Gazi University Journal of Science
GU J Sci
29(1):213-221 (2016)

A Class of Ratio Cum Dual to Ratio Estimator in Presence of Non-Response Using Two Phase Sampling Scheme

Bhupendra Kumar SINGH [1], Waikhom Warseen CHANU [1,♠], Yater TATO[1]

[1]*Department of Mathematics, North Eastern Regional Institute of Science and technology Nirjuli-791109, Papum Pare District Arunachal Pradesh*

Received: 22/04/2015 *Accepted: 04/01/2016*

ABSTRACT

In the present paper, we have proposed a class of ratio cum dual to ratio estimator in double sampling scheme to estimate the population mean of the study variable y in two different cases of non-response. The bias and mean square error (MSE) of the proposed class have been obtained for both the cases of non-response. The asymptotically optimum estimator (AOE) of it has also been obtained along with its bias and MSE. Comparisons of the class have been made with the mean per unit estimator and estimators belonging to it theoretically and numerically to demonstrate the superiority of the proposed estimator over them.

Keywords: *Ratio, Dual to ratio, Non-response, Bias, Mean square error, Asymptotically optimum estimator (AOE), Efficiency*.

1. INTRODUCTION

In many human surveys, information is in most cases not obtained from all the units in the surveys even after some call-backs. An estimate obtained from such incomplete data may be misleading and biased. Hansen and Hurwitz (1946) proposed a technique for adjusting the non-response to address the bias problem. Their idea is to take a sub-sample from the non-respondents to get an estimate for the sub-population represented by the non-respondents. In estimating population parameters such as the mean, total or ratio, sample survey experts

♠Corresponding author, e-mail: *warcn2013@gmail.com*

sometimes use auxiliary information to improve precision of the estimates. Sodipo (2007) have considered the problem of estimating the population mean in presence of non response with full response of an auxiliary character *x*. Other authors such as Bouza (2010), Cochran (1977), Khare (1993, 1995, 1997), Okafor (2000), Rao (1986), Singh (2010, 2008), Singh (2009), Tabasum (2006, 2004), Agarwal et.al. (2012) and Riaz et.al. (2014) have studied the problem of non-response under double sampling scheme.

Let us assume a finite population of N distinct units. Let y and x be the study and auxiliary variables having values y_i and x_i , $i=1,2,...,N$. Let x is correlated with y and is used to estimate the unknown population mean $\bar{Y}$. When the mean of the auxiliary variable x is available, the ratio, product and regression estimators are used to increase the efficiency of the estimates of $\bar{Y}$. When $\bar{X}$ unknown, two-phase sampling scheme is used, the first phase estimates $\bar{X}$ and the second phase is devoted for the estimation of $\bar{Y}$.

Let a large sample of size n' $(n'<N)$ is drawn by simple random sampling without replacement (SRSWOR) to collect information on the auxiliary variable X . It is assumed that all n' units provide complete information on X . In the second phase, a smaller sample of size n of n' units $(n<n')$ is drawn by SRSWOR for obtaining information of the study variable y . Suppose the non-response is present in second phase, in this situation a subset of size n_1 supplies information on Y and the remaining $n_2=n-n_1$ units are non-respondents. Following the technique of Hansen and Hurwitz (1946), a sub-sample of size $r=\frac{n_2}{k};\quad k>1$ is selected from the n_2 non-responded units where r would be a rounded off integer and k is the inverse sampling rate at the second phase sample of size n. Assuming that all r selected units show full response on second call. Consequently, the whole population is said to be stratified into two strata of size N_1 that would give response on first call and of size N_2 which would not respond on first call but will cooperate on the second call.

An unbiased estimator for the population mean $\bar{Y}$ of the study variable y proposed by Hansen and Hurwitz (1946), when non-response occurs on y is defined by

$$\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_{2r}$$

where, $w_1=n_1/n$ and $w_2=n_2/n$.

