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Research Article

Global Criterion Approach for the Solution of Multiple Criteria Data Envelopment Analysis Model: An Application at Packaging Waste Collection and Separation Facilities

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ABSTRACT

Adverse effects of packaging waste on the environment and economic losses resulting from the use of untouched resources have made the recycling process compulsory. The success of the collection and separation of packaging waste, which are the most important phases in the recycling process, depends on the effective management of these processes. In this paper, we analyzed the efficiency of 14 collection and separation facilities. A Global- Multiple Criteria Data Envelopment Analysis model (G-MCDEA) based on a global criterion method was proposed for the solution of the multiple criteria data envelopment analysis model, which is organized as a three-objective multiple objective linear programming model. With the proposed model, the three objective functions were transformed into a single objective function, and the normalized grades of the distance from the ideal solution was calculated for each decision making unit. In this was it was determined which objective was closer to the global activity achieved.

Keywords: Data Envelopment Analysis, Multiple Criteria Data Envelopment Analysis, Global Criterion Method, Global- Multiple Criteria Data Envelopment Analysis, Packaging Waste

1. Introduction

Social and economic changes occurring in the recent years have considerable effects on consumption. The high standards of living in the western world and the desire of developing countries to meet these standards have led to an increase in demand for consumer's goods. With the improvement of economic prosperity and the increase of the demand for consumer's goods, the amount of waste generated by people has reached considerable levels. These wastes do not only include the final products we consume but also consist of packing materials used in the packaging of the final products. Additionally, as the increase of international trade and the trend towards urbanization have extended the distance between producers and consumers, the need for appropriate packaging of goods has also grown. As all these developments contribute to the amount of packing waste, such waste is becoming more threatening to the environment day by day.

A package can be defined as "any material used for protection, transfer, transportation, marketing and presentation of products". Packing waste is one of the biggest environmental problems because of large quantities and non-biodegradable materials preferred for packaging. Recycling and reuse of packaging materials are very important for saving nature and energy resources and reducing the waste sources on the earth (Han et al. 2010). The logistics chain created for the recycling of packaging waste is quite complex. Establishing an efficient system requires high initial installation costs (new infrastructure investments for sorted stream collection and separation) and additional transportation costs (Cruz et al. 2012).

In Turkey, the sorted stream collection and separation procedures are undertaken by companies authorized by the relevant ministry and local authorities. The facilities, which are called Collection and Separation Facility (CSF), collect the wastes separated at the point of discard and subject them to a sorting process. The sorted packaging wastes provide input for recycling facilities. In this paper, the data including an output variable and six input variables of CSFs was arranged in the form of Multiple Objective Linear Programming (MOLP) problem based on the model proposed by Li and Reeves (1999) and positive ideal solutions (PIS) were determined for each objective function in line with the definition of ideal solutions. Subsequently, Then, the Global Multiple Criteria Data Envelopment Analysis (G-MCDEA) model was proposed to solve the model proposed by Li and Reeves (1999) according to the Global Criterion Method and a 4-step solution procedure was introduced.

2. Literature Review

It is possible to mention economic benefits of recycling packaging wastes as it reduces the use of virgin raw materials and provides environmental benefits through the reuse of materials which do not dissolve in nature for many years, such as plastic, paper, and metal. Therefore, there are studies in the literature that analyze the economic aspect of recycling packaging waste besides those with environmental approaches. McCarthy (1993), Dewees and Hare (1998), Dixon-Hardy and Curran (2009) and Yıldız-Geyhan et al. (2016) examined the environmental effects of packaging wastes. On the other hand, Metin et al. (2003), Marques et al. (2014), Ramos et al. (2014), Cruz et al. (2014), Rigamonti et al. (2015) and Cimpan et al.

(2016) investigated the packaging wastes regarding their economical aspects. Since countries are aware of the environmental and economic effects of packaging waste, they have guaranteed the management of packaging waste with legislations. In Europe, "green dot labeled production" has become mandatory for many sectors and the responsibility of producers has been expanded. Millock (1994), Matsueda and Nagase (2012), Cruz et al. (2012) and Dace et al. (2014) addressed the packaging waste management legislations in their studies.

