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Shehu Conformable Fractional Transform, Theories and Applications

Mohamed Elarbi Benattia * [1] and Kacem Belghaba [2]

[1]High school of Economics., Laboratory of Mathematics and its Applications(LAMAP), Oran, Algeria
[2]Department of Mathematics, Laboratory of Mathematics and its Applications(LAMAP), University of Oran 1, Oran, Algeria

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Abstract

The study of famous properties of fractional derivative and their proof has gained a lot of attention recently. In present work, we have been interested to generalizing the definition and some rules and important properties of the Shehu transform to the conformable fractional order which have been demonstrated. We use some properties of the conformable fractional Shehu transform to find the general analytical solutions of linear and nonlinear conformable fractional differential equations in the case homogeneous and nonhomogeneous based on the new transform and Adomain polynomial method. The two illustrative examples indicate that the used transform is powerful, effective and applicable for the both linear and nonlinear problems.

1. Introduction

Mathematics researchers have been interested in differential equations, because they govern many physical, chemical, biological and even economic aspects. Recently, after the tremendous development brought about by fractional calculus, their interest in researching fractional differential equations has become more and more interesting due to their applications in the field of visco-elasticity, mechanics, electroanalytical chemistry, electrical circuits, etc... [1]. There are many definitions of fractional derivatives that have been used in many applications and natural phenomena such as Riemann-Liouville, Caputo, Hadamard, Grunwald, Letnikov and Riesz. In 2014, Khalil et al [2, 3] gave the new definition of the fractional derivative called the conformable fractional derivative (CFD), and it can be easily found compared to other previous definitions. In this paper, we expanded the definition of the Shehu transform [4–7] into a fractional order and deduced a list of important rules and properties of this extension. As we note, there is a very important relationship between the conformable Shehu transform (CFSHT) and the conformable Sumudu (CFST) and conformable Laplace transform (CFLT) transform. The main advantages of the (CFSHT) can be summarized as follow [8]:

- It satisfies the all concepts and rules of an ordinary derivative such as: quotient, product and chain rules while the other fractional definitions fail to meet these rules.
- It can be extended to solve exactly and numerically fractional differential equations and systems easily and efficiently.
- It generalizes the well-known transforms such as: Laplace and Sumudu conformable transforms and used as tools for solving some singular conformable fractional differential equations.
- It creates new comparisons of (CFSHT) and other previous conformable fractional definitions in many applications.

In addition, we present two important applications for Conformable Fractional Shehu transform. First, we apply it to obtain the general solutions of some linear and homogeneous equations, secondly, we apply the conformable

*Corresponding author: mohamed.benattia74@yahoo.com
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fractional Shehu transformation with Adomain decomposition method to find the general analytical solution of the singular and nonlinear conformable fractional Riccati equation. Finally, the proposed method helped us greatly in finding general solutions for all cases of conformable fractional differential equations [9].
This document can be described as follows. In section 2, some basic properties of fractional calculus are given. In section 3, we present the new fractional transformation with some new concepts and interesting relationships. In section 4, the transformation is used to obtain analytical solutions of fractional models. the paper ends with a conclusion in section 5.

2. Preliminary

Definition 1 *We consider functions in the set F defined by [4]:*

$$F=\left\{v(t):\exists N,k_1,k_2>0\,,\left|v(t)\right|<N\exp\left(\frac{|t|}{k_j}\right),\ if\ t\in(-1)^j\times\left[0;\infty\right[\right\},\tag{1}$$

the Shehu transform is defined for functions of exponential order is defined over functions in F by the following integral:

$$V(s,u)=Sh\left[f(t)\right]=\int_0^\infty \exp\left(\frac{-st}{u}\right)f(t)dt,\ s>0,\ u>0.\tag{2}$$