The variance of $\bar{y}^*$ is given by

$$Var\left(\bar{y}^*\right)=\left(\frac{1-f}{n}\right)S_y^2+\frac{W_2(k-1)}{n}S_{2y}^2 . \quad (1)$$

where $f=n/N$, $S_y^2=\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2/\left(N-1\right)$,

$$S_{2y}^2=\sum_{i=1}^{N_2}\left(y_i-\bar{Y}_2\right)^2/\left(N_2-1\right),$$

$$\bar{Y}=\sum_{i=1}^{N}y_i/N \; ,\bar{Y}_2=\sum_{i=1}^{N_2}y_i/N_2 \, ,$$

$$W_2=N_2/N \; , \; \bar{y}_1=\sum_{i=1}^{n_1}y_i/n_1 \, ,$$

$\bar{y}_{2r}=\sum_{i=1}^{r}y_i/r$, N_1 and $N_2(=N-N_1)$ are the sizes of the responding and non-responding units from the finite population N .

It is well known that in estimating the population mean $\bar{Y}$, sample surveys experts sometimes use auxiliary information to improve the precision of the estimates. Let x denote an auxiliary variable with population mean $\bar{X}=\sum_{i=1}^{N}x_i/N$. Let

$\bar{X}_1=\sum_{i=1}^{N_1}y_i/N_1$ and $\bar{X}_2=\sum_{i=1}^{N_2}x_i/N_2$ denotes the population means of the response and non-response groups. Let $\bar{x}=\sum_{i=1}^{n}x_i/n$ denotes the mean of all n units, $\bar{x}_1=\sum_{i=1}^{n_1}x_i/n_1$ and $\bar{x}_2=\sum_{i=1}^{n_2}x_i/n_2$ denotes the means of the n_1 responding units and n_2 non-responding units. Further, $\bar{x}_{2r}=\sum_{i=1}^{r}x_i/r$ denotes the mean of the $r(=\frac{n_2}{k},k>1)$ sub sampled units.

We define an unbiased estimator of population mean $\bar{X}$ when non-response occur on x as

$$\bar{x}^*=w_1\bar{x}_1+w_2\bar{x}_{2r}$$

The variance of $\bar{x}^*$ is given by

$$Var\left(\bar{x}^*\right)=\left(\frac{1-f}{n}\right)S_x^2+\frac{W_2(k-1)}{n}S_{2x}^2$$

where $S_x^2=\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2/\left(N-1\right)$,

$$S_{2x}^2=\sum_{i=1}^{N_2}\left(x_i-\bar{X}_2\right)^2/\left(N_2-1\right)$$

Using the transformation

$$\bar{x}_i^\sigma=\left(N\bar{X}-nx_i\right)/\left(N-n\right),$$

$i=\left(1,2,3,...,N\right)$, Srivenkataramana(1980) obtained dual to ratio estimator as

$$\bar{y}_{dR}=\bar{y}\left(\frac{\bar{x}^\sigma}{\bar{X}}\right)$$

where $\bar{x}^\sigma=\left(N\bar{X}-n\bar{x}\right)/\left(N-n\right)$

Using the transformation

$$\bar{x}_i^*=\left(n'\bar{x}'-n\bar{x}_i\right)/\left(n'-n\right),\left(i=1,2,...,N\right),$$

Kumar (2006) obtained dual to ratio estimator in double sampling as

$$\bar{y}_{dR}^d=\bar{y}\frac{\bar{x}^*}{\bar{x}'}.$$

where $\bar{x}^*=\left(n'\bar{x}'-n\bar{x}\right)/\left(n'-n\right)$ is unbiased estimator of $\bar{X}$ and $corr\left(\bar{y},\bar{x}^*\right)=(-)ve$.

Chanu and Singh (2012) suggested an efficient class of double sampling dual to ratio estimators in sample surveys as

$$\bar{y}_{dR}^{d\alpha}=\bar{y}\left[\frac{\bar{x}^{*\sigma}}{\bar{x}'}\right]^\alpha$$

where $\bar{x}^{*\sigma}=\left(n'\bar{x}'-n\bar{x}\right)/\left(n'-n\right)$ and

$$\bar{x}'=\sum_{i=1}^{n'}x_i/n'$$

Sharma (2010) suggested a new ratio cum dual to ratio estimator in simple random sampling as

$$\bar{y}_{bk1}=\bar{y}\left[\alpha\frac{\bar{X}}{\bar{x}}+\left(1-\alpha\right)\frac{\bar{x}^\sigma}{\bar{X}}\right] \quad (2)$$

Chanu and Singh (2015) studied Sharma (2010) estimator in presence of non-response. In the present paper, we have studied Chanu and Singh (2015) estimator in two phase sampling scheme. The expressions of bias and mean square error have been obtained to the first degree of approximation. Comparisons of the proposed estimator have been carried out with the usual unbiased estimator and other estimators under the class. An empirical study is presented to judge the performance of the proposed class of estimators.

