



## MIXED CONVECTION FLOW MICROPOLAR FLUID OVER A VERTICAL PLATE SUBJECT TO HALL AND ION-SLIP CURRENTS

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### **Abstract**

*This paper analyses the mixed convection flow of an incompressible micropolar fluid along a semi-infinite vertical plate with uniform heat and mass flux in the presence of transverse magnetic field. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using local similarity transformations and then solved numerically using the Keller-box method. The non-dimensional velocity components, microrotation, temperature and concentration are presented graphically for various values of magnetic parameter, Hall parameter, Ion-slip parameter, Dufour and Soret numbers. In addition, the Nusselt number, the Sherwood number, the skin-friction coefficient, the wall couple stress are shown in a tabular form.*

**Key words:** Mixed convection, Micropolar fluid, Hall and ion-slip, Soret and Dufour, Heat and mass transfer.

### **1. Introduction**

Convection flows driven by temperature and concentration differences have been studied vastly in the past and various extensions of these problems have been reported in the literature. Simultaneous interactions of both temperature and concentration on different geometries have been studied by several Researchers among others, Beg[1] studied free convection heat and mass transfer in Darcian porous regime with chemical reaction, Anna[2] studied heat and mass transfer between air and falling-film desiccant in a parallel-plate using simplified model, Lloy [3] studied combined forced and free convection flow on vertical surfaces, Mahdy [4] examined mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media. The contribution of mixed convection along a vertical plate has received a considerable attention because of wide-spread application in geophysical and engineering. Examples include cooling of electronic equipment, thermal insulation heating of the Trombe wall system and many other.

The Soret and Dufour effects have garnered considerable interest in both Newtonian and non-Newtonian convective heat and mass transfer. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology,

geosciences, etc. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary systems, often encountered in chemical process engineering and also in high-speed aerodynamics. Soret and Dufour effects are also critical in various flow regimes occurring in chemical and geophysical systems. Alam and Rahman [5] have investigated the Dufour and Soret effects on mixed convection flow past a vertical porous at plate with variable suction. Gorla [6] studied mixed convection in a micropolar fluid along a vertical surface with uniform heat flux. Kafoussiassetal [7] have investigated Dufour and Soret effects on mixed free forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Awad and Sibanda [8] examined Dufour and Soret effects on heat and mass transfer in a micropolar fluid in a horizontal channel. Srinivasacharya and Ramreddy [9] studied mixed convection heat and mass transfer in a micropolar fluid with soret and Dufour effect over semi-infinite vertical plate.

The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of Magneto hydrodynamic(MHD) heat transfer. This MHD heat transfer has gained significance owing to recent advancement of space technology. The MHD heat transfer can be divided into two sections. One contains problems in which the heating is an incidental by product of the electromagnetic fields as in MHD generators and pumps etc. and the second consists of problems in which the primary use of lectromagnetic fields is to control the heat transfer. Several investigators have studied the effects of magnetic fields on the convection heat and mass transfer by ignoring the Hall and Ion-slip terms in Ohms law were ignored. However, in the presence of strong magnetic field, the influence of Hall current and Ion-slip are important. Hossian and Ahmed [10] studied MHD forced and free convection flow of an electrically conducting viscous incompressible fluid past a vertical flat plate with uniform heat flux. Chamkha [11] numerically examined MHD-free convection from a vertical plate embedded in a thermally stratified porous medium with hall effects. Seddeek [12] studied numerically under the assumption small magnetic Reynolds number the effects of Hall and ion-slip currents on a magneto-micropolar fluid and the heat transfer over a non-isothermal stretching sheet with suction and blowing. Shateyietal [13] investigated the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surfaces in the presence of Hall, radiation, Soret , and Dufour effects. Salem and Abd El-Aziz [14] examine effect of hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation/absorption. Motsa and Shateyi [15] studied the effects of chemical reaction, Hall, ion-slip currents, and variable thermal diffusivity on the magnetomicropolar fluid flow, heat, and mass transfer with suction through a porous medium using successive linearization method together with the Chebyshev collocation method. Elgazery [16] analyzed numerically the problem of magneto-micropolar fluid flow, heat and mass transfer with suction and blowing through a porous medium under the effects of chemical reaction, Hall, ion-slip currents, variable viscosity and variable thermal diffusivity.

It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. The study of non-Newtonian fluid flows has gained much attention from the researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications; for instance, the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. A number of mathematical models have been proposed to explain the rheological behaviour of non-Newtonian fluids. Further, there exist several approaches to study the mechanics of fluids

with a substructure. Ericson [17, 18] derived field equations which account for the presence of substructures in the fluid. It has been experimentally demonstrated by Hoyt[19] fluids containing small amount of polymeric additives display a reduction in skin friction. Eringen [20] first formulated the theory of micropolar fluids which display the effects of local rotary inertia and couple stresses. This theory can be used to explain the flow of colloidal fluids, liquid crystal, animal blood, etc. Eringen [21] extended the micropolar fluid theory and developed the theory of thermo-micropolar fluids. Physically, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical element, polymer fluids, fluid suspension, etc. The presence of dust or smoke, particularly in a gas, may also be modelled using micropolar fluid dynamics.

The objective of the present work is to study the effects of Hall current, ion-slip, Soret and Dufour on boundary layer mixed convection MHD flow, heat and mass transfer along a vertical plate with uniform heat and mass flux conditions embedded in a micropolar fluid. The governing equations are solved numerically using a very efficient finite-difference method known as Keller-box method Cebeci [22]. The results obtained under special cases are then compared with that of Lin and Lin [23] and Yih[24] and found to agree very favourably.

## 2. Mathematical formulation

Consider a steady, laminar, incompressible, mixed convective heat and mass transfer along a semi infinite vertical plate embedded in a free stream of electrically conducting micropolar fluid under the influence of a transversally applied magnetic field. The free stream velocity which is parallel to the vertical plate is  $u_\infty$ , temperature is  $T_\infty$  and concentration is  $C_\infty$ . Choose the coordinate system such that x-axis is along the vertical plate and y-axis normal to the plate. The plate is maintained at uniform and constant heat and mass fluxes  $q_w$  and  $q_m$  respectively. A uniform magnetic field of magnitude  $B_0$  is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect and the ion slip cannot be neglected. Further, assume that all the fluid properties are constant except the density in the buoyancy term of the balance of momentum equation. In addition, the Soret and Dufour effects are considered.