The inverse Shehu transform is given by

$$Sh^{-1}\left[V(s,u)\right]=v(t).\tag{3}$$

Definition 2 *[9](The conformable fractional integral). Let $0<\alpha\leq 1$, and $f:\ \left[0,\infty\right)\to\mathbb{R}$. The conformable fractional integral of f of order α from 0 to x is defined as:*

$$I^\alpha f(x)=\int_0^x f(t)d_\alpha t=\int_0^x f(t)t^{\alpha-1}dt,\tag{4}$$

where the above integral is the usual improper Riemann integral.
Furthermore, if $f(x)$ is an α- differentiable function in some $\left[0,\infty\right)$ and $0<\alpha\leq 1$, then:

$$D^\alpha f(x)=x^{1-\alpha}\frac{df(x)}{dx}.$$
$$D^\alpha I^\alpha f(x)=f(x).\tag{5}$$

3. Conformable fractional Shehu transform

Definition 3 *(The conformable fractional Shehu transform (CFSHT)):*
Let $0<\alpha\leq 1$, and $f:\ \left[0,\infty\right)\to\mathbb{R}$ be a real value function. Then, the conformable fractional Shehu of order α is defined by:

$$Sh_\alpha\left[f(t)\right]=V_\alpha(s;u)=\int_0^\infty \exp\left(\frac{-s}{u}\frac{t^\alpha}{\alpha}\right)f(t)t^{\alpha-1}dt,\tag{6}$$

provided the integral exists.

Theorem 1 *Let $f:\ \left[0,\infty\right)\to\mathbb{R}$ be a real value function and $0<\alpha\leq 1$, then:*

$$V_\alpha\left[D^\alpha f(t)\right]=\frac{s}{u}V_\alpha(s;u)-f(0).\tag{7}$$

Proof 1 *By using the definition (3), we have:*

$$Sh_\alpha\left[D^\alpha f(t)\right]=\int_0^\infty \exp\left(\tfrac{-st^\alpha}{u\alpha}\right)D^\alpha f(t)t^{\alpha-1}dt=\int_0^\infty \exp\left(\tfrac{-st^\alpha}{u\alpha}\right)t^{1-\alpha}f'(t)t^{\alpha-1}dt=\int_0^\infty \exp\left(\tfrac{-st^\alpha}{u\alpha}\right)f'(t)dt,$$

and with integration by parts, we obtain :

$$Sh_\alpha\left[D^\alpha f(t)\right]=\left[\exp\left(\tfrac{-st^\alpha}{u\alpha}\right)f(t)\right]_0^\infty+\frac{s}{u}\int_0^\infty \exp\left(\tfrac{-st^\alpha}{u\alpha}\right)f(t)t^{\alpha-1}dt$$

$$=\lim_{t\to\infty}\left(\exp\left(\tfrac{-st^\alpha}{u\alpha}\right)f(t)\right)-f(0)+\frac{s}{u}V_\alpha(s;u)$$

$$\Rightarrow V_\alpha\left[D^\alpha f(t)\right]=\frac{s}{u}V_\alpha(s;u)-f(0).$$

Theorem 2 *Let $f:\ [0,\infty)\to\mathbb{R}$ be a real value function and $0<\alpha\leq 1$, then:*

$$V_\alpha(s;u)=V\left\{f(\alpha t)^{\frac{1}{\alpha}}\right\}(s,u). \tag{8}$$

Proof 2 *If we take $x=\dfrac{t^\alpha}{\alpha}$, we have:*

$$V_\alpha(s;u)=\int_0^\infty \exp\left(\tfrac{-st^\alpha}{u\alpha}\right)f(t)t^{\alpha-1}dt=\int_0^\infty \exp\left(-\frac{s}{u}x\right)f\left[(\alpha x)^{\frac{1}{\alpha}}\right]dx=V\left(f\left((\alpha x)^{\frac{1}{\alpha}}\right)\right).$$