2. THE PROPOSED ESTIMATOR

In this section, utilizing information on the auxiliary variable x with unknown population mean $\bar{X}$, we will study Chanu and Singh (2015) estimator in double sampling scheme for the population mean $\bar{Y}$ in two different situations of non-response.

The following two cases will be considered separately.

Case I: When non-response occurs only on y .

Case II: When non-response occurs on both y and x .

2.1 Case I: Non-response only on y

We define Chanu and Singh (2015) estimator for $\bar{Y}$ in the presence of non response as

$$\left(\bar{y}_{RdR}^{d*}\right)_I=\bar{y}^*\left[\alpha_1\frac{\bar{x}'}{\bar{x}}+\left(1-\alpha_1\right)\frac{n'\bar{x}'-n\bar{x}}{\left(n'-n\right)\bar{x}'}\right]. \quad (3)$$

where α_1 is a scalar constant.

To obtain the bias and MSE of $\left(\bar{y}_{RdR}^{d*}\right)_I$, we write

$e_0^*=\left(\bar{y}^*-\bar{Y}\right)/\bar{Y}$, $e_1=\left(\bar{x}-\bar{X}\right)/\bar{X}$ and $e_2=\left(\bar{x}'-\bar{X}\right)/\bar{X}$.

Expressing $\left(\bar{y}_{RdR}^{d*}\right)_I$ in terms of e's, we obtain

$$\left(\bar{y}_{RdR}^{d*}\right)_I=\bar{Y}\left(1+e_0^*\right)\left[\alpha_1\left(1+e_2\right)\left(1+e_1\right)^{-1}\right.$$

$$\left.+\left(1-\alpha_1\right)\left\{\left(1+h\right)-h\left(1+e_1\right)\left(1+e_2\right)^{-1}\right\}\right]$$

where $h=\frac{n}{n'-n}$.

Assuming the sample size to be large enough, so that $|e_1|<1$ and $|e_2|<1$, therefore $(1+e_1)^{-1}$ and $(1+e_2)^{-1}$ are expandable in Taylor series.

Expanding the right hand side of the above equation, multiplying out and retaining terms of e's up to second degree, we obtain

$$\left(\bar{y}_{RdR}^{d*}\right)_I-\bar{Y}\cong\bar{Y}\Big[e_0^*-h\left(e_1-e_2+e_2^2-e_1e_2+e_0^*e_1-e_0^*e_2\right)$$
$$+\alpha_1\Big\{(1-h)e_2-(1-h)e_1+e_1^2+he_2^2-(1+h)e_1e_2$$
$$-(1-h)e_0^*e_1+(1-h)e_0^*e_2\Big\}\Big] \quad (4)$$

To obtain the bias and MSE of $\left(\bar{y}_{RdR}^{d*}\right)_I$, the following notations are used

$C_y^2=S_y^2/\bar{Y}^2$ $C_x^2=S_x^2/\bar{X}^2$ and $\rho=S_{xy}/S_yS_x$

where C_y and C_x are the coefficients of variation of the study variable y and the auxiliary variable x respectively.

$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$ and

$S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$ are population variances of study variable y and auxiliary variable x respectively and $S_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(y_i-\bar{Y}\right)$ is the covariance between y and x.

3. BIAS, MSE AND OPTIMUM VALUE OF $\left(\bar{y}_{RdR}^{d*}\right)_I$ IN CASE I

In this case, we have

$$E\left(e_0^*\right)=E\left(e_1\right)=E\left(e_2\right)=0$$

$$E\left(e_0^{*2}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\frac{W_2(k-1)}{n}C_{2y}^2,$$

$$E\left(e_1^2\right)=\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2,\ E\left(e_2^2\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2,$$

$$E\left(e_0^*e_1\right)=\left(\frac{1}{n}-\frac{1}{N}\right)C_x^2K_{yx},$$

$$E\left(e_0^*e_2\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2K_{yx}$$

and $$E\left(e_1e_2\right)=\left(\frac{1}{n'}-\frac{1}{N}\right)C_x^2 \quad (5)$$

where $K_{yx}=\rho_{yx}\frac{C_y}{C_x}$.

