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ORIGINAL ARTICLE

Effect of Ranking Selection on the Weibull Modulus Estimation

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Abstract

Determination of mechanical properties of materials requires repeatable and consistent results. Therefore, the Weibull statistical distribution is widely used to verify the confidence levels of the test results where probability of failure is predicted. The reliability of results is characterised by Weibull distribution where Weibull parameters are determined and compared with the benchmarks. Recent studies showed that the size of the population and the method chosen to estimate the Weibull modulus play an important role. Therefore, in this study, the effect of ranking selection over the Weibull parameters (*alpha*-characteristic life, *beta*-shape parameter, R^2 and survival probability) was investigated. The data from authors' recent research were re-evaluated. In addition, randomly generated data with population from 5 to 50 sample sizes were studied. The results showed that the mean rank regression to estimate the failure probability had the highest R^2 and the lowest shape parameter. It was also found that the results were independent of sample size.

Keywords: *Estimators, mechanical test, prediction limit, Weibull distribution, Weibull modulus*

1. INTRODUCTION

Increasing demand for reliable performance of materials requires that the strength should be measured accurately within the range of intended use. As it is well-known, due to the defects that may be present in the material, the mechanical test results may vary significantly depending on the size and population of these defects. Therefore it is important to produce materials with minimal defects and more reliable mechanical properties.

For this purpose, in engineering applications, Weibull distributions [1] are widely used to study the distribution and magnitude of scatter of independent results obtained from experimental findings.

The basic Weibull distribution is a plot between random variables, *x*, and its cumulative probability, *F(x)*; in such a way that the plot against *x* of its cumulative probability, *F(x)*, appears as a straight line.

The two-parameter form of the Weibull distribution is widely adopted and can be expressed as follows:

$$F(x) = 1 - e^{-\left(\frac{x}{x_o}\right)^m} \quad (1)$$

where *F(X)* is the fraction of specimens that fail at or below a given value of *x* (e.g. a measured tensile strength), x_o is a characteristic value of *x* at which 63%

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of the population of specimens have failed (also known as '*alpha* (α) - characteristic life') and m is the Weibull modulus (also known as '*beta* (β) - shape parameter') which is a constant that characterises the spread of the failure data with respect to the x axis. A high Weibull modulus is desirable since it indicates an increased homogeneity, less spread and more predictable failure behaviour.

When it comes to examination of mechanical tests results, Equation (1) can be re-written as follows:

$$P(\sigma) = 1 - e^{-\left(\frac{\sigma}{\sigma_o}\right)^m} \qquad (2)$$

where P is probability and σ is the strength. Linear regression is used to determine the model parameters, where m is now simply the slope of the graph:

$$\ln\left(\ln\left\{\frac{1}{1-P(\sigma)}\right\}\right) = m.\ln(\sigma) - m\ln(\sigma_o) \qquad (3)$$

Since σ is simply obtained from the experimental results, the most important parameter is the plotting positions given as $P(\sigma)$. The strength values (σ) are ordered from low to high and each has to be assigned to a probability of failure based on its ranking. There are several probability estimators available in the literature. These probability estimators can all be written in the form:

$$P = \frac{i-a}{n+b} \qquad (4)$$

where i is the rank of the data point in the sample in ascending order, n represents the sample size, and a and b are generic coefficients. If a and b are taken as zero, then this gives 100% at the last point which is off scale. The main purpose of the Weibull distribution function is that it makes engineering data possible to estimate a population of infinite size from small amounts of data. Therefore, to correct this problem, different probability estimators have been investigated in the literature. Table 1 summaries the common probability estimators also known as ranking methods.

It is important to use the most accurate approximation to rank the failure probability, in order to have reliable parameters comparable with other benchmark tests. In general, means ranks method [3] is preferable and more widely used for estimation of mechanical tests. In this study, the aim was targeted to investigate the effect of selection of ranking method on the Weibull distribution parameters. The data from authors' recent published works [8,9] has been used to re-evaluate the Weibull parameters.

While the Weibull statistics are commonly accepted as a good correlation for the distribution of independent variables, it still doesn't reveal whether the material meets the reliability goal. For this purpose, a reliability plot of the Weibull distribution has also been investigated as well:

$$R = \exp - \left(\frac{\sigma}{\alpha}\right)^{\beta}$$

where R is the reliability, σ is strength, α and β are the characteristic life and shape parameters respectively.

Table 1. Common probability estimators [2-7]

Ranking regression	Estimator
Benard [2]	$\frac{i-0.3}{n+0.4}$
Mean [3]	$\frac{i}{n+1}$
Hazen [4]	$\frac{i-0.5}{n}$
Filliben [5]	$\frac{i-0.3175}{n+1.635}$
Blom [6]	$\frac{i-0.5}{n+0.25}$
Gringorten [7]	$\frac{i-0.44}{n+0.12}$

2. EXPERIMENTAL PROCEDURE

The Weibull analysis was carried out on Ring-on-Ring (ROR) test results of glass and tensile test of cast aluminium bars. The details of the experimental procedure can be found at previous works of Kirtay [9] and Dispinar [8].

Basically, glass samples were coated at different coating rate (5, 10, 20, 30 cm/min) in water-based and alcohol-based SiO_2-TiO_2 containing ormosil base solutions by sol-gel method. ROR tests were carried out to compare the effect of coating rate on strength of glass with respect to the reference and original uncoated glass. All the indented specimens were named as original, some of the "original" specimens were heat-treated at 200 °C for 1 h in an electrically heated furnace and this group was named as "reference". The coating rate is a factor of coating thickness which is the measure of how much a crack has been filled.

For the aluminium alloys, tensile test bars were produced with different mould patterns in order to investigate the effect of runner design on the mechanical properties.