Theorem 3 *Let a, c and $p\in\mathbb{R}$ and $0<\alpha\leq 1$, then:*

$$1)\quad V_\alpha\{c\}(s;u)=c\frac{u}{s}. \tag{9}$$

$$2)\quad V_\alpha\{t^p\}(s;u)=\alpha^{\frac{p}{\alpha}}\left(\frac{u}{s}\right)^{\frac{p}{\alpha}+1}\Gamma(1+\frac{p}{\alpha}). \tag{10}$$

$$3)\quad V_\alpha\left\{\exp(a\frac{t^\alpha}{\alpha})\right\}(s;u)=\frac{u}{s-au},\ \frac{s}{u}>0. \tag{11}$$

$$4)\quad V_\alpha\left\{\sin(a\frac{t^\alpha}{\alpha})\right\}(s;u)=\frac{au^2}{s^2+a^2u^2},\ \frac{s}{u}>0. \tag{12}$$

$$5)\quad V_\alpha\left\{\cos(a\frac{t^\alpha}{\alpha})\right\}(s;u)=\frac{su}{s^2+a^2u^2},\ \frac{s}{u}>0. \tag{13}$$

$$6)\quad V_\alpha\left\{\sinh(a\frac{t^\alpha}{\alpha})\right\}(s;u)=\frac{au^2}{s^2-a^2u^2},\ \frac{s}{u}>|a|. \tag{14}$$

$$7)\quad V_\alpha\left\{\cosh(a\frac{t^\alpha}{\alpha})\right\}(s;u)=\frac{su}{s^2-a^2u^2},\ \frac{s}{u}>|a|. \tag{15}$$

Proof 3 *1) proof follow easily by the aid of definition.*

2) $V_\alpha\{t^p\}(s;u)=V_\alpha\left\{(\alpha t)^{\frac{p}{\alpha}}\right\}(s;u)=\alpha^{\frac{p}{\alpha}}V(t^{\frac{p}{\alpha}})=\alpha^{\frac{p}{\alpha}}\left(\frac{u}{s}\right)^{\frac{p}{\alpha}+1}\Gamma(1+\frac{p}{\alpha})$.

If we put $p=n\alpha$, then $V_\alpha\{t^{n\alpha}\}(s;u)=\alpha^n\left(\frac{u}{s}\right)^{n+1}\Gamma(1+n)$. So

$$V_\alpha\left\{\frac{t^{n\alpha}}{\alpha^n}\right\}(s;u)=\left(\frac{u}{s}\right)^{n+1}\Gamma(1+n)=\left(\frac{u}{s}\right)^{n+1}n!.$$

3) $V_\alpha\left\{\exp(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V_\alpha\left\{\exp(a\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right\}(s;u)=V\left\{\exp(at)\right\}=\frac{u}{s-au}.$

4) $V_\alpha\left\{\sin(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V_\alpha\left\{\sin(a\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right\}(s;u)=V\left\{\sin(at)\right\}=\frac{au^2}{s^2+a^2u^2}.$

5) $V_\alpha\left\{\cos(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V_\alpha\left\{\cos(a\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right\}(s;u)=V\left\{\cos(at)\right\}=\frac{su}{s^2+a^2u^2}.$

6) $V_\alpha\left\{\sinh(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V_\alpha\left\{\sinh(a\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right\}(s;u)=V\left\{\sinh(at)\right\}=\frac{au^2}{s^2-a^2u^2}.$

7) $V_\alpha\left\{\cosh(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V_\alpha\left\{\cosh(a\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right\}(s;u)=V\left\{\cosh(at)\right\}=\frac{su}{s^2-a^2u^2}.$

Theorem 4 *(Some Properties of the Conformable Fractional Shehu Transform)*

1) The conformable fractional Shehu transform is linear operator:

$$V_\alpha\left[\mu f(t)\pm\lambda g(t)\right]=\mu V_\alpha\left[f(t)\right]\pm\lambda V_\alpha\left[g(t)\right], \tag{16}$$

where μ and λ are constants.