Taking expectations on both the sides of (4) and using the results of (5), we get the bias of the estimator $\left(\bar{y}_{RdR}^{d*}\right)_I$ to the first degree of approximation as

$$B\left(\bar{y}_{RdR}^{d*}\right)_I=\bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left[-hK_{yx}+\alpha_1\left\{1-(1-h)K_{yx}\right\}\right] \quad (6)$$

The bias in (6) is zero, when

$$\alpha_1=\frac{hK_{yx}}{1-(1-h)K_{yx}}$$

Squaring and taking expectations on both the sides of (4) and using the results of (5), we obtain the MSE of $\left(\bar{y}_{RdR}^{d*}\right)_I$ up to the first degree of approximation as

$$MSE\left(\bar{y}_{RdR}^{d*}\right)_I=\bar{Y}^2\Bigg[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+\frac{W_2(k-1)}{n}C_{2y}^2$$
$$+\alpha_1^2(1-h)^2\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2$$
$$+2h\alpha_1(1-h)\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left\{1-\frac{1}{h}K_{yx}\right\}$$
$$-2h\left(\frac{1}{n}-\frac{1}{n'}\right)K_{yx}C_x^2+h^2\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\Bigg] \quad (7)$$

Minimizing (7) with respect to α yields its optimum value

$$\alpha_1=\frac{K_{yx}-h}{1-h}=\alpha_{1(opt)} \quad (say) \quad (8)$$

Substituting the value of α_1 from (8) in (3) gives the asymptotically optimum estimator (AOE) as

$$\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}=\bar{y}^*\left[\alpha_{1(opt)}\frac{\bar{x}'}{\bar{x}}+\left(1-\alpha_{1(opt)}\right)\frac{n'\bar{x}'-n\bar{x}}{(n'-n)\bar{x}'}\right] \quad (9)$$

Therefore, the resulting bias and MSE of $\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}$ are

$$B\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)} = \bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left[\frac{K_{yx}-h}{1-h}-K_{yx}^2\right]$$

and

$$MSE\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)} = \bar{Y}^2\left[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2 + \frac{W_2(k-1)}{n}C_{2y}^2 - \left(\frac{1}{n}-\frac{1}{n'}\right)K_{yx}^2C_x^2\right] \quad (10)$$

which is the same as the variance of the linear regression estimator $\bar{y}_{dlr} = \bar{y}^* + b\left(\bar{x}' - \bar{x}\right)$ in two phase sampling, where b is the sample regression coefficient of y on x.

3.1. Remarks:

1. When $\alpha_1 = 0$, the proposed class of estimators reduces to dual to ratio estimator in double sampling as

$$\left(\bar{y}_{dR}^{d*}\right)_I = \bar{y}^* \frac{n'\bar{x}' - n\bar{x}}{\left(n'-n\right)\bar{x}'}$$

The bias and MSE of $\left(\bar{y}_{dR}^{d*}\right)_I$ can be obtained by putting $\alpha_1 = 0$ in (6) and (7) respectively as

$$B\left(\bar{y}_{dR}^{d*}\right)_I = -\bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)hK_{yx}C_x^2$$

and

$$MSE\left(\bar{y}_{dR}^{d*}\right)_I = \left[\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2 + \frac{W_2(k-1)}{n}S_{2y}^2 + \left(\frac{1}{n}-\frac{1}{n'}\right)h^2R^2S_x^2\left(1-\frac{2}{h}K_{yx}\right)\right] \quad (11)$$

2. When $\alpha_1 = 1$, the proposed class of estimators reduces to ratio estimator in double sampling as

$$\left(\bar{y}_R^{d*}\right)_I = \bar{y}^* \frac{\bar{x}'}{\bar{x}}$$

The bias and MSE of $\left(\bar{y}_R^{d*}\right)_I$ can be obtain by putting $\alpha_1 = 1$ in (6) and (7) respectively as

$$B\left(\bar{y}_R^{d*}\right)_I = \bar{Y}\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2\left(1-K_{yx}\right)$$

and

$$MSE\left(\bar{y}_R^{d*}\right)_I = \left[\left(\frac{1}{n}-\frac{1}{N}\right)S_y^2 + \frac{W_2(k-1)}{n}S_{2y}^2 + \left(\frac{1}{n}-\frac{1}{n'}\right)R^2S_x^2\left(1-2K_{yx}\right)\right] \quad (12)$$

