafi [8], Matsunaga [9] and Kameswara et al. [10]. In this paper, discrete singular convolution method technique is presented for computation of the free vibration analysis of a pile embedded in elastic foundation. The method of DSC is used for vibration response of tapered piles in elastic foundation [29].

2. Discrete Singular Convolution (DSC)

Discrete singular convolution (DSC) method is a recently proposed numerical method in science and applied mechanics [11-14]. The method of discrete singular convolution (DSC) was proposed to solve linear and nonlinear differential equations by Wei [15, 16], and later it was introduced to solid and fluid [17-19]. It has been also successfully employed for different vibration problems of structural members such as plates and shells [20–23]. For more details of the mathematical background and application of the DSC method in solving problems in engineering, the readers may refer to some recently published reference [24-38]. In the context of distribution theory, a singular convolution can be defined by [11]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

where T is a kind of singular kernel such as Hilbert, Abel and delta type, and $\eta(t)$ is an element of the space of the given test functions. In the present approach, only singular kernels of delta type are chosen. This type of kernel is defined by [12]

$$T(x) = \delta^{(r)}(x); \qquad (r = 0,1,2,...,). \tag{2}$$

where subscript r denotes the rth-order derivative of distribution with respect to parameter x. In order to illustrate the DSC approximation, consider a function $F(x)$. In the method of DSC, numerical approximations of a function and its derivatives can be treated as convolutions with some kernels. According to DSC method, the r*th* derivative of a function $F(x)$ can be approximated as [13]

$$F^{(r)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(r)}(x_i - x_k) f(x_k); \quad (r=0,1,2,...,). \tag{3}$$

where Δ is the grid spacing, x_k are the set of discrete grid points which are centered around x, and $2M+1$ is the effective kernel, or computational bandwidth. It is also known, the regularized Shannon kernel (RSK) delivers very small truncation errors when it use the above convolution algorithm. The regularized Shannon kernel (RSK) is given by [14]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \ \sigma > 0 \tag{4}$$

The researchers is generally used the regularized delta Shannon kernel by this time. The required derivatives of the DSC kernels can be easily obtained using the below formulation [15]

$$\delta_{\Delta,\sigma}^{(r)}(x - x_j) = \frac{d^r}{dx^r}\left[\delta_{\Delta,\sigma}(x - x_j)\right]\Big|_{x = x_i}, \tag{5}$$

3. Fundamental Equations

The governing equations for free vibration of tapered beam-column embedded in Winkler foundation (Fig. 1) using the Euler-Bernoulli beam theory can be written as:

$$EI(x)\frac{\partial^4 w}{\partial x^4} + p\frac{\partial^2 w}{\partial x^2} + kw - \rho A\frac{\partial^2 w}{\partial t^2} = 0, \quad \text{for } 0 < x < L_s \tag{6}$$

$$EI(x)\frac{\partial^4 w}{\partial x^4} + p\frac{\partial^2 w}{\partial x^2} - \rho A\frac{\partial^2 w}{\partial t^2} = 0. \quad \text{for } L_s < x < L \tag{7}$$

The transverse displacement w is assumed to be

$$w(x,t) = W(x)e^{i\omega t} \tag{8}$$

Substituting expression (8) into equations (6-7) and normalizing the equation yields

$$\frac{EI(x)}{L^4}\frac{d^4W}{dX^4}+\frac{p}{L^2}\frac{d^2W}{dX^2}+kW-\rho A\omega^2 W=0, \tag{9}$$

$$\frac{EI(x)}{L^4}\frac{d^4W}{dX^4}+\frac{p}{L^2}\frac{d^2W}{dX^2}-\rho A\omega^2 W=0. \qquad \gamma < X < 1 \tag{10}$$

in which EI is the flexural rigidity of beam-column, w is the transverse deflection, p is the applied axial load, k is the Winkler parameter, ρ is the mass density, A is the cross-sectional area, I the second moment of area of cross-section, E the Young's modulus, and ω is the circular frequency.