2) Shifting property :

$$V_\alpha\left\{\exp(-a\frac{t^\alpha}{\alpha})f(t)\right\}(s;u)=V\left(f(\alpha t)^{\frac{1}{\alpha}}\right)|_{s=s+au}=V_\alpha(s+au;u). \tag{17}$$

3) The conformable fractional Shehu transform of the conformable fractional integral:

$$V_\alpha\left[I^\alpha f(t)\right]=\frac{u}{s}V_\alpha(s;u). \tag{18}$$

4) Convolution property:

$$V_\alpha\left\{(f\star g)(t)\right\}(s;u)=V_\alpha\left\{f(t)\right\}(s;u).V_\alpha\left\{g(t)\right\}(s;u). \tag{19}$$

Proof 4 *1) Trivial by definition(2).*

2) we have:

$$V_\alpha\left\{\exp(-a\frac{t^\alpha}{\alpha})f(t)\right\}(s;u)=V\left\{\exp(-a\frac{(\alpha t)^{\frac{1}{\alpha}}}{\alpha})f(\alpha t)^{\frac{1}{\alpha}}\right\}(s;u)$$

$$=V\left\{\exp(-at)f(\alpha t)^{\frac{1}{\alpha}}\right\}(s;u)$$

$$=\int_0^\infty\exp\left(-(\frac{s}{u}+a)t\right)f\left[(\alpha t)^{\frac{1}{\alpha}}\right]dt$$

$$=V\left\{f(\alpha t)^{\frac{1}{\alpha}}\right\}(s+au;u).$$

For example:

$$V_\alpha\left\{\exp(-a\frac{t^\alpha}{\alpha})\sinh(a\frac{t^\alpha}{\alpha})\right\}(s;u)=V\left\{\sinh(at)\right\}(s+au;u)$$

$$=\frac{au^2}{(s+au)^2-a^2u^2}=\frac{au^2}{s^2+2asu}.$$

3) By the theorem (1), we have :

$$V_\alpha\left[D^\alpha I^\alpha f(t)\right]=\frac{s}{u}V_\alpha\left[I^\alpha f(t)\right](s;u)-I^\alpha f(0).$$

But $D^\alpha I^\alpha f(t)=f(t)$ *and* $I^\alpha f(0)=0$, *so we obtain:*

$$V_\alpha\left[f(t)\right](s;u)=\frac{s}{u}V_\alpha\left[I^\alpha f(t)\right](s;u)$$

$$\Rightarrow V_\alpha\left[I^\alpha f(t)\right](s;u)=\frac{u}{s}V_\alpha(s;u).$$

4) By the theorem (2) and the convolution definition, we have:

$$V_\alpha\left\{(f\star g)(t)\right\}(s;u)=V\left\{(f\star g)(\alpha t)^{\frac{1}{\alpha}}\right\}(s;u)$$

$$=\int_0^\infty \exp\left(-\frac{s}{u}t\right)(f\star g)\left[(\alpha t)^{\frac{1}{\alpha}}\right]dt$$

$$=\int_0^\infty \exp\left(-\frac{s}{u}t\right)\left(\int_0^t f\left[(\alpha(t-z))^{\frac{1}{\alpha}}\right]g\left[(\alpha z)^{\frac{1}{\alpha}}\right]dz\right)dt.$$