The values were plotted as *ln*-*ln* plot of failure probability versus *ln* property according to Equation 3. For each set of mechanical test results, different probability estimators were used according to Table 1. An example of set of a test matrix is given in Table 2.

From each set of data and each set of tests, least square regression analysis was carried out to determine the parameters. The parameters determined were alpha characteristic life, beta shape parameter (i.e. '*m*' Weibull modulus) and regression coefficient (R^2). The parameters obtained from different rankings were cross analyzed and compared with each other. Additionally, survival probability plots (Equation 5) were also compared.

Table 2. An example of set of a data used in Weibull analysis: Data taken from ROR test results of uncoated reference glass [9]

	Strength (σ)	Benard	Mean	Hazen	Filliben	Blom	Gringorten
1	53.0	0.03	0.05	0.02	0.03	0.02	0.03
2	61.8	0.08	0.09	0.07	0.07	0.07	0.07
3	62.5	0.13	0.14	0.12	0.12	0.12	0.12
4	62.9	0.17	0.18	0.17	0.16	0.16	0.17
5	63.6	0.22	0.23	0.21	0.21	0.21	0.22
6	65.9	0.27	0.27	0.26	0.25	0.26	0.26
7	71.2	0.31	0.32	0.31	0.30	0.31	0.31
8	71.6	0.36	0.36	0.36	0.34	0.35	0.36
9	74.7	0.41	0.41	0.40	0.38	0.40	0.41
10	76.1	0.45	0.45	0.45	0.43	0.45	0.45
11	76.8	0.50	0.50	0.50	0.47	0.49	0.50
12	77.6	0.55	0.55	0.55	0.52	0.54	0.55
13	78.5	0.59	0.59	0.60	0.56	0.59	0.59
14	79.2	0.64	0.64	0.64	0.60	0.64	0.64
15	79.2	0.69	0.68	0.69	0.65	0.68	0.69
16	81.4	0.73	0.73	0.74	0.69	0.73	0.74
17	83.5	0.78	0.77	0.79	0.74	0.78	0.78
18	86.6	0.83	0.82	0.83	0.78	0.82	0.83
19	86.9	0.87	0.86	0.88	0.83	0.87	0.88
20	87.4	0.92	0.91	0.93	0.87	0.92	0.93
21	92.6	0.97	0.95	0.98	0.91	0.96	0.97

As an additional study to check the effect of sample size on Weibull parameter, randomly generated numbers with different sample sizes (5, 10, 15, 20, 25, 30, 35, 40, 45 and 50) were used. The range of the random numbers was selected to be between 100 and 200 which corresponded to the strength values of cast aluminium bars [8]. The range was also selected to give three different Weibull modulus values: low (5), medium (10) and high (50).

3. RESULTS and DISCUSSION

The effect of different coating rates on the scatter of strength of glass is given in Figure 1.

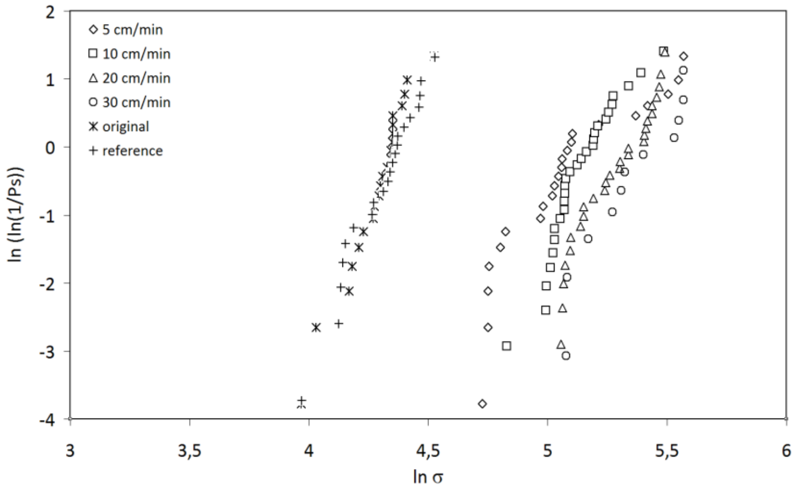
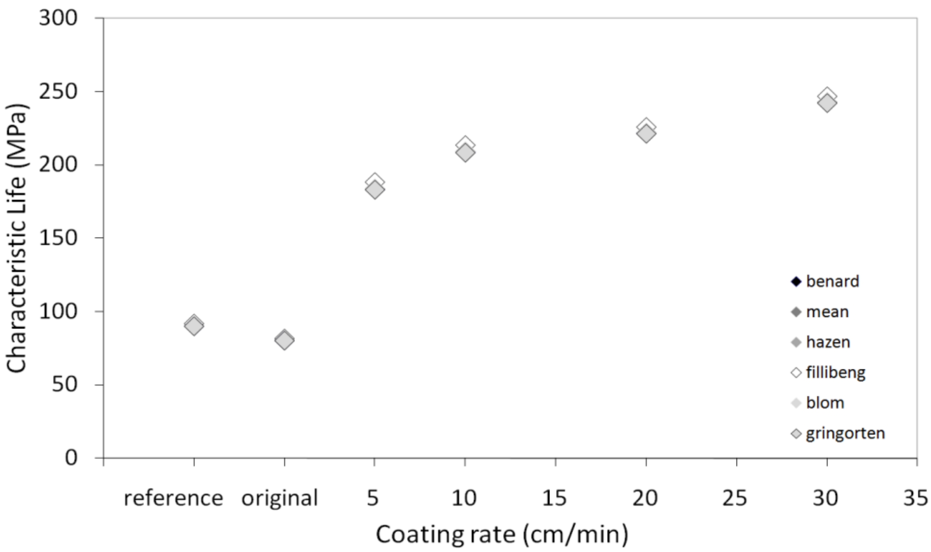


Figure 1. A typical Weibull analysis [9]

For the same data used in Figure 1, different ranking methods were applied and their effects on the Weibull parameters were investigated. The results of alpha characteristic life, beta shape factor, regression coefficient, survival probability and survivability limit are given in Figures 2-6 respectively.