4. EFFICIENCY COMPARISONS OF $\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}$ IN CASE I

4.1. Comparison with the Mean per Unit Estimator

From equation (1) and (10), we observe that

$$V\left(\bar{y}^*\right) - M\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)} = \left(\frac{1}{n}-\frac{1}{n'}\right)R^2S_x^2K_{yx}^2 > 0. \quad (13)$$

4.2. Comparison with the Usual Ratio Estimator in Double Sampling

From (12) and (10), we observe that

$$MSE\left(\bar{y}_R^{d*}\right)_I - MSE\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)} = \left[\left(\frac{1}{n}-\frac{1}{n'}\right)R^2S_x^2\left(1-K_{yx}\right)^2\right] > 0 \quad (14)$$

4.3. Comparison with the Dual to Ratio Estimator in Double Sampling

From (11) and (10), we observe that

$$MSE\left(\bar{y}_{dR}^{d*}\right)_I - M\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)} = \left(\frac{1}{n}-\frac{1}{n'}\right)R^2h^2S_x^2\left(1-\frac{1}{h}K_{yx}\right)^2 > 0 \quad (15)$$

5. CASE II: NON-RESPONSE ON BOTH y AND x

In this case, the suggested estimator is given as follows

$$\left(\bar{y}_{RdR}^{d**}\right)_{II} = \bar{y}^{*}\left[\alpha_2 \frac{\bar{x}'}{\bar{x}^{*}} + (1-\alpha_2)\frac{n'\bar{x}' - n\bar{x}^{*}}{\left(n'-n\right)\bar{x}'}\right] \tag{16}$$

where α_2 is a scalar constant.

In this case, we have

Where $E\left(e_0^*\right) = E\left(e_1^*\right) = E\left(e_2\right) = 0$

$$E(e_0^{*2}) = \left(\frac{1}{n} - \frac{1}{N}\right)C_y^2 + \frac{W_2(k-1)}{n}C_{2y}^2,$$

$$E(e_1^{*2}) = \left(\frac{1}{n} - \frac{1}{N}\right)C_x^2 + \frac{W_2(k-1)}{n}C_{2x}^2,$$

$$E(e_2^2) = \left(\frac{1}{n'} - \frac{1}{N}\right)C_x^2,$$

$$E(e_0^*e_1^*) = \left(\frac{1}{n} - \frac{1}{N}\right)C_x^2 K_{yx} + \frac{W_2(k-1)}{n}C_{2x}^2 K_{2yx}$$

$$E(e_0^*e_2) = \left(\frac{1}{n'} - \frac{1}{N}\right)C_x^2 K_{yx},$$

$$E(e_1^*e_2) = \left(\frac{1}{n'} - \frac{1}{N}\right)C_x^2$$

where $e_1^* = \left(\bar{x}^* - \bar{X}\right)/\bar{X}$ (17)

Replacing e_1 by e_1^* and taking expectations in (4) and using the results of (17), we get the bias of $\left(\bar{y}_{RdR}^{d**}\right)_{II}$ to the first degree of approximation as

$$B\left(\bar{y}_{RdR}^{d**}\right)_{II} = \bar{Y}\left[-h\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)K_{yx}C_x^2 + \lambda' K_{2yx}C_{2x}^2\right\} + \alpha_2\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \lambda' C_{2x}^2\right\}\right. \tag{18}$$

Replacing e_1 by e_1^*, squaring and taking expectations on both the sides of (4) and using the results of (17), we obtain the MSE of $\left(\bar{y}_{RdR}^{d**}\right)_{II}$ to the first degree of approximation as

$$MSE\left(\bar{y}_{RdR}^{d**}\right)_{II} = \left[C + h^2R^2A + (1-h)^2\alpha_2^2R^2A - 2hRB - 2\alpha_2(1-h)RB + 2\alpha_2 h(1-h)R^2A\right] \tag{19}$$