Change the order of integration, we get:

$$V_\alpha\left\{(f\star g)(t)\right\}(s;u)=\int_0^\infty\int_z^\infty \exp\left(-\frac{s}{u}t\right)f\left[(\alpha(t-z))^{\frac{1}{\alpha}}\right]g\left[(\alpha z)^{\frac{1}{\alpha}}\right]dtdz,$$

if we put $t-z=v$, *then* $dt=dv$, *and:*

$$V_\alpha\left\{(f\star g)(t)\right\}(s;u)=\int_0^\infty\int_0^\infty \exp\left(-\frac{s}{u}(v+z)\right)f\left[(\alpha v)\right]g\left[(\alpha z)^{\frac{1}{\alpha}}\right]dvdz$$

$$=\left(\int_0^\infty \exp\left(-\frac{s}{u}v\right)f\left[(\alpha v)^{\frac{1}{\alpha}}\right]dv\right)\left(\left(\int_0^\infty \exp\left(-\frac{s}{u}z\right)g\left[(\alpha z)^{\frac{1}{\alpha}}\right]dz\right)\right)$$

$$=V\left\{f(\alpha v)^{\frac{1}{\alpha}}\right\}(s;u)\times V\left\{g(\alpha z)^{\frac{1}{\alpha}}\right\}(s;u)$$

$$=V_\alpha\left\{f(t)\right\}(s;u)\times V_\alpha\left\{g(t)\right\}(s;u).$$

4. Applications

In this section, we study two examples using the conformable fractional Shehu transform to solve linear and nonhomogeneous conformable fractional differential equations.

Example 4.1 *Consider the following linear initial value problem [2]:*

$$D_t^\alpha f(t)=f(t)+1,\ 0<\alpha\leq 1, \tag{20}$$

subject to the initial condition

$$f(0)=0, \tag{21}$$

we applying the conformable fractional Shehu transform of Eq (20), and with the theorem (1), we get :

$$V_\alpha\left[D^\alpha f(t)\right]=V_\alpha(s;u)+V_\alpha(1)(s;u)$$

$$\Rightarrow \frac{s}{u}V_\alpha(s;u)-f(0)=V_\alpha(s;u)+V_\alpha(1)(s;u)$$

$$\Rightarrow V_\alpha(s;u)\left(\frac{s}{u}-1\right)=\frac{u}{s}$$

$$\Leftrightarrow V_\alpha(s;u) = \frac{u^2}{s(s-u)}. \tag{22}$$

By taking V_α^{-1} *of both sides of Eq(22), we get:*

$$f(t) = V_\alpha^{-1}\left[\frac{u^2}{s(s-u)}\right] = V_\alpha^{-1}\left[\frac{u}{(s-u)} - \frac{u}{s}\right] = \exp(\frac{t^\alpha}{\alpha}) - 1.$$

The exact solution, when $\alpha = 1$ *is* $f(t) = \exp(t) - 1$.

Example 4.2 *Consider the following fractional Riccati equation [10] :*

$$D_t^\alpha f(t) = -f^2(t) + 1, \quad t \geq 0, 0 < \alpha \leq 1, \tag{23}$$

with initial condition

$$f(0) = 0, \tag{24}$$

we applying the conformable fractional Shehu transform of Eq (20), and with the theorem (1), we obtain:

$$V_\alpha\left[D^\alpha f(t)\right] = -V_\alpha\left\{f^2(t)\right\}(s;u) + V_\alpha(1)(s;u)$$

$$\frac{s}{u}V_\alpha(s;u) - f(0) = -V_\alpha\left\{f^2(t)\right\}(s;u) + \frac{u}{s}$$

$$\Rightarrow V_\alpha(s;u) = -\frac{u}{s}V_\alpha\left\{f^2(t)\right\}(s;u) + \left(\frac{u}{s}\right)^2. \tag{25}$$

By taking V_α^{-1} *of both sides of Eq(25), we obtain:*

$$f(t) = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{f^2(t)\right\}(s;u)\right] + V_\alpha^{-1}\left[\left(\frac{u}{s}\right)^2\right] = \frac{t^\alpha}{\alpha} - V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{f^2(t)\right\}(s;u)\right]. \tag{26}$$

Suppose that $f(t)$ *and the nonlinear function* $f^2(t)$ *are defined by the following infinite series:*

$$f(t) = y(t) = \sum_{n=0}^{\infty} y_n(t), \qquad f^2(t) = \sum_{n=0}^{\infty} A_n.$$