Non-dimensional variables are given below:

X=x/L, W=w/L and γ=L_s/L

By using these non-dimensional quantities, Eqs. (9-10) can be written as

$$\frac{d^4W}{dX^4}+P\frac{d^2W}{dX^2}+KW-\Omega^2 W=0, \tag{11a}$$

$$\frac{d^4W}{dX^4}+P\frac{d^2W}{dX^2}-\Omega^2 W=0. \tag{11b}$$

where

$$P=pL^2/EI_0;\ K=kL^4/EI_0;\ \Omega^2=\omega L^2\sqrt{\rho A_0/EI_0} \tag{12}$$

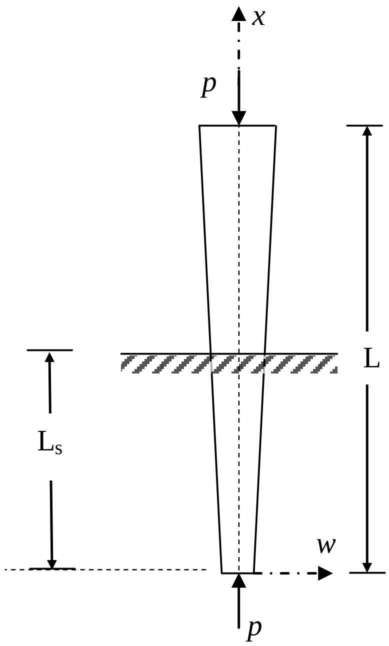

Figure 1: Tapered piles and cross-section embedded in elastic foundation

The taper ratios are given as

$$\alpha = h_1 / h_0 \text{ and } \beta = b_1 / b_0 \tag{13}$$

Using DSC discretization the Eq. (8) takes the form

$$\sum_{j=1}^{N} \delta_{\pi/\Delta,\sigma}^{(4)}(\Delta x) W(x_i) + P \sum_{j=1}^{N} \delta_{\pi/\Delta,\sigma}^{(2)}(\Delta x) W(x_i) + K W(x_j) = \Omega^2 W_i , \tag{14a}$$

$$\sum_{j=1}^{N} \delta_{\pi/\Delta,\sigma}^{(4)}(\Delta x) W(x_i) + P \sum_{j=1}^{N} \delta_{\pi/\Delta,\sigma}^{(2)}(\Delta x) W(x_i) = \Omega^2 W_i \tag{14b}$$

Pinned boundary conditions are considered for both edges. Related equations are given as

$$W = 0 \quad \text{and} \quad EI_0 \frac{\partial^2 W}{\partial x^2} = 0 \tag{15}$$

After implementation of the given boundary conditions in Eqs. (14a) and (14b) can be expressed by

$$[\mathbf{R}]\{\mathbf{U}\} = \omega^2 \{\mathbf{U}\}, \tag{16}$$

where $\mathbf{U}$ is the displacements vector, $\mathbf{R}$ is the stiffness matrix.

4. Numerical Examples

Some results for mode shapes are provided in Figs. 2-5 and Table 1. In these results the value P_{cr} is the critical buckling load. In order to comparison with the results calculated by SAP

2000 structural analysis program, the results provided in dimensional form in this last example. The dimensions of the beam-column embedded in Winkler foundation studied in this example are: the length is $L=4$ m; the mass density is $\rho = 7849\ kg/m^3$; the elasticity modulus is $E=2\times10^8$ kN/m^2. 41-Node frame elements are used for modeling of beam-column via SAP 2000 [28].

Table 1. Comparison study of circular frequency parameters (rad/sec) of Euler-Bernoulli beam column embedded in Winkler foundation (γ=0.75; P=P_{cr}; α=β=1.0)

k	SAP-2000 (41 nodes) Frame element	Present DSC N=9	Present DSC N=11	Present DSC N=15
		Mode 1 (ω_1)		
1	89.81	89.826	89.818	89.818
10	90.45	90.472	90.459	90.459
100	96.04	96.069	96.042	96.042
1000	140.02	140.102	140.023	140.023
		Mode 2 (ω_2)		
1	358.40	358.429	358.421	358.421
10	358.53	358.568	358.56	358.56
100	359.73	359.749	359.746	359.746
1000	371.49	371.501	371.495	371.495
		Mode 3 (ω_3)		
1	803.18	803.189	803.182	803.182
10	803.23	803.240	803.232	803.231
100	803.73	803.744	803.735	803.734
1000	808.71	808.733	808.728	808.726