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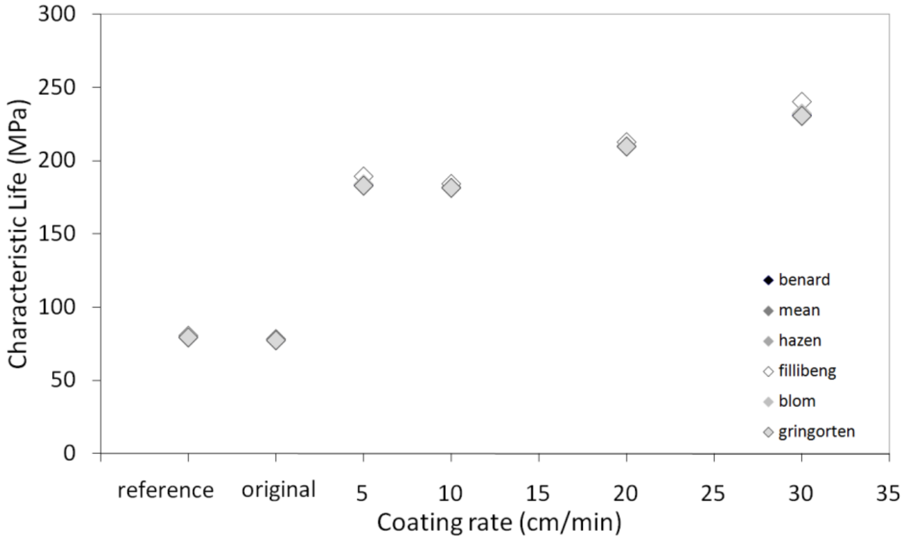
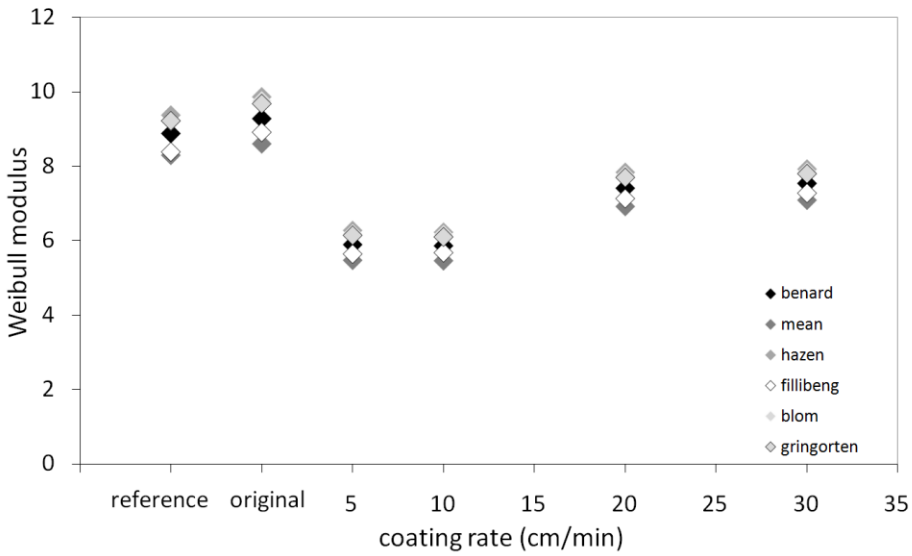


(b)

Figure 2. Alpha characteristic life comparison

(a) alcohol based solution, (b) water based solution



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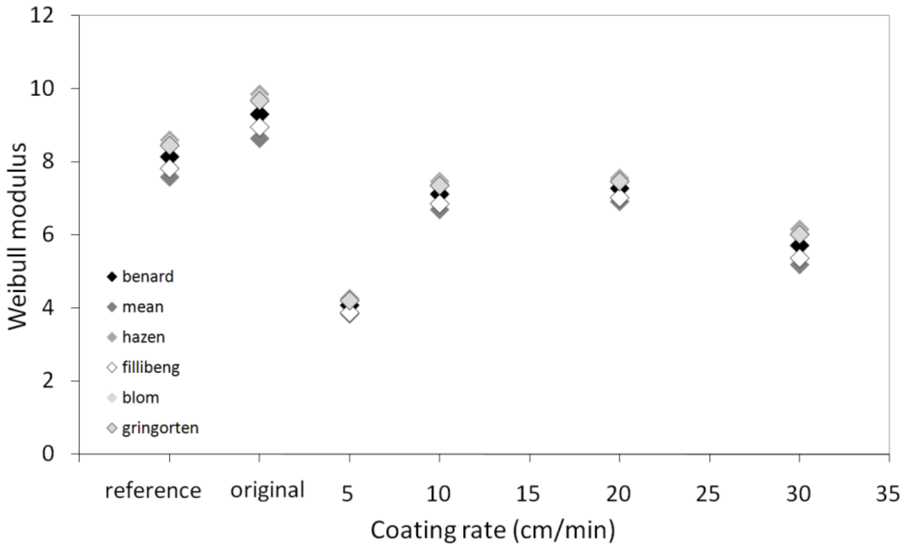
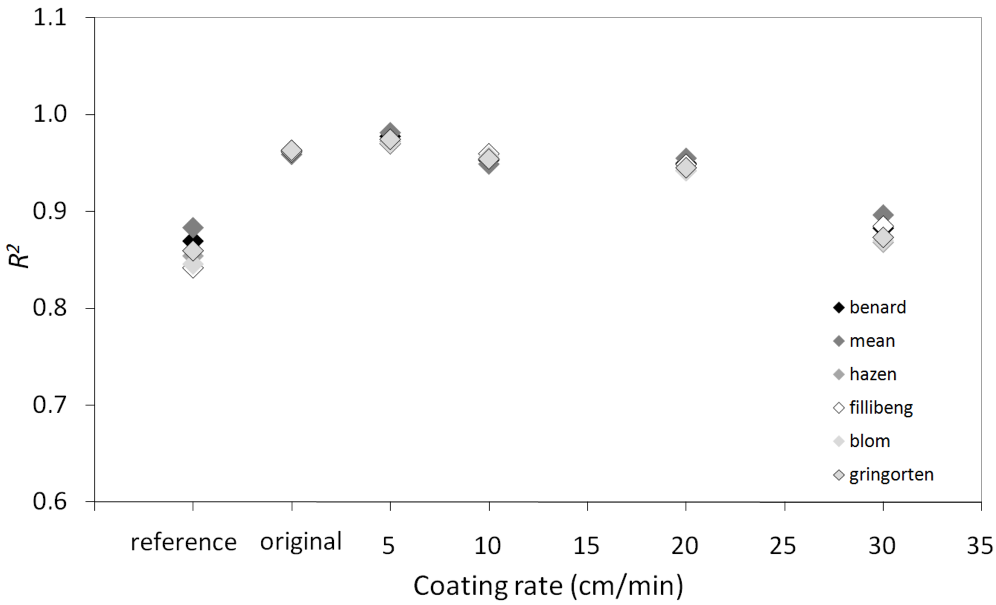