Using the Boussinesq, MHD and boundary layer approximations, the governing equations for micropolar fluid in the presence of Soret and Dufour effect are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{\kappa}{\rho} \frac{\partial \Gamma}{\partial y} - \frac{(\mu + \kappa)}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho \{(1 + \beta_i B_e)^2 + B_e^2\}} \{B_e u + (1 + \beta_i B_e) w\} - g^* \{\beta_T (T - T_0) + \beta_c (C - C_0)\} = 0 \quad (2)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} - \frac{(\mu + \kappa)}{\rho} \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^2}{\rho \{(1 + \beta_i B_e)^2 + B_e^2\}} \{-B_e w + (1 + \beta_i B_e) u\} = 0 \quad (3)$$

$$j \left\{ u \frac{\partial \Gamma}{\partial x} + v \frac{\partial \Gamma}{\partial y} \right\} + 2 \frac{\kappa}{\rho} \Gamma + \frac{\kappa}{\rho} \frac{\partial u}{\partial y} - \frac{\gamma}{\rho} \frac{\partial^2 \Gamma}{\partial y^2} = 0 \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \alpha \frac{\partial^2 T}{\partial y^2} - \frac{D_f}{C_s C_p} \frac{\partial^2 C}{\partial y^2} - \frac{1}{\rho C_p} \frac{\sigma B_0^2}{(1 + \beta_i B_e)^2 + B_e^2} \{u^2 + w^2\} = 0 \quad (5)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} - D \frac{\partial^2 C}{\partial y^2} - \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} = 0 \quad (6)$$

where  $u, v, w$  are velocity components  $\Gamma$  is microrotaion,  $T$  is the temperature,  $C$  is the concentration,  $\rho$  and  $j$  are the fluid density and gyration parameter,  $\mu$  dynamic coefficient of viscosity,  $\kappa$  is vortex viscosity,  $\gamma$  is the spin gradient viscosity,  $g^*$  is the acceleration due to gravity,  $\beta_T$  is the coefficient of thermal expansion,  $\beta_c$  is the coefficient of solutal expansion,  $\alpha$  is the thermal diffusivity,  $D$  is the mass diffusivity,  $C_p$  is the specific heat capacity,  $C_s$  is the concentration susceptibility, and  $T_m$  is the mean fluid temperature,  $Be = \sigma\beta B_0$  is Hall parameter.

The boundary conditions are given by

$$\text{At } y = 0: \quad u = 0, \quad v = 0, \quad \Gamma = 0, \quad q_w = -k \frac{\partial T}{\partial y}, \quad q_m = -D \frac{\partial C}{\partial y} \quad (7a)$$

$$\text{As } y \rightarrow \infty: \quad u = u_\infty, \quad w = 0, \quad \Gamma = 0, \quad T = T_\infty, \quad C = C_\infty \quad (7b)$$

In view of equation (1) we introduce a stream function  $\psi$  by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (8)$$

substituting (8) in (2) to (6) and using the following local similarity transformations

$$\begin{aligned} \psi &= v Re_x^{\frac{1}{2}} f(\eta), \quad w = U_\infty h(\eta), \quad \Gamma = \frac{v}{x^2} Re_x^{\frac{3}{2}} g(\eta), \\ \theta(\eta) &= \frac{T - T_\infty}{q_w x} Re_x^{\frac{1}{2}}, \quad \phi(\eta) = \frac{C - C_\infty}{q_m x} Re_x^{\frac{1}{2}}, \quad \eta = \frac{y}{x} Re_x^{\frac{1}{2}} \end{aligned} \quad (9)$$

where  $Re_x = \frac{U_\infty x}{\nu}$  is the local Reynolds number, we get the following dimensionless equations

$$\begin{aligned} f'''' + \frac{1}{2}(1 - N)ff' + N g' + (1 - N)X^{3/2}[g_s\theta + g_c\phi] \\ - (1 - N) \frac{X}{Re} \frac{H_a^2}{\{(1 + \beta_i B_e)^2 + B_e^2\}} \{B_e h + (1 + \beta_i B_e)f'\} = 0 \end{aligned} \quad (10)$$

$$h'' + \frac{1}{2}(1 - N)fh' + (1 - N) \frac{X}{Re} \frac{H_a^2}{\{(1 + \beta_i B_e)^2 + B_e^2\}} \{B_e f' - (1 + \beta_i B_e)h\} = 0 \quad (11)$$

$$\lambda g'' + \frac{1}{2}gf' + \frac{1}{2}g'f - JX \frac{N}{1 - N}(2g + f'') = 0 \quad (12)$$

$$\frac{1}{Pr}\theta'' + \frac{1}{2}f\theta' - \frac{1}{2}\theta f' + D_f \phi'' + \frac{X^{1/2}Ec}{Re^{1/2}} \frac{H_a^2}{\{(1 + \beta_i B_e)^2 + B_e^2\}} \{(f')^2 + h^2\} = 0 \quad (13)$$

$$\frac{1}{Sc}\phi'' + \frac{1}{2}f\phi' - \frac{1}{2}\phi f' + Sr \theta'' = 0 \quad (14)$$

where primes denote differentiation with respect to the variable  $\eta$ ,  $N = \frac{\kappa}{\kappa + \mu}$  is coupling number,  $J = \frac{\nu L}{ju_\infty}$  is the micro inertia density,  $X = \frac{x}{L}$  is the dimensionless coordinate along the

plate,  $D_f = \frac{DK_T q_m k}{\nu C_s c_p q_w D}$  is the Dufour number,  $Sr = \frac{DK_T q_w}{\nu T_m q_m k}$  is the Soret number,  $Gr = \frac{g^* \beta_T q_w L^4}{k \nu^2}$  is thermal Grashof number,  $Gc = \frac{g^* \beta_c q_m L^4}{D \nu^2}$  is the solutal Grashof number,  $Re = \frac{u_\infty L}{\nu}$  is the Reynolds number,  $g_s = \frac{Gr}{Re^{5/2}}$  is the temperature bouncy parameter,  $g_c = \frac{Gc}{Re^{5/2}}$  is the mass bouncy parameter,  $\lambda = \frac{\gamma}{j \rho \nu}$  is the spin gradient viscosity,  $Ec = \frac{u_\infty^2 k}{c_p q_w L}$  is the Eckert number,  $H_a^2 = \frac{\sigma B_0^2 L^2}{\rho \nu}$  is Hartman number,  $Sc = \frac{\nu}{D}$  is the Schmidt number,  $Pr = \frac{\nu}{\alpha}$  is the Prandtl number.