Where A_n *the Adomian polynomial of* $f^2(t)$ *.*

A few components of Adomian polynomials above are as follows:

$$A_0 = f_0^2$$

$$A_1 = 2f_0 f_1$$

$$A_2 = 2f_0 f_2 + f_1^2$$

$$\vdots$$

Thus, the general solution of Eq(23) is :

$$\begin{cases} y_0 = \dfrac{t^\alpha}{\alpha} \\ y_{n+1}(t) = -V_\alpha^{-1}\left[\dfrac{u}{s}V_\alpha\left\{A_n\right\}(s;u)\right], \quad n \geqslant 0. \end{cases} \tag{27}$$

So, for $n = 0$,we have:

$$y_1(t) = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\{A_0\}(s;u)\right] = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{\frac{t^{2\alpha}}{\alpha^2}\right\}(s;u)\right]$$

$$= -2!V_\alpha^{-1}\left[\left(\frac{u}{s}\right)^3\right] = -\frac{2}{3!}\frac{t^{3\alpha}}{\alpha^3} = -\frac{1}{3}\frac{t^{3\alpha}}{\alpha^3}.$$

and, for $n = 1$,we have:

$$y_2(t) = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\{A_1\}(s;u)\right] = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{-\frac{2}{3}\frac{t^{4\alpha}}{\alpha^4}\right\}(s;u)\right]$$

$$= \frac{2}{3}V_\alpha^{-1}\left[\left(\frac{u}{s}\right)^6 4!\right] = \frac{4!}{5!}\frac{2}{3}\frac{t^{5\alpha}}{\alpha^5} = \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5}.$$

and, for $n = 2$,we have:

$$y_3(t) = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\{A_2\}(s;u)\right] = -V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{\frac{4}{15}\frac{t^{6\alpha}}{\alpha^6}.+\frac{1}{9}\frac{t^{6\alpha}}{\alpha^6}\right\}(s;u)\right]$$

$$= -\frac{17}{45}V_\alpha^{-1}\left[\frac{u}{s}V_\alpha\left\{\frac{t^{6\alpha}}{\alpha^6}\right\}(s;u)\right] = -\frac{17}{45}V_\alpha^{-1}\left[\left(\frac{u}{s}\right)^8 6!\right] = -\frac{17}{45}\frac{6!}{7!}\frac{t^{7\alpha}}{\alpha^7} = -\frac{17}{315}\frac{t^{7\alpha}}{\alpha^7}.$$