(b)

Figure 3. Beta shape factor (Weibull modulus) comparison
(a) alcohol based solution, (b) water based solution



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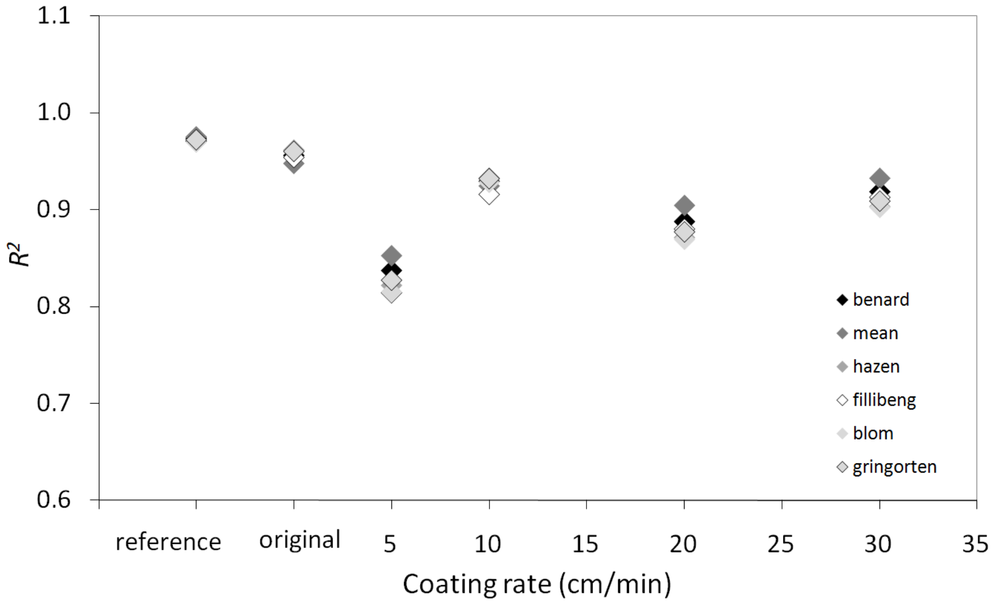

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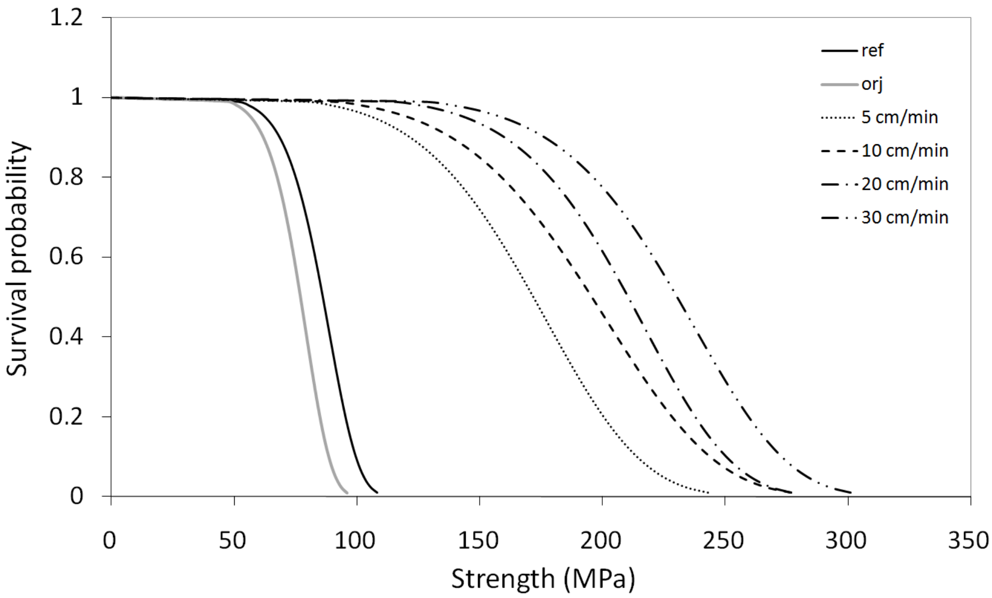
Figure 4. Regression coefficient (R^2) comparison
(a) alcohol based solution, (b) water based solution