The corresponding boundary conditions in dimensionless form are:

$$\eta = 0: \quad f(0) = 0, \quad f'(0) = 0, \quad h(0) = 0, \quad g(0) = 0, \quad \theta(0) = -1, \quad \phi(0) = -1 \quad (15a)$$

$$\eta \rightarrow \infty: \quad f'(\infty) = 1, \quad h(\infty) = 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \quad (15b)$$

The wall shear stress, the local Nusselt number ( $Nu_x$ ) and the local Sherwood number ( $Sh_x$ ), respectively are given by

$$\tau_w = [(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa \Gamma]_{y=0} \quad (16a)$$

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} \quad (16b)$$

$$Sh_x = \frac{q_m x}{k(C_w - C_\infty)} \quad (16c)$$

The non-dimensional skin friction  $C_f$ , the local Nusselt number  $Nu_x$  and Sherwood number  $Sh_x$  respectively are

$$C_f Re_x^{\frac{1}{2}} = \frac{2}{1-N} f''(0) \quad (17a)$$

$$\frac{Nu_x}{Re_x^{1/2}} = \frac{1}{\theta(0)} \quad (17b)$$

$$\frac{Sh_x}{Re_x^{1/2}} = \frac{1}{\phi(0)} \quad (17c)$$

### 3. Results and discussion

The flow equations (10) and (12) which are coupled, together with the energy and concentration equations (13) and (14), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence, these equations (10) to (14) are solved numerically using the Keller-box implicit method discussed in Cebeci and Bradshaw [22]. This method has been proven to be adequate and give accurate results for boundary layer equations. A uniform grid was adopted, which is concentrated towards the wall. The calculation are repeated until some convergence criterion is satisfied and the calculations were stopped when  $\delta f_0'' \leq 10^{-8}$ ,  $\delta h_0' \leq 10^{-8}$ ,  $\delta g_0' \leq 10^{-8}$ ,  $\delta \theta_0' \leq 10^{-8}$ ,  $\delta \phi_0' \leq 10^{-8}$

In the present study the boundary conditions for  $\eta$  at  $\infty$  are replaced by a sufficiently large value of  $\eta$  where the velocity, microrotation, temperature and concentration profiles approach zero. In order to see the effects of step size ( $\Delta\eta$ ) the code ran for our model with three different step sizes  $\Delta\eta = 0.001$ ,  $\Delta\eta = 0.01$  and  $\Delta\eta = 0.05$  and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of  $\eta$  at  $\infty$  of 10 and a grid size of  $\eta$  as 0.01.

In the limit, as  $N \rightarrow 0$ , the governing Eqs. (10-14) reduce to the corresponding equations for a mixed convection heat and mass transfer in viscous fluids. Hence, in the absence of coupling number  $N$ , Soret number  $Sr$  and Dufour number  $Df$  with  $Ha=0$ ,  $G_s = G_c = 0$  and  $J=0$  the results have been compared with the case Lin and Lin [23] also it is compared the present value of  $\frac{1}{2}C_f Re_x^{1/2}$  with Yih [24] for  $Ha = 0$ ,  $N = 0$ ,  $G_s = G_c = 0$ ,  $Df = 0$  and  $Sr = 0, X = 1$  it was found that they are in good agreement, as shown in table (1) and table (2) respectively.

Table 1: A comparison of values of Pr obtained by the viscous fluid without Ha, Df and Sr effects

Pr	Lin[23]	Present
0.1	0.20065	0.20062
1	0.45897	0.45882
10.0	0.99788	0.99776
100	2.15196	2.15459

Table 2: A comparison of values of present value of  $\frac{1}{2}C_f Re_x^{1/2}$  with Yih [24]

Yih[24]	Present
0.332057	0.33206

In the present study, we have adopted the following default parameter values for the numerical computations: Pr is taken equal to 0.71,  $G_s = 1.0$ ,  $G_c = 0.1$ ,  $j = 0.1$ ,  $Re = 10$ ,  $Sc = 0.22$ ,  $Ec = 0.1$ ,  $X = 0.5$ ,  $\lambda = 1.0$  and the values of Dufour number  $Df$  and Soret number  $Sr$  are chosen in such a way that their product is constant assuming that the mean temperature is constant are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen[21]. These values are used throughout the computations, unless otherwise indicated.