Wh

ere $R = \frac{\bar{Y}}{\bar{X}}$

$$A = \left(\frac{1}{n} - \frac{1}{n'}\right)S_x^2 + \frac{W_2(k-1)}{n}S_{2x}^2$$

$$B = \left(\frac{1}{n} - \frac{1}{n'}\right)S_{xy} + \frac{W_2(k-1)}{n}S_{2xy}$$

$$C = \left(\frac{1}{n} - \frac{1}{N}\right)S_y^2 + \frac{W_2(k-1)}{n}S_{2y}^2$$

Minimizing (19) with respect to α_2 yields its optimum value

$$\alpha_2 = \frac{B}{(1-h)A} - \frac{h}{1-h} = \alpha_{2(opt)} \quad (say) \tag{20}$$

Substituting the value of α_2 form (20) in (3) gives the asymptotically optimum estimator (AOE) as

$$\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)} = \bar{y}^{*}\left[\alpha_{2(opt)}\frac{\bar{x}'}{\bar{x}} + \left(1-\alpha_{2(opt)}\right)\frac{n'\bar{x}' - n\bar{x}}{\left(n'-n\right)\bar{x}'}\right] \tag{21}$$

Therefore, the resulting bias and MSE of $\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)}$ are

$$B\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)} = \left[\frac{R^2(B-hA)}{(1-g)} - \frac{RB^2}{A}\right]$$

and

$$MSE\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)} = \left[C - \frac{B^2}{A}\right] \tag{22}$$

5.1. Remarks:

1. When $\alpha_2 = 0$, the proposed class of estimators reduces to dual to ratio estimator in double sampling as

$$\left(\overline{y}_{dR}^{d**}\right)_{II} = \overline{y}^* \frac{n'\overline{x}' - n\overline{x}^*}{(n'-n)\overline{x}'}$$

The bias and MSE of $\left(\overline{y}_{dR}^{d**}\right)_{II}$ can be obtained by putting $\alpha_2 = 0$ in (18) and (19) respectively as

$$B\left(\overline{y}_{dR}^{d**}\right)_{II} = \overline{Y}\left[-h\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)K_{yx}C_x^2 + \lambda' K_{2yx}C_{2x}^2\right\}\right]$$

and $MSE\left(\overline{y}_{dR}^{d**}\right)_{II} = C + h^2R^2A - 2hRB$.

(23)

2. When $\alpha_2 = 1$, the proposed class of estimators reduces to ratio estimator in double sampling as

$$\left(\overline{y}_{R}^{d**}\right)_{II} = \overline{y}^* \frac{\overline{x}'}{\overline{x}^*}$$

The bias and MSE of $\left(\overline{y}_{R}^{d**}\right)_{II}$ can be obtained by putting $\alpha_2 = 1$ in (18) and (19) respectively as

$$B\left(\overline{y}_{R}^{d**}\right)_{II} = \overline{Y}\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)C_x^2 + \lambda' C_{2x}^2\right\}$$

$$-\left\{\left(\frac{1}{n}-\frac{1}{n'}\right)K_{yx}C_x^2 + \lambda' K_{2yx}C_{2x}^2\right\}\Bigg]$$

and $MSE\left(\overline{y}_{R}^{d**}\right)_{II} = \left[C + R^2A - 2RB\right]$

(24)

6. EFFICIENCY COMPARISON OF $\left(\overline{y}_{RdR}^{d**}\right)_{II(opt)}$ IN CASE II

6.1 Comparison with the Mean per Unit Estimator

From equation (1) and (22), we observe that

$$V\left(\overline{y}^*\right) - M\left(\overline{y}_{RdR}^{d**}\right)_{II(opt)} = \frac{B^2}{A} > 0. \qquad (25)$$

6.2. Comparison with the Usual Ratio Estimator in Double Sampling

From (24) and (22), we observe that

$$M\left(\overline{y}_{R}^{d**}\right)_{II} - M\left(\overline{y}_{RdR}^{d**}\right)_{II(opt)} = \frac{(RA-B)^2}{A} > 0 \qquad (26)$$

6.3. Comparison with the Dual to Ratio Estimator in Double Sampling

From (23) and (22), we observe that

$$MSE\left(\overline{y}_{dR}^{d**}\right)_{II} - M\left(\overline{y}_{RdR}^{d**}\right)_{II(opt)} = \frac{(hRA-B)^2}{A} > 0 \qquad (27)$$

7. EMPIRICAL STUDY

To illustrate the properties of the proposed estimator of the population mean $\overline{Y}$, we consider a real data set considered by Khare (1993). The description of the sample is given below.