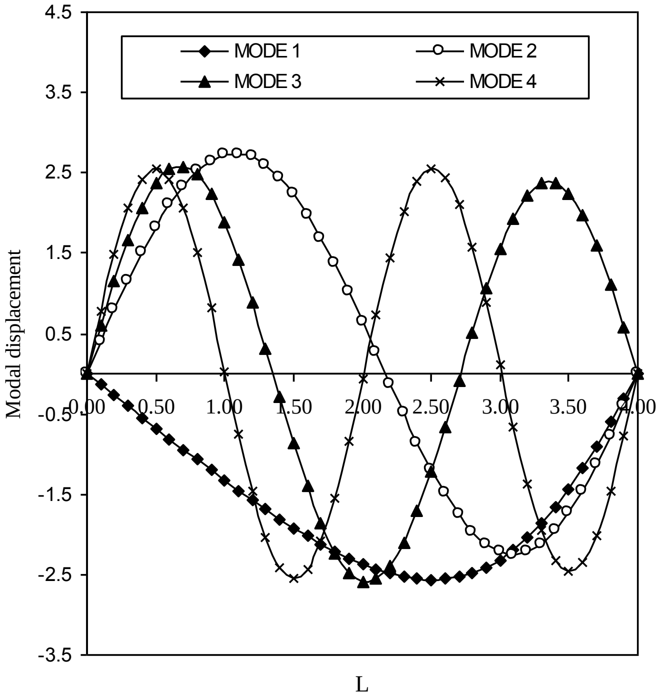

Figure 2: Variation of modal displacements with the length of beam-column (α=β=1; γ=0.75; k=10000)

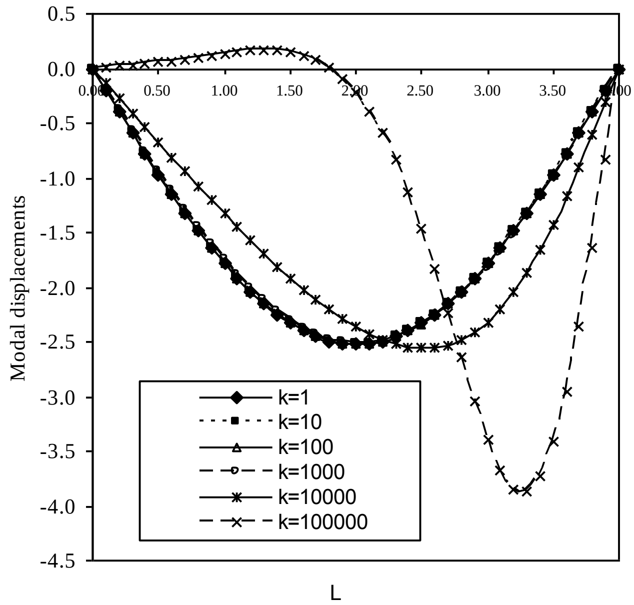

Figure 3: Variation of modal displacements with the length of beam-column for different Winkler parameters (α=β=1; γ=0.75; Mode 1)

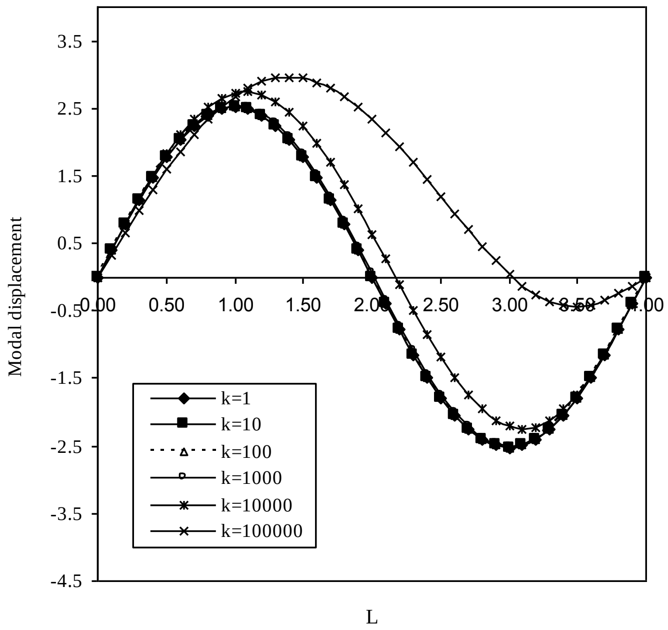

Figure 4: Variation of modal displacements with the length of beam-column for different Winkler parameters (α=β=1; γ=0.75; Mode 2)