The effect of magnetic parameter on non-dimensional velocity components, microrotation, temperature and concentration is depicted in Fig.(1) It is observed from Fig. (1a) that velocity decreases as the magnetic parameter ( $Ha$ ) increases. It is found from Fig. (1b) that the non-dimensional fluid velocity component ( $h$ ) increases with increasing values of magnetic parameter. It is also note that when  $Ha$  is zero the velocity  $h$  will be zero hence no flow in the  $z$  direction. From Fig. (1c), it is clear that the microrotation component increases near the plate and decreases far away from the plate for increasing values of  $Ha$ . It is noticed from Fig. (1d) that the non-dimensional fluid temperature increases with increasing values of magnetic parameter. It is clear from Fig. (1e) that the non-dimensional fluid concentration increases with increasing values of  $Ha$ . Application of a uniform magnetic field normal to the flow direction produces a force which acts in the negative direction of flow. This force is called the

Lorentz force which tends to slow down the movement of the electrically conducting fluid in the vertical direction. This retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration. These behaviours are clearly depicted in Figs. (1)

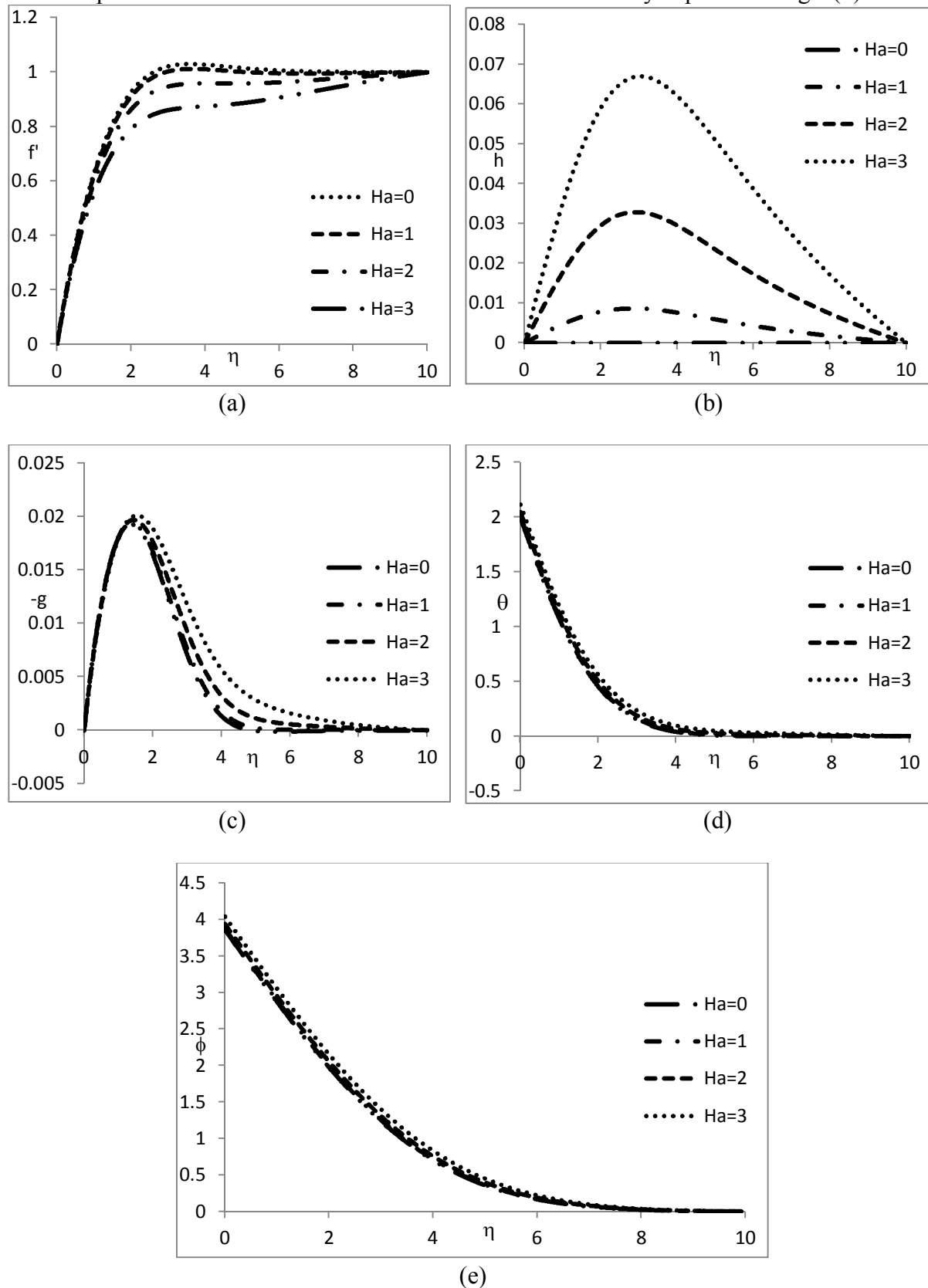


Figure 1: Effect of Magnetic parameter on (a) velocity( $f'$ ) (b) induced velocity( $h$ ) (c) microrotation (d) temperature and (e) concentration for  $N = 0.4$ ,  $Df = 2.0$ ,  $Sr = 0.03$ ,  $\beta_i = 2$ ,  $Be = 2$

The variation of non-dimensional velocity components, microrotation, temperature and concentration with Hall parameter is shown in Fig.(2). From Fig.(2a) it is noticed that an increase in the Hall parameter increases the velocity  $f'$ . As Hall parameter increases the effective conductivity also increases, in turn, decreases the damping force on velocity, and hence the velocity increases. It is observed from Fig.(2b) the velocity component  $h$  is increasing with the increase of Hall parameter. When there hall parameter is zero then there is no cross flow. The microrotation component is increasing away from the plate as the hall parameter is increasing as depicted in Fig.(2c). Figs (2d) and (2e) shows that increase in the Hall parameter decreases the temperature and concentration throughout the boundary layer.