The sample of 100 consecutive trips (after omitting 20 outlier's values) measured by two fuel meters for a small family car in normal usage given by Lewis et.al.(1991) has been taken into consideration. The measurement of tribune meter (in ml) is considered as main character y and the measurement of displacement meter (in cm^3) is considered as auxiliary character x. We treat the last 25% values as non-response values. The values of the parameters are as follows:

$\overline{Y} = 3500.12$, $\overline{X} = 260.84$,
$S_y = 2079.30$, $S_x = 156.40$,
$\overline{Y}_2 = 3401.08$, $\overline{X}_2 = 259.96$,
$S_{2y} = 1726.02$, $S_{2x} = 134.36$,
$\rho_{yx} = 0.985$, $\rho_{2yx} = 0.995$, $N = 100$,
$n' = 50$, $n = 30$.

Here, we have computed the percent relative efficiencies (PRE) of different estimators w.r.t the usual unbiased estimator $\overline{y}^*$ for different values of k .

Table 1. PRE of the different estimators $\left(\bar{y}_{R}^{d*}\right)_{I}$, $\left(\bar{y}_{dR}^{d*}\right)_{I}$ and $\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}$ with respect to $\bar{y}^{*}$

W_2	k	$\bar{y}^{*}$	$\left(\bar{y}_{R}^{d*}\right)_{I}$	$\left(\bar{y}_{dR}^{d*}\right)_{I}$	$\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}$
0.1	1.5	100	211.41	160.06	212.06
	2.0	100	192.29	148.86	192.84
	2.5	100	176.35	139.12	176.81
	3.0	100	162.92	130.63	163.31
0.2	1.5	100	192.29	148.86	192.84
	2.0	100	162.92	130.63	163.31
	2.5	100	141.27	116.34	141.56
	3.0	100	124.75	106.73	124.98
0.3	1.5	100	176.35	139.12	176.81
	2.0	100	141.27	116.34	141.56
	2.5	100	117.84	100.00	118.04
	3.0	100	101.07	87.63	101.22

Table 2. PRE of the different estimators $\left(\bar{y}_{R}^{d**}\right)_{II}$, $\left(\bar{y}_{dR}^{d**}\right)_{II}$ and $\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)}$ with respect to $\bar{y}^{*}$

W_2	k	$\bar{y}^{*}$	$\left(\bar{y}_{R}^{d**}\right)_{II}$	$\left(\bar{y}_{dR}^{d**}\right)_{II}$	$\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)}$
0.1	1.5	100	234.31	168.99	235.05
	2.0	100	234.04	165.14	234.78
	2.5	100	233.50	161.46	234.51
	3.0	100	233.50	157.96	234.24
0.2	1.5	100	234.04	165.14	234.78
	2.0	100	233.50	157.96	234.24
	2.5	100	232.97	151.36	233.72
	3.0	100	232.43	145.30	233.19
0.3	1.5	100	233.77	161.46	234.51
	2.0	100	232.97	151.36	233.72
	2.5	100	232.17	142.44	232.94
	3.0	100	231.37	134.52	232.16

8. CONCLUSION

Table 1 suggests that the proposed estimator $\left(\bar{y}_{RdR}^{d*}\right)_{I(opt)}$ is performing much better than the usual unbiased estimator $\bar{y}^{*}$ and the estimator $\left(\bar{y}_{dR}^{d*}\right)_{I}$ and slightly better than $\left(\bar{y}_{R}^{d*}\right)_{I}$ in its optimality when there is non-response on y. Also, when there is non-response on both y and x, Table 2 shows the same pattern for the proposed estimator $\left(\bar{y}_{RdR}^{d**}\right)_{II(opt)}$ in case of the usual unbiased estimator $\bar{y}^{*}$ and estimators belonging to its class. It is also observed from the tables that the efficiency of the suggested estimator decreases with the increase in rate of non-response.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Agarwal, G.K., Allende, S.M. and Bouza C.N. Double sampling with ranked set selection in the second phase with non-response: analytical results and Monte Carlo experiences. *Journal of*

[2] *Probability and Statistics*, (2012), Article ID 214959, 12 pages. doi:10.1155/2012/214959

[3] Bouza, C.N. Ranked set sampling procedures for the estimation of the population mean under non responses: a comparison. *Revista Investigacion Operacional*, 30(2):150-160, (2010).