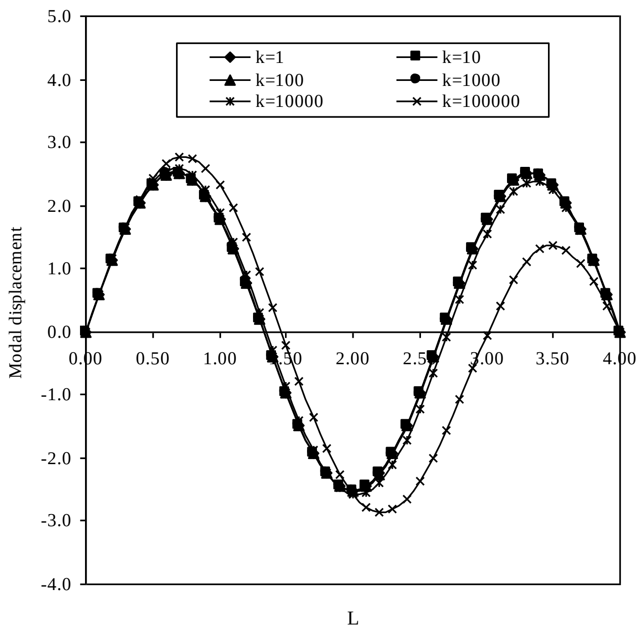

Figure 5: Variation of modal displacements with the length of beam-column for different Winkler parameters (α=β=1; γ=0.75; Mode 3)

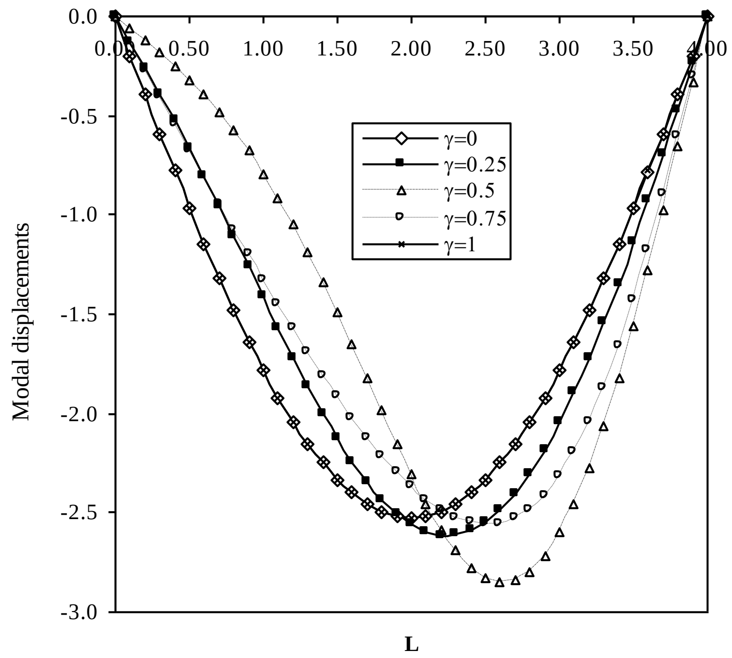

Figure 6: Variation of modal displacements with the length of beam-column for different ratio of supported length to total length of beam-column (k =10000; α=β=1; Mode 1)