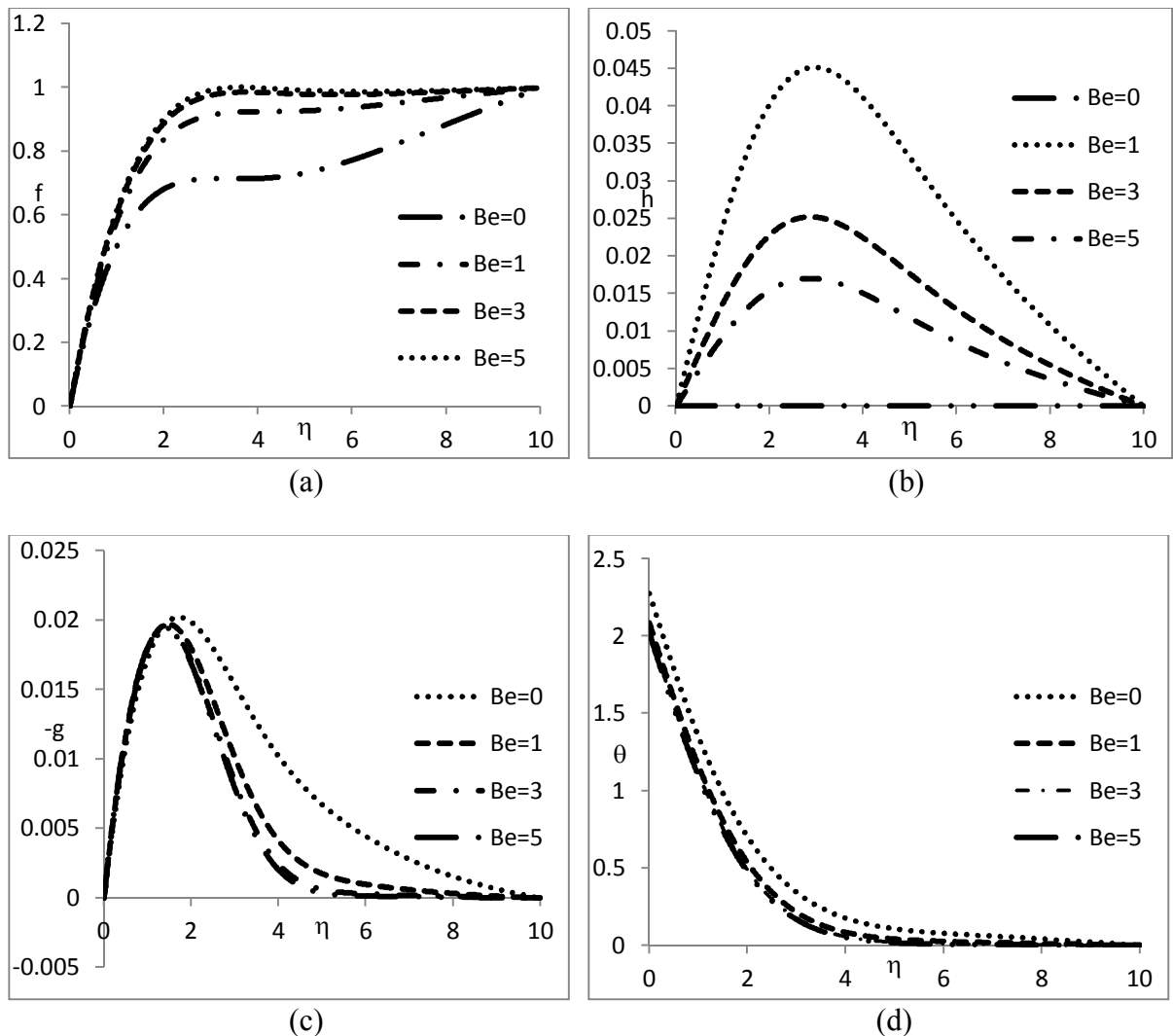


Figure 2: Effect of Hall parameter on (a) velocity( $f'$ ) (b) induced velocity( $h$ ) (c) microrotation (d) temperature and for  $N = 0.4$ ,  $Df = 2.0$ ,  $Sr = 0.03$ ,  $\beta_i = 2$ ,  $Ha = 2$



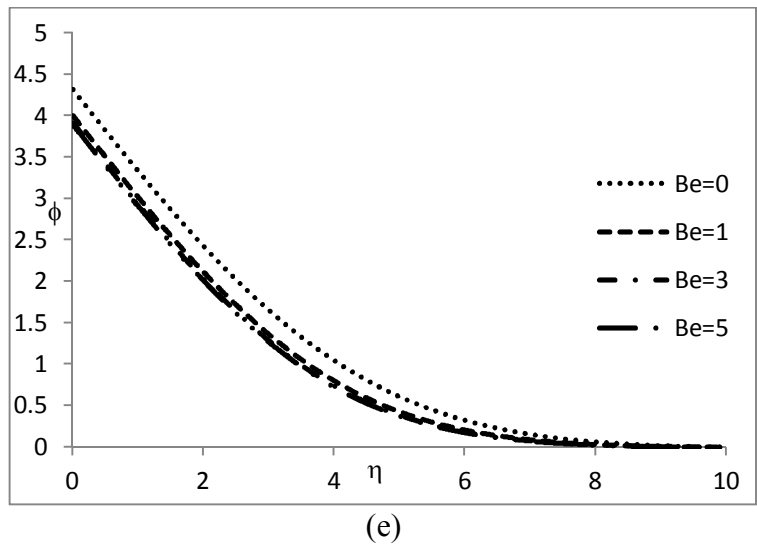


Figure 2(e): Effect of Hall parameter on concentration for  $N = 0.4, Df = 2.0, Sr = 0.03, \beta_i = 2, Ha = 2$

The ion-slip effect ( $\beta_i$ ) on the non-dimensional velocity components, microrotation, temperature and concentration is depicted in Fig Fig.(3). It is observed from Fig.(3a) that increase in  $\beta_i$  increases the velocity component ( $f'$ ). As Ion slip parameter increases the effective conductivity also increases, in turn, decreases the damping force on velocity, and hence the velocity increases. From Fig (3b) it is noticed that increase in the ion-slip parameter decrease the velocity component  $h$ . It is clear from Fig. (3c) that increase in ion-slip parameter decreases the microrotation away from the plate and the effect of this parameter on microrotation near the plate is negligible. It is seen from Figs.(3d) and (3e) that the effect of ion-slip on temperature and concentration is less pronounced. The increase in the ion-slip parameter decreases the fluid temperature and concentration.