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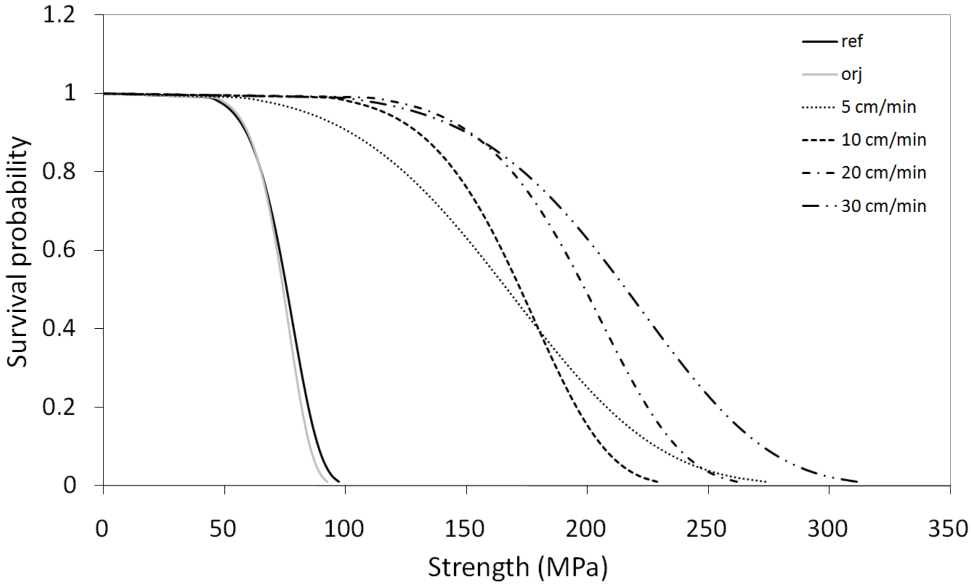
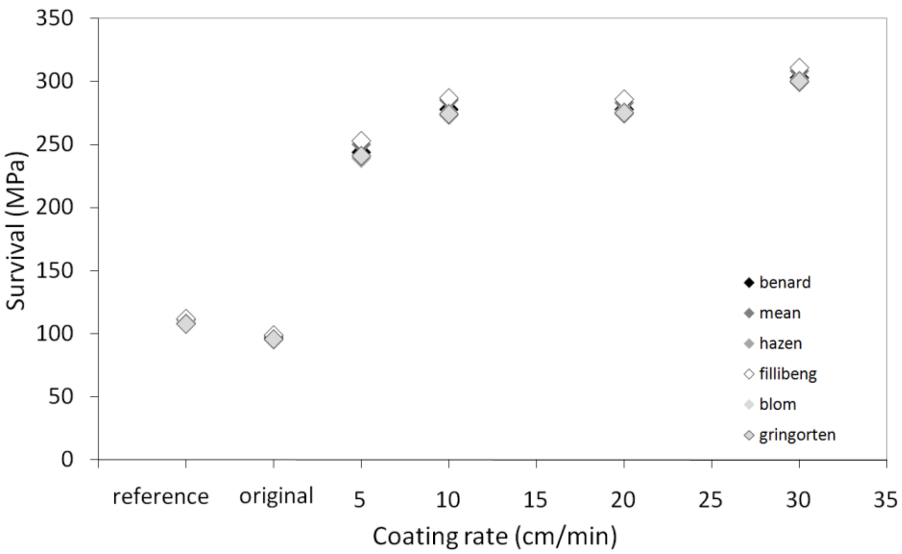


(b)

Figure 5. survival probability comparison

(a) alcohol based solution, (b) water based solution



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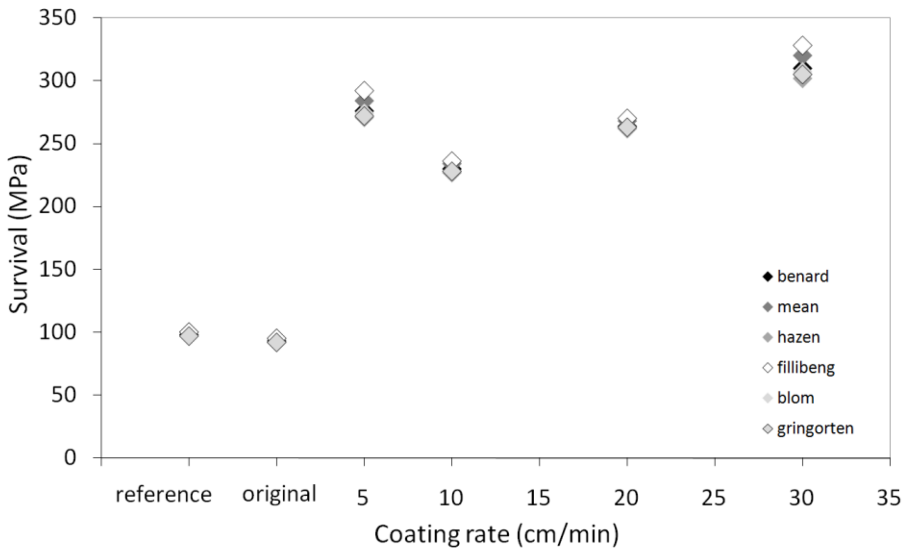
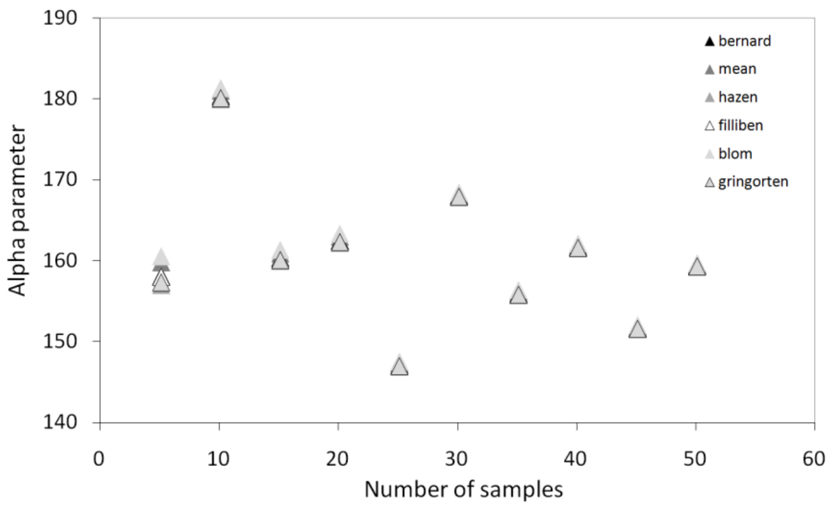