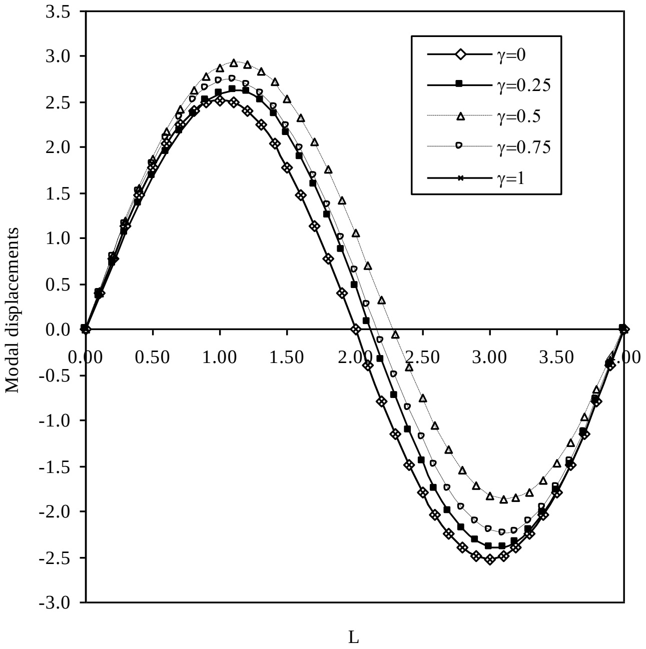

Figure 7: Variation of modal displacements with the length of beam-column for different ratio of supported length to total length of beam-column (k =10000;α=β=1; Mode 2)

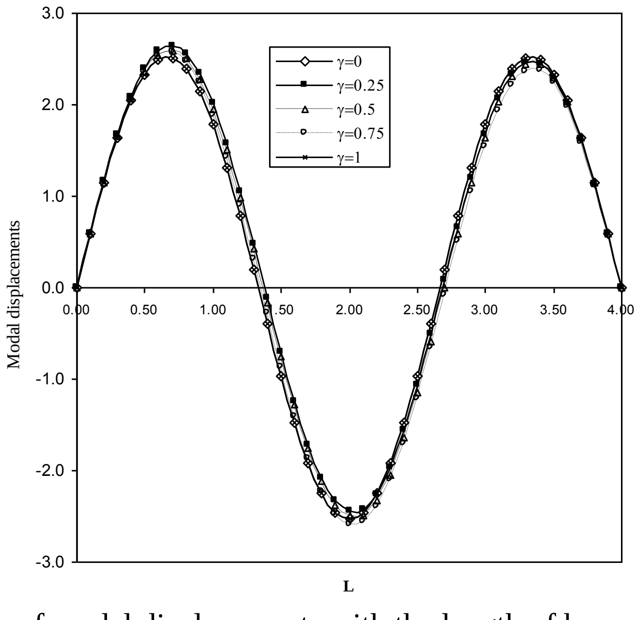

Figure 8: Variation of modal displacements with the length of beam-column for different ratio of supported length to total length of beam-column (k =10000; α=β=1; Mode 3)

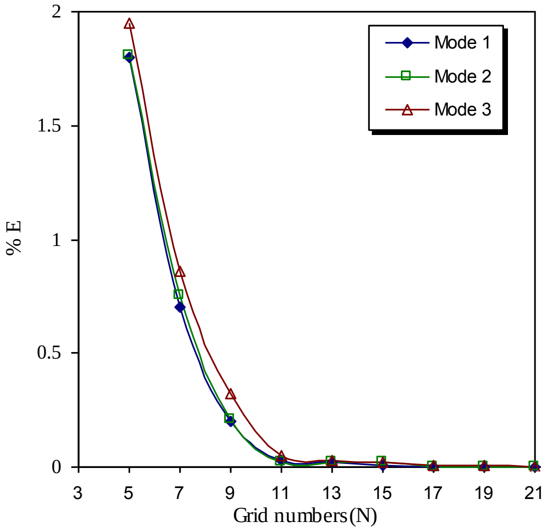

Figure 9: The variation of errors with grid numbers

It is concluded from these figures that, the effect of *k* is significant for first three mode shapes. It is clearly shown in Table 1 that, the DSC results are very good. The reasonable accurate results can be obtained for *N=11* in DSC analysis. It is also shown from these figures (Figs. 2-8), the mode shapes have a less effect for small Winkler parameter ($k \leq 1000$) and then rapidly changes with increasing value of *k*. The effect of γ on mode shapes has also been investigated and results presented in Figs. 6-8. It is concluded that, with the increase of mode numbers the effect of the γ on the mode shapes is insignificant. The effect of supported length to total length γ is more effective for the first two modes. Fig. 9 depicted the variation of errors with grid numbers for first three modes. In general, the errors are decreased with increasing value of *N*. The accurate results are obtained for *N*=9 for first second mode. For higher modes however, the accurate results are obtained for *N*=11.