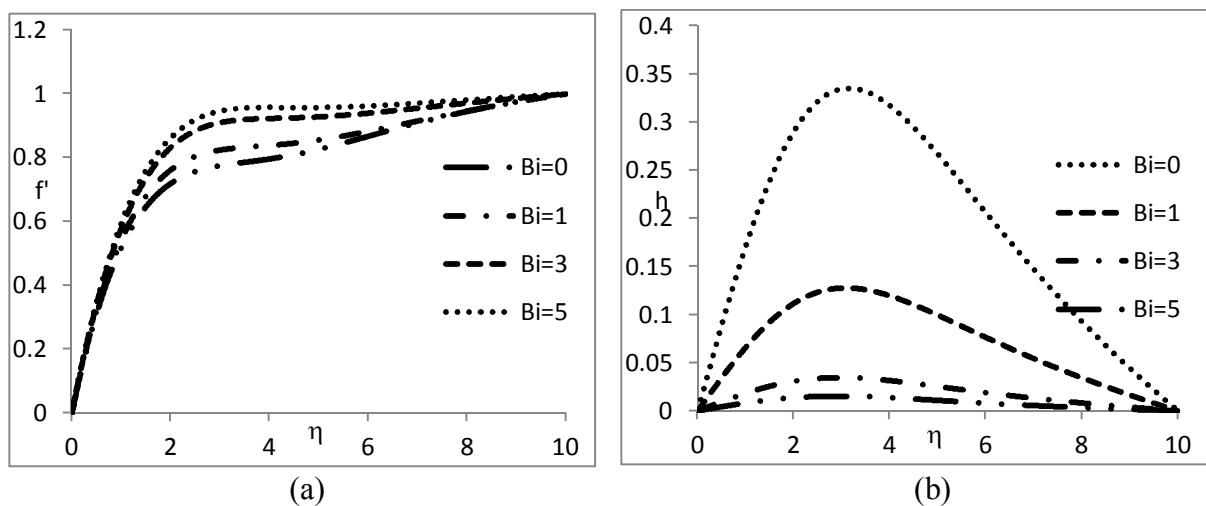


Figure 3: Effect of ion-slip parameter on (a) velocity( $f'$ ) (b) induced velocity( $h$ ) for  $N = 0.4, Df = 2.0, Sr = 0.03, Ha = 2, Be = 2$

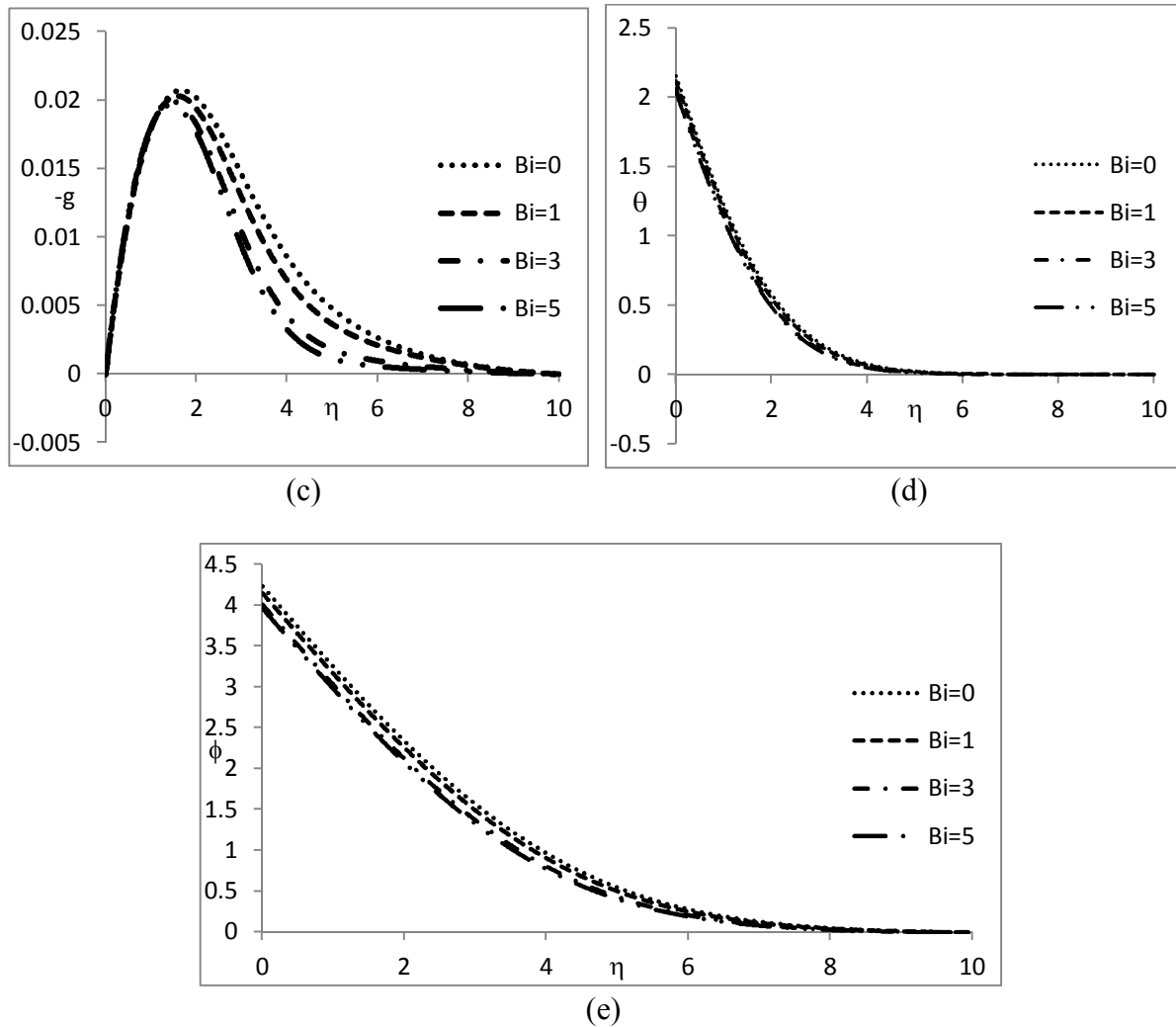


Figure 3: Effect of ion-slip parameter on (c) microrotation (d) temperature and (e) concentration for  $N = 0.4$ ,  $Df = 2.0$ ,  $Sr = 0.03$ ,  $Ha = 2$ ,  $Be = 2$