(b)

Figure 6. survivability limit comparison

(a) alcohol based solution, (b) water based solution

In Figures 7 to 9, the effect of number of samples over the alpha, beta parameter and regression coefficient has been investigated. The data were randomly generated as was described before.



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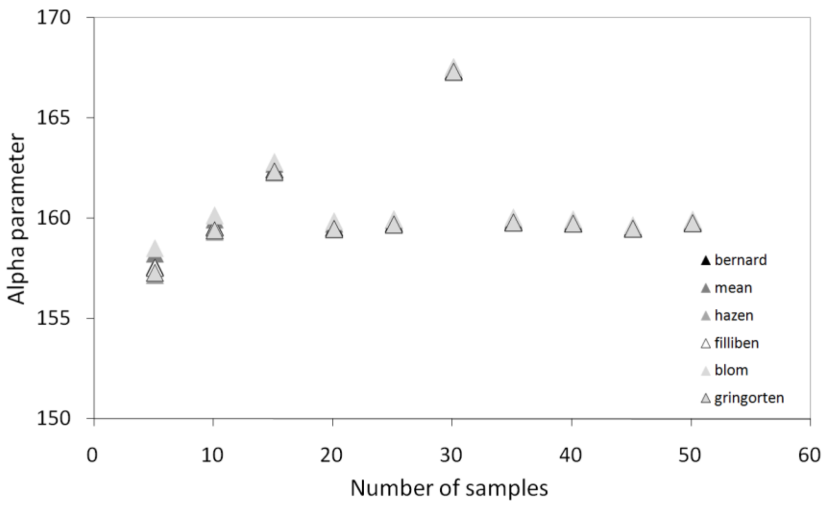

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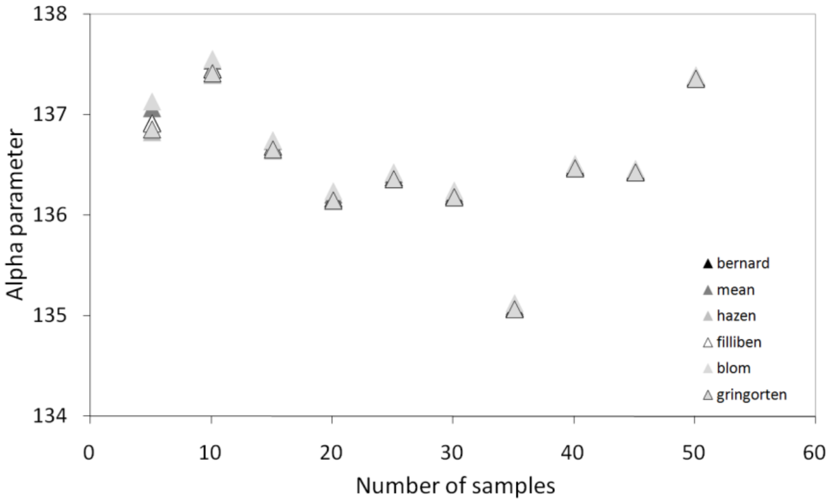
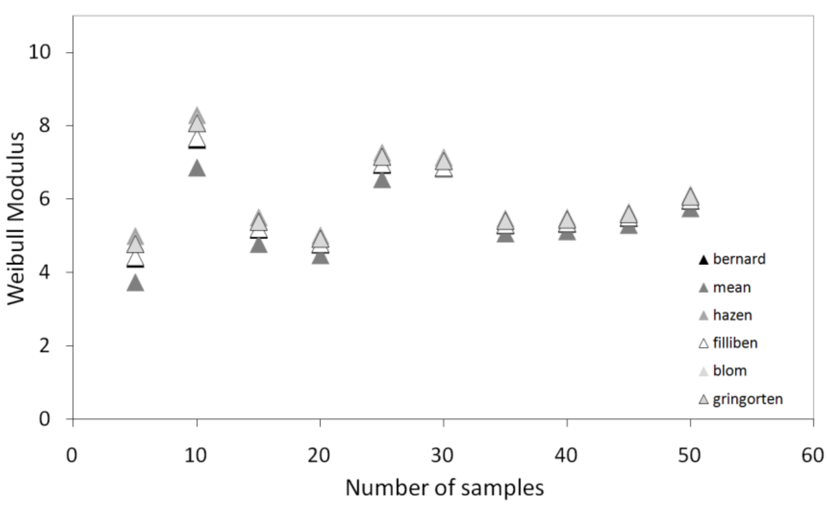


(c)