5. Concluding remarks

Modal analysis of tapered piles embedded in Winkler foundation is the investigated. The efficiency and accuracy of the present method have been demonstrated on the basis of presented numerical examples.

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References

[1] Zhaohua, F., Cook, R.D., Beams elements on two-parameter elastic foundations. *ASCE Journal of Engineering Mechanics*, 109, 1390-1401, 1983.

[2] Yankelevsky, D.Z., Eisenberger, M., Analysis of a beam-column on elastic foundations. *Computers and Structures,* 23, 351-56, 1986.

[3] Doyle, P.F., Pavlovic, M.N., Vibration of beams on partial elastic foundations. *Earthquake Engineering and Structural Dynamics*, 10, 663-674, 1982.
[4] Yokoyama, T., Vibrations of Timoshenko beam-columns on two-parameter elastic foundations. *Earthquake Engineering and Structural Dynamics,* 20, 355-370, 1991.
[5] Valsangkar, A.J., Pradhanang, R., Vibrations of beam-columns on two-parameter elastic foundations. *Earthquake Engineering and Structural Dynamics,* 16, 217-225, 1988.
[6] De Rosa, M.A., Maurizi, M.J., Dynamic analysis of multistep piles on Pasternak soil subjected to axial tip forces. *Journal of Sound and Vibration*, 219, 771-783, 1999.
[7] Halabe, U.B., Jain, S.K., Lateral free vibration of a single pile with or without an axial load. *Journal of Sound and Vibration*, 195, 531-544, 1996.
[8] West, H.H., Mafi, M., Eigenvalues for beam-columns on elastic supports, *ASCE Journal of Structural Engineering*, 110, 1305-1320, 1984.
[9] Matsunaga, H., Vibration and buckling of deep beam-columns on two parameter elastic foundations, *Journal of Sound and Vibration*, 228, 359-376, 1999.
[10] Kameswara Rao, N.S.V., Das, Y.C., Anandakrishnan, M., Dynamic response of beams on generalized elastic foundation. *International Journal of Solids and Structures*, 11, 255-73, 1975.
[11] Wei, G.W., A new algorithm for solving some mechanical problems, *Computer Methods in Applied Mechanics and Engineering*, 190, 2017-2030, 2001.
[12] Wei, G.W., Vibration analysis by discrete singular convolution, *Journal of Sound and Vibration*, 244, 535-553, 2001.
[13] Wei, G.W., Discrete singular convolution for beam analysis, *Engineering Structures*, 23, 1045-1053, 2001.
[14] Wei, G.W., Zhao, Y.B., Xiang, Y., Discrete singular convolution and its application to the analysis of plates with internal supports. Part 1: Theory and algorithm, *International Journal for Numerical Methods in Engineering*, 55, 913-946, 2002.
[15] Wei, G.W., Zhao, Y.B., Xiang, Y., A novel approach for the analysis of high-frequency vibrations, *Journal of Sound and Vibration*, 257, 207-246, 2002.
[16] Zhao, Y.B., Wei, G.W., Xiang, Y., Discrete singular convolution for the prediction of high frequency vibration of plates, *International Journal of Solids and Structures*, 39, 65-88, 2002.
[17] Zhao, Y.B., Wei, G.W., Xiang, Y., Plate vibration under irregular internal supports, *International Journal of Solids and Structures*, 39, 1361-1383, 2002.
[18] Zhao, Y.B., Wei, G.W., DSC analysis of rectangular plates with non-uniform boundary conditions, *Journal of Sound and Vibration*, 255, 203-228, 2002.
[19] Civalek, Ö., An efficient method for free vibration analysis of rotating truncated conical shells, *International Journal of Pressure Vessels and Piping*, 83, 1-12, 2006.
[20] Civalek, Ö., The determination of frequencies of laminated conical shells via the discrete singular convolution method, *Journal of Mechanics of Materials and Structures,* 1, 165-192, 2006.
[21] Civalek, Ö., Nonlinear analysis of thin rectangular plates on Winkler-Pasternak elastic foundations by DSC-HDQ methods, *Applied Mathematical Modelling*, 31, 606-624, 2007.
[22] Civalek, Ö., Free vibration analysis of composite conical shells using the discrete singular convolution algorithm, *Steel and Composite Structures*, 6, 353-366, 2006.
[23] Civalek, Ö., Three-dimensional vibration, buckling and bending analyses of thick rectangular plates based on discrete singular convolution method, *International Journal of Mechanical Sciences*, 49, 752-765, 2007.