Fig.(4) displays the non-dimensional velocity components, microrotation, temperature and concentration for different values of Soret number  $Sr$  and Dufour number  $Df$ . It is observed from Fig.(4a) that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). It is clear from Fig.(4b) that the velocity component  $h$  is increasing slightly near the plate and is decreasing away from the plate as Dufour number increases (or Soret number decrease). It is seen from Fig.(4c) that there is no effect of Dufour (or Soret) number on the microrotation near the plate but away from the plate the magnitude of the microrotation is decreasing with the increase in Dufour number (or decrease in Soret number). The dimensionless temperature for different values of Soret and Dufour number is shown in Fig.(4d). The Dufour number signifies the contribution of the concentration gradients to the thermal energy flux in the flow. It is found that an increase in the Dufour number (decrease in Soret number) causes a rise in the temperature throughout the boundary layer i.e., the raise of Dufour number encourages heat transfer. Fig.(4e) demonstrates the dimensionless concentration for different values of Soret and Dufour number. It is seen that the concentration of the fluid increases with increase of Soret number.

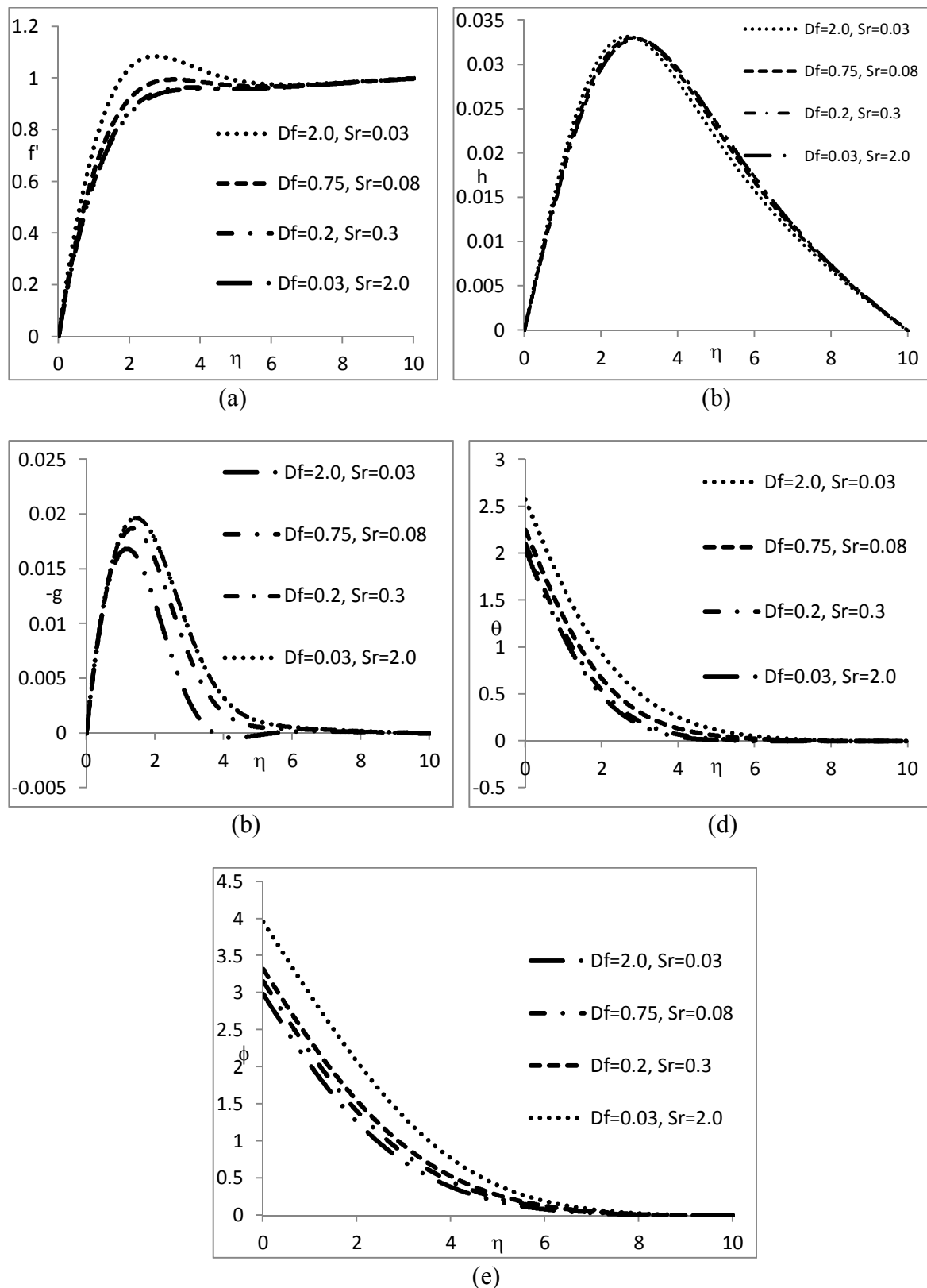


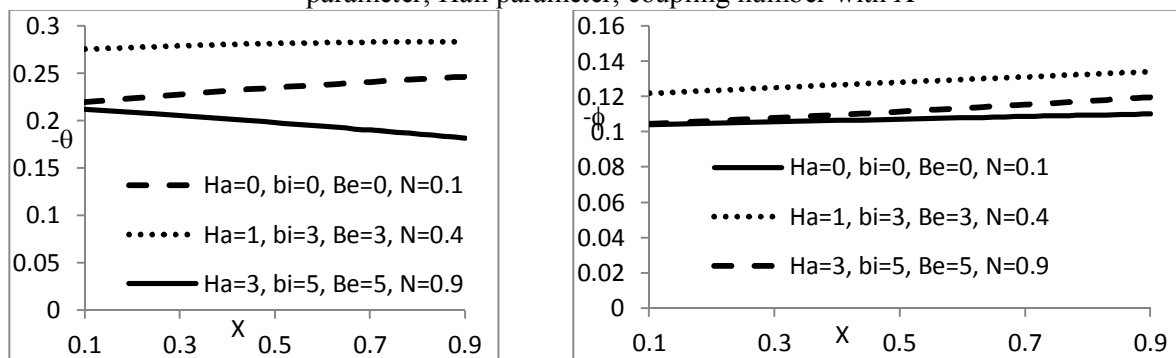
Figure 4: Effect of Soret and Dufour numbers on (a) velocity( $f'$ ) (b) induced velocity( $h$ ) (c) microrotation (d) temperature and (e) concentration for  $N = 0.4, Ha = 2, \beta_i = 2, Be = 2$