Figure 7. alpha characteristic life comparison

(a) low Weibull modulus, (b) medium Weibull modulus, (c) high Weibull modulus



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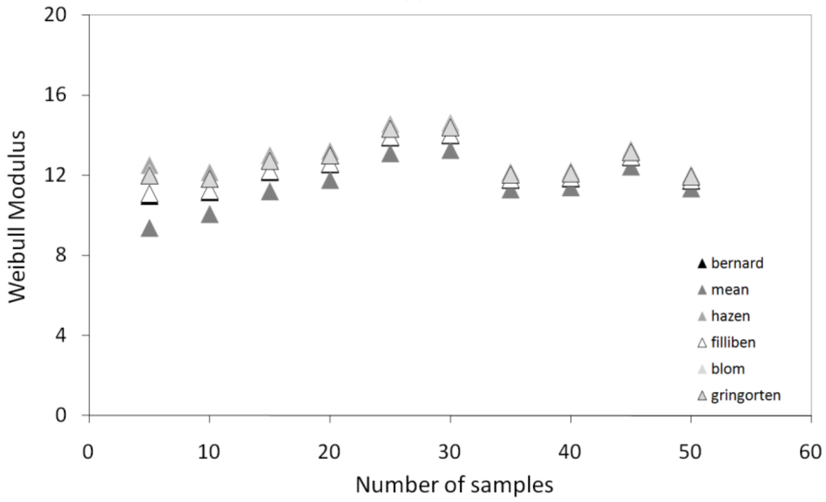

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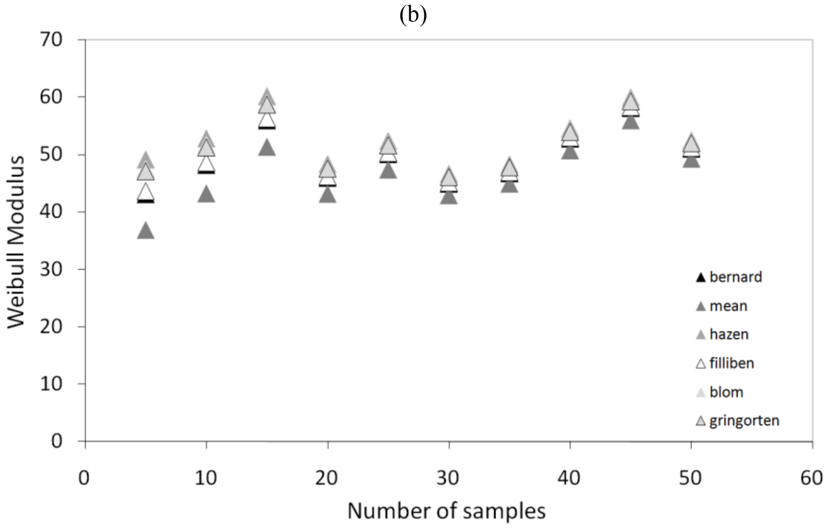
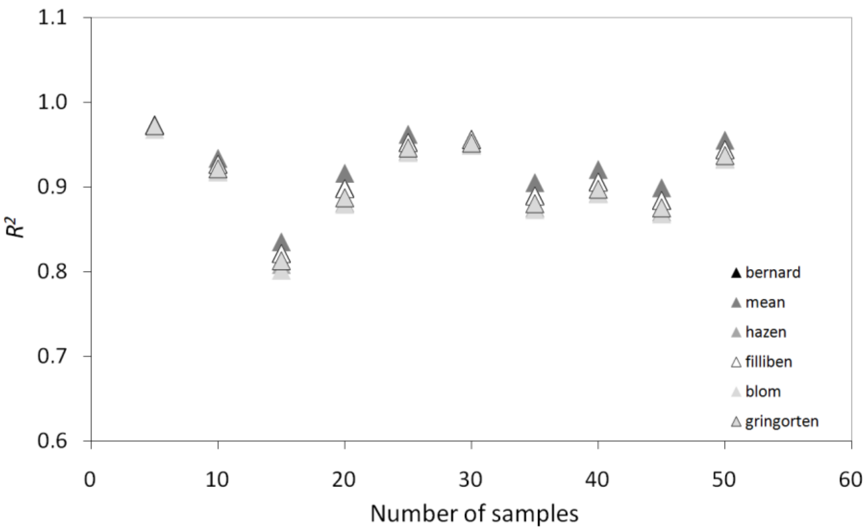


(c)

Figure 8. beta shape factor comparison
(a) low Weibull modulus, (b) medium Weibull modulus, (c) high Weibull modulus



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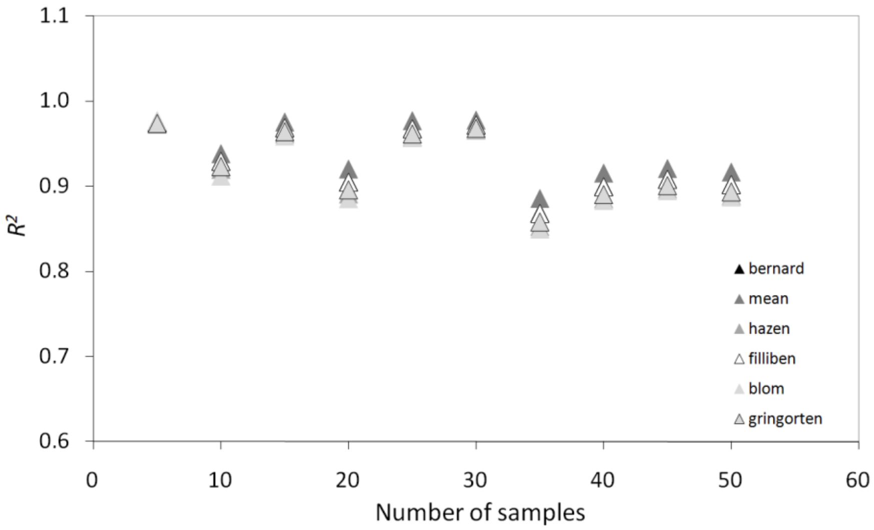