[24] Civalek, Ö., Numerical analysis of free vibrations of laminated composite conical and cylindrical shells: discrete singular convolution (DSC) approach, *Journal of Computational and Applied Mathematics*, 205, 251 – 271, 2007.
[25] Civalek, Ö., Kiracioglu, O., Numerical approach for the analysis of anisotropic rectangular plates using discrete singular convolution, *Asian Journal of Civil Engineering*, 8, 647-658, 2007.
[26] Civalek, Ö., Gürses, M., Frequency Analysis of Trapezoidal Plates and Membrane Using Discrete Singular Convolution, *Asian Journal of Civil Engineering*, 9, 593-605, 2009.
[27] Civalek, Ö., Ersoy, H., Frequency analysis of moderately thick uniform isotropic annular plates by discrete singular convolution method, *International Journal of Structural Engineering and Mechanics*, 29, 411-422, 2008.
[28] Wilson, E., Habibullah, A., SAP2000NL. Integrated finite element analysis and design of structures basic analysis, Reference Manual, *Computers and Structures*, Berkeley, CA, 1998.
[29] Civalek, Ö., Öztürk, B., Free vibration analysis of tapered beam-column with pinned ends embedded in Winkler-Pasternak elastic foundation, *Geomechanics and Engineering*, 2, 45-56, 2010.
[30] Civalek, Ö., Fundamental frequency of isotropic and orthotropic rectangular plates with linearly varying thickness by discrete singular convolution method, *Appl. Math. Model.*, 33(10), 3825-3835, 2009.
[31] Civalek, Ö., Free vibration analysis of symmetrically laminated composite plates with first-order shear deformation theory (FSDT) by discrete singular convolution method. *Finite Elem. Anal. Des.*, 44(12-13), 725-731, 2008.
[32] Civalek, Ö. Geometrically non-linear static and dynamic analysis of plates and shells resting on elastic foundation by the method of polynomial differential quadrature (PDQ), Ph. D. Thesis, Firat University, Elazig, 2004 (in Turkish), 2004.
[33] Civalek, Ö. Finite Element analysis of plates and shells, Elazığ: Fırat University, 1998.
[34] Baltacıoğlu, A.K., Civalek, Ö., Akgöz B., Demir F., Large deflection analysis of laminated composite plates resting on nonlinear elastic foundations by the method of discrete singular convolution. *Int. J. Press. Vessels Pip.*, 88 (8), 290-300, 2011.
[35] Akgöz, B., Civalek, Ö., Modeling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory. *Meccanica*, 48, 863-873, 2013.
[36] Civalek, Ö., Akgöz, B., Vibration analysis of micro-scaled sector shaped graphene surrounded by an elastic matrix. *Comput. Mater. Sci.*, 77, 295-303, 2013.
[37] Baltacıoğlu, A.K., Akgöz, B., Civalek, Ö., Nonlinear static response of laminated composite plates by discrete singular convolution method. *Compos. Struct.*, 93(1), 153-161, 2010.
[38] Civalek, Ö., Demir, C., Buckling and bending analyses of cantilever carbon nanotubes using the euler-bernoulli beam theory based on non-local continuum model, *Asian Journal of Civil Engineering*, 12, 651-661, 2011.