Table 2 shows the local skin friction coefficient, rate of heat and mass transfer for different values of coupling number, Hartman number, Hall and ion-slip parameters. As it can be seen from this table the proportional skin-friction coefficient  $f''(0)$  increases with both the ion-slip and Hall parameters increases where as it decreases with Hartman, coupling number and X-distance increases. Here it is noted that increasing the coupling number  $N \rightarrow 1$  (microstructure is significant) the skin friction decreases, hence micropolar fluid may be used as a lubricant. The quantity of heat exchanged between the body and the fluid is given by the temperature gradient Nu (Nusselt number) which is given in the same table inferred that the higher values of Hartmann number, ion-slip parameter and coupling number decreases Nu and increasing in Hall parameter and X-distance Nu increases. The rate of mass transfer Sh (Sherwood number) found to be increasing with increasing the X-location,  $\beta_i$  and Be where as it decreases with increase in coupling number and Hartmann number. Fig 5 shows the Effects of heat and mass transfer coefficients for some values of Hartman number, ion-slip parameter, Hall parameter and coupling number with X.

Table 2: Effects of skin friction, heat and mass transfer coefficients for varying values of Hartman number, ion-slip parameter, Hall parameter, coupling number and x-location

Ha	$\beta_i$	Be	N	X	$f''$	$-\theta$	$-\phi$
0.0	2.0	2.0	0.4	0.5	1.03603	0.39542	0.34143
1.0	2.0	2.0	0.4	0.5	1.02992	0.39296	0.34006
2.0	2.0	2.0	0.4	0.5	1.01225	0.38583	0.33598
3.0	2.0	2.0	0.4	0.5	0.98505	0.37478	0.32934
2.0	0.0	2.0	0.4	0.5	0.99951	0.38227	0.33326
2.0	1.0	2.0	0.4	0.5	1.00373	0.38253	0.33399
2.0	3.0	2.0	0.4	0.5	1.01774	0.38803	0.33727
2.0	5.0	2.0	0.4	0.5	1.02374	0.39045	0.33865
2.0	2.0	0.0	0.4	0.5	0.92779	0.34987	0.31328
2.0	2.0	1.0	0.4	0.5	0.99606	0.37922	0.33208
2.0	2.0	3.0	0.4	0.5	1.01917	0.38863	0.33760
2.0	2.0	5.0	0.4	0.5	1.02541	0.39114	0.33904
2.0	2.0	2.0	0.1	0.5	1.27676	0.40787	0.35454
2.0	2.0	2.0	0.4	0.5	1.01225	0.38583	0.33598
2.0	2.0	2.0	0.7	0.5	0.66810	0.34929	0.30502
2.0	2.0	2.0	0.9	0.5	0.32935	0.29848	0.26180
2.0	2.0	2.0	0.4	0.1	0.36273	0.30426	0.26609
2.0	2.0	2.0	0.4	0.4	0.84766	0.36901	0.32132
2.0	2.0	2.0	0.4	0.7	1.33856	0.41466	0.36156
2.0	2.0	2.0	0.4	0.9	1.47944	0.49229	0.42540

Fig 5. Effects of heat and mass transfer coefficients for some values of Hartman number, ion-slip parameter, Hall parameter, coupling number with X



#### 4. Conclusions

In this paper the flow of heat and mass transfer character of the mixed convection flow of micropolar fluid is studied. Numerical solutions (Killer box method) are presented for the fluid flow and heat transfer characteristics and their dependence on the pertinent material parameters is discussed. It is observed decrease in Soret number enhance the temperature inclusion of hall and ion-slip parameter decrease the temperature and concentration. It is also observed micropolar fluids reduces skin friction, heat and mass transfer rates hence may be use as a lubricant. It is hoped that the findings of this investigation may be useful for magnetohydrodynamic (MHD) energy generators, materials processing, geophysical hydromagnetics, etc.

#### Nomenclature

$q_w$	heat fluxes
$q_m$	mass fluxes
$T_m$	mean fluid temperature
$K_T$	thermal diffusion ratio
$u, v, w$	velocity components (in x, y, z directions respectively)
$u_\infty$	free stream velocity
$T_\infty$	free stream temperature
$C_\infty$	free stream concentration
$\Gamma$	microrotaion
$T$	temperature
$C$	concentration
$\rho$	fluid density
$j$	gyration parameter
$\mu$	dynamic coefficient of viscosity
$\kappa$	vortex viscosity
$\gamma$	spin gradient viscosity
$g^*$	acceleration due to gravity
$\beta_T$	coefficient of thermal expansion
$\beta_c$	coefficient of solutal expansion
$\alpha$	thermal diffusivity
$D$	mass diffusivity
$C_p$	specific heat capacity
$C_s$	concentration susceptibility
$T_m$	mean fluid temperature
$Be$	Hall parameter
$\sigma$	electric conductivity of the fluid
$B_0$	applied magnetic field
$\beta_i$	ion-slip parameter
$\nu$	kinematic viscosity
$D$	solutal diffusivity of the medium
$D_f$	Dufour number,
$Sr$	Soret number
$Gr$	thermal Grashof number
$Gc$	solutal Grrashof number
$Re$	Reynolds number
$g_s$	temperature bouncy parameter

$g_c$	mass bouncy parameter
$X$	dimensionless coordinate along the plate
$\lambda$	spin gradient viscosity
$Ec$	Eckert number
$H_a^2$	Hartman number
$Sc$	Schmidt number
$Pr$	Prandtl number

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