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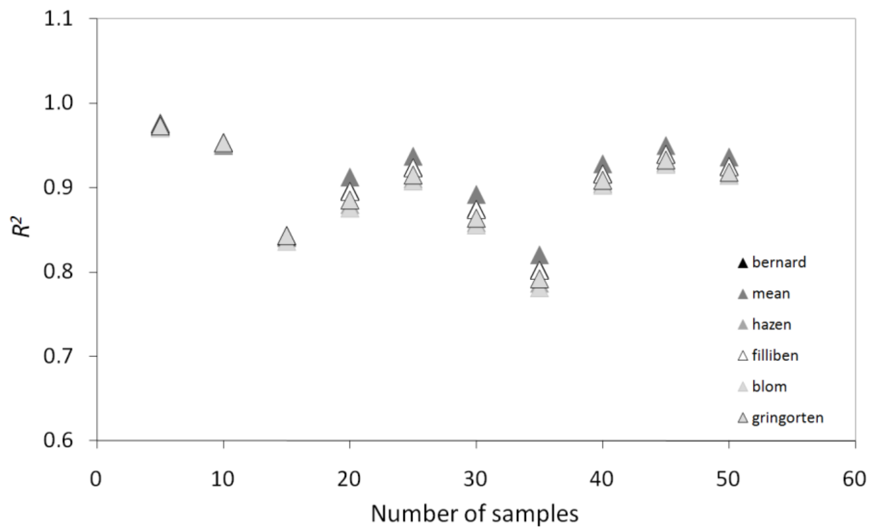

(c)

Figure 9. regression coefficient comparison

(a) low Weibull modulus, (b) medium Weibull modulus, (c) high Weibull modulus

4. DISCUSSION

A typical Weibull distribution function would look like straight line as seen in Figure 1. The general way to analyse this is to measure the slope by a regression method and compare it with different test results or the benchmark tests. In this example in Figure 1, the effect of different coating rates on the scatter of strength of glass is given. As can be seen from the figure, as the coating rate is increased the strength is increased. The scatter of the results is the important parameter as well as the average strength. This can be seen from the Weibull modulus values that ranges from 3.8 to 8.6. From this Weibull analysis, it can simply be concluded that 5 cm/min coating will have the highest scatter meaning that it is the least reliable method with lowest strength. On the other hand, 30 cm/min have the highest strength however 20 cm/min is the most reliable method with 6.9 highest Weibull modulus value.

Hazen probability estimator was used to plot the data in Figure 1. This is the typical choice of estimator when it comes to assessing the mechanical properties of ceramics and metals etc. Since Weibull modulus is a function of this estimator (i.e. the y-axis of the Weibull plot), it is critical that these values should represent consistency. The independent test results by the researchers are usually compared with the Weibull Modulus. Therefore in this work, the existing estimators were analyzed.

In Figure 2, the changes in alpha characteristic life with different ranking methods are shown. The same data was used as in Figure 1. It can be clearly seen that the alpha characteristic life parameter does not change with the selection of ranking method. All data is exactly on top of each other. Therefore it only looks like there is one data in the graph although there are six legends. It is only Filliben ranking that seems to be slightly higher than the rest of the results which is shown by the empty diamonds (Fig 2). However the standard deviation is so small that it can easily be concluded that alpha characteristic parameter is independent of ranking method.

The situation for beta shape factor was quite different in comparison to alpha characteristic life parameter. The Weibull modulus was strongly affected by the selection of ranking method. The results in Figure 3 showed that Mean ranking had the lowest Weibull modulus whereas Hazen ranking had the highest. It is also important to note that Benard ranking was always in the average of all ranking systems.

The general trend appeared that mean ranking probability estimator had the highest regression coefficient in most of the cases as seen in Figure 4. However there is no certain ranking system which is always the lowest.

Since mean ranking regression had the highest regression coefficients (Fig 4), for the survival probability plots, only mean ranking data were used for Equation 5. And as seen in Figure 5a, the survival probability of sol-gel coating in alcohol based solution clearly showed an orderly systematic increase with increasing coating rate. On the other hand, in coatings in water based solutions (Figure 5b), the survivability plots were more complex. However, when survivability limits are compared (Figure 6), it was found that Hazen ranking had the lowest survivability limits whereas Fillibeng ranking had the highest values.

So far, all of these findings were cross examined from experimental findings carried out with ROR testing of glass [9]. As an additional work, the effect of population size on the Weibull parameters was investigated. Different sample sizes from 5 to 50 with randomly selected numbers were studied.

The change in the alpha parameter with increasing sample size is given in Figure 7. It can be clearly seen that when the sample size is greater than 20, the dependency of the results by the ranking method is reduced. The scatter is highest when sample size is five.

Weibull modulus was strongly affected by ranking selection, particularly for the low number of samples. As seen in Figure 8, mean ranking had the lowest beta shape parameter and Hazen ranking had the highest for all cases.

On the other hand, for the regression coefficients, mean ranking had the highest value for all populations and Hazen and Blom ranking were the lowest.

5. CONCLUSIONS

1. Irrespective of sample size and Weibull modulus, mean ranking has the highest regression coefficient and Hazen ranking has lowest which indicated that the use of mean ranking gives the closest Weibull estimator for any set of data.
2. Regardless of the sample size, mean ranking gives the lowest Weibull modulus.
3. As the sample size increases from five to fifty, Weibull modulus values that are calculated by various ranking methods merge together. Thus, the scatter and dependence on ranking selection is decreased.
4. For samples sizes above 10, alpha characteristic parameter does not change either by population size or ranking method.
5. Irrespective of sample size, mean ranking gives the highest survival limit and Hazen has the lowest.

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