[4] Chanu, W.W. and Singh, B.K. An efficient class of double sampling dual to ratio estimators of population mean in sample surveys. *International Journal of Statistics and Economics*, 14(2), (2012).

[5] Chanu, W.W. and Singh, B.K. A class of ratio cum dual to ratio estimator of finite population mean in presence of non-response. *International Journal of Mathematics and Computation*, 26(3), (2015).

[6] Cochran, W. Sampling techniques. *New York, Wiley and Sons*, 98:259-261, (1977).

[7] Hansen, M.N. and Hurwitz, W. N. The problem of non-response in sample surveys. *Journal of the American Statistical Association,* 41(236):517-529, (1946).

[8] Khare, B.B. and Srivastava, S. Estimation of population mean using auxiliary character in

presence of non-response. *National Academy Science Letters-India*, 16(3):111-114, (1993).

[9] Khare, B.B. and Srivastava, S. Study of conventional and alternative two phase sampling ratio, product and regression estimators in presence of non-response. *Proceedings-national Academy Science, India, Section A*, 65:195-204, (1995).

[10] Khare, B.B. and Srivastava, S. Transformed ratio types estimators for the population mean in the presence of non-response. *Communications in Statistics-Theory and Methods*, 26(7):1779-1791, (1997).

[11] Kumar, M. and Bahl, S. Class of dual to ratio estimators for double sampling. *Statistical Papers*, 47(2):319-326, (2006).

[12] Lewis, P. A., Jones, P.W., Polak, J .W. and Tillotson, H. T. The problem of conversion in method comparison studies. *Applied Statistics*, 105-112, (1991).

[13] Neyman, J. Contribution to the theory of sampling human populations. *Journal of the American Statistical Association*, 33(201):101-116, (1938).

[14] Okafor, F. C. and Lee, H. Double sampling for ratio and regression estimation with sub-sampling the non-respondents. *Survey Methodology*, 26(2):183-188, (2000).

[15] Rao, P.S.R.S. Ratio estimation with sub sampling the non respondents. *Survey Methodology*, 12(2):217-230, (1986).

[16] Sharma, B. and Tailor, R. A new ratio-cum-dual to ratio estimator of finite population mean in simple random sampling. *Global Journal of Science Frontier Research*, 10(1), (2010).

[17] Singh, H. P. and Kumar, S. Estimation of mean in presence of non-response using two phase sampling scheme. *Statistical Papers*, 51(3):559-582, (2010).

[18] Singh, H.P. and Kumar, S. A regression approach to the estimation of the finite population mean in the presence of non-response. *Australian & New Zealand Journal of Statistics*, 50(4):395-408, (2008).

[19] Singh, H.P. and Kumar, S. A general procedure of estimating the population mean in the presence of non-response under double sampling using auxiliary information. *Statistics and Operations Research Transactions*, 33(1):71-84, (2009).

[20] Sodipo, A. A. and Obisesan, K. O. Estimation of the population mean using difference cum ratio estimator with full response on the auxiliary character. *Research Journal of Applied Sciences,* 2(6):769-772, (2007).

[21] Srivenkataramana, T. A dual to ratio estimator in sample surveys. *Biometrika*, 67(1):199-204, (1980).

[22] Riaz, S., Diana, G., & Shabbir, J. A general class of estimators for the population mean using multi-phase sampling with the non-respondents. *Hacettepe Journal of Mathematics Statistics*, *43*(3), 511-527, (2014).

[23] Tabasum, R. and Khan, I. A. Double sampling ratio estimator for the population mean in presence of non-response. *Assam Statistics Review*, 20(1):73-83, (2006).

[24] Tabasum, R. and Khan, I.A. Double sampling for ratio estimation with non-response. *Journal of Indian Society and Agricultural Statistics*, 58(3):300-306, (2004).