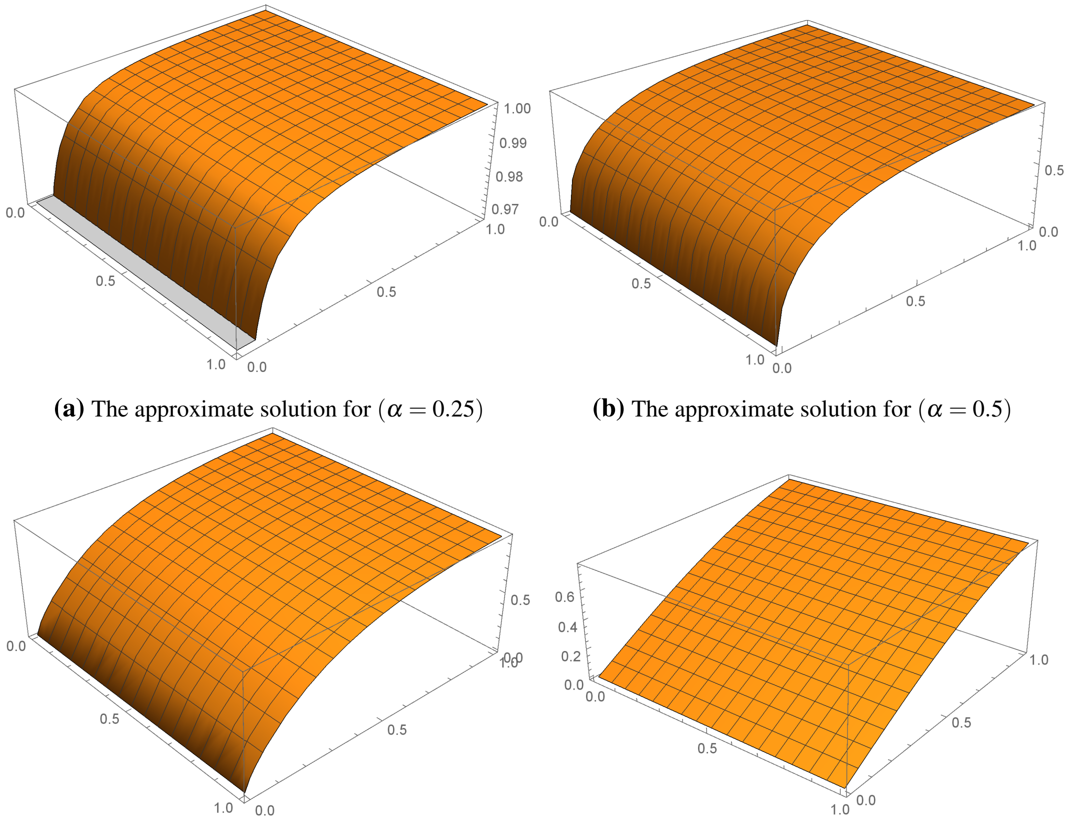
Therefore, the approximate solution of Eq(23) is given as:

$$y(t) = \frac{t^\alpha}{\alpha} - \frac{1}{3}\frac{t^{3\alpha}}{\alpha^3} + \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5} - \frac{17}{315}\frac{t^{7\alpha}}{\alpha^7} + \dots\dots\dots\dots$$

$$= \tanh\left(\frac{t^\alpha}{\alpha}\right) = \frac{\exp\left(2\frac{t^\alpha}{\alpha}\right)-1}{\exp\left(2\frac{t^\alpha}{\alpha}\right)+1}.$$

If we put $\alpha = 1$, the exact solution for Eq(23) is:

$$f(t) = \frac{\exp(2t)-1}{\exp(2t)+1}.$$



(a) The approximate solution for $(\alpha = 0.25)$ **(b)** The approximate solution for $(\alpha = 0.5)$

(c) The approximate solution for $(\alpha = 0.75)$ **(d)** The exact solution for $(\alpha = 1)$

Figure 1: The approximate solution $f(t)$ for different α values.

5. Conclusion

In the present article, we presented some important and interesting results of the (CFSHT) which are playing a central role for solving linear and nonlinear conformable fractional differential equations (CFDEs). Furthermore, we apply this method to linearized conformable fractional-order Riccati equations. Differences between the solutions of the model with integer derivatives and conformable factional derivatives are graphically investigated. We observe, in the various graphs studied, that the different values of the fractional-order of the derivative allow very different behaviors of the solution, especially in the time of convergence to the exact solution. The present study confirms previous findings in case of $a = 1$. Some illustrative examples are given to show the effectiveness of the contributed results. We notice that the analytic expressions of the conformable fractional Shehu transforms are generalizations of those of the conformable fractional Laplace transform ($V_\alpha(s;1) = \mathscr{L}_\alpha(s)$) and conformable fractional Sumudu transform ($V_\alpha(1;u) = u\mathscr{S}_\alpha(u)$).

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Declaration of Competing Interest

The author(s), declares that there is no competing financial interests or personal relationships that influence the work in this paper.

Authorship Contribution Statement

Mohamed Elarbi Benattia: Conceptualization, Methodology, Validation, Formal Analysis, Writing Original Draft

Kacem Belghaba: Software, Investigation, Resources, Visualization, Supervision

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