The foundations of the hybrid model proposed for the analysis of the study data and of the Multiple Criteria Data Envelopment Analysis (MCDEA), the first phase of the solution procedure, were laid with the classic Data Envelopment Analysis. Charnes et al. (1978) further extended Farrell's (1957) theoretical work on technical efficiency and developed a linear programming based approach, which was termed as Data Envelopment Analysis (DEA). In the literature, the Charnes, Cooper and Rhodes (CCR) model is used to examine input-oriented efficiency or output-oriented efficiency. Charnes et al. (1982) proposed a model in which the data is transformed using a logarithmic structure in the multiplicative model. Banker et al. (1984) developed the Banker, Charnes and Cooper (BCC) model employed to analyze input-oriented efficiency. In another variation of the model introduced by Charnes et al. (1985), slack variables were added to the objective function.

MCDEA was first introduced by Li and Reeves (1999). Zhao et al. (2006) applied that model to assess the environmental impact for a dam design. Moreover, San Cristobal (2011) used the MCDEA model to analyze thirteen renewable energy technologies. Moheb-Alizadeh et al. (2011) suggested the use of the MCDEA model for the solution of the positioning and assignment problems in a fuzzy environment. Yadav et al. (2012) used the MCDEA model to measure the regional effectiveness of coal-fired thermal power plants. Rubem and Brandao (2015) assessed the performance of national teams competing in UEFA EURO 2012 with the MCDEA model. Moreover, Verma et al. (2016) used hierarchical genetic algorithms and the MCDEA model to plan the distribution network layout in a new industrial area.

The method of global criterion was first unveiled by Yu (1973) and Zeleny (1973) and then further extended and put into its current form referred as the Global Criterion Method by Hwang and Masud (1979). In the following years, Shih and Chang (1995), Mahapatra (2009), Costa and Pereira (2010), De Freitas Gomes et al. (2012), Saraj and Safaei (2012) and Umarusman and Türkmen (2013) also carried out theoretical studies on the global criterion method.

Although the use of DEA methodology is limited in literature regarding packaging wastes, De Jaeger and Rogge (2014) investigated the income-expenditure efficiency using household packaging waste collection costs. Marques et al. (2012) employed DEA to determine the effectiveness of recycling facilities for packaging wastes in Portugal. In our literature review, we did not come across any study applying Global Criterion Method and/ or MCDEA for investigating packaging wastes.

3. Theoretical Framework

'Packaging' expresses all products made of any materials of any nature to be used for the containment, protection, handling, delivery and presentation of goods, from raw materials to processed goods, from the producer to the user or the consumer (European Parliament and Council Directive on Packaging and Packaging waste 94/62/EC 2004). Packing waste is defined as the waste of sales packaging, secondary packaging and transport packaging which are used for the presentation and delivery of the products or any material to the consumer or to the end user or the waste which is thrown or discharged into the environment after the use of the product. The definition includes reusable packaging with expired lifetime but excludes production residues (Regulation on Control of Packaging Waste in Turkey 2011: 4/1a). As it is mentioned in the definition, there are three types of packaging (Dixon-Hardy and Curran 2009):

- Sales packaging (primary packaging) is a sales unit to the final user or consumer at the point of purchase like a container for the product or a type of material wrapped around the product.
- Grouped packaging (secondary packaging) is the general term for large containers or boxes in which the product with primary packaging are placed with the aim of delivery or presentation.
- Transport packaging or (tertiary packaging), is defined as the packaging used to facilitate handling and transport of a number of sales units or grouped packagings in order to prevent physical handling and transport damage. Transport packaging does not include road, rail, ship and air containers.

As a waste management option, packaging waste recycling has valuable benefits over final disposal including (Nahman 2010);

- Savings in natural resources and energy
- Reduction of production costs as a result of the use of recyclable materials instead of raw materials
- Decrease of the cost resulting from waste management
- Reduction in the environmental effects of the wastes
- Reduction of costs associated with waste disposal and other storage practices
- Income and employment opportunities for the poor and the unemployed

In Turkey, the facilities that are authorized to collect packaging wastes and carry out the waste sorting procedure to provide input for recycling premises are named as CSF. In this paper, we examined the income and expense items of the CSFs and measured the efficiency of the facilities using the Global- Multiple Criteria Data Envelopment Analysis. The MCDEA model is an extension of the classical DEA model. The DEA model developed by Charnes et al. (1978) is in the form of a linear programming problem defined to identify the efficiency of each decision making unit (DMU).

$$\text{Max } h_0 = \sum_{r=1}^{s} u_r y_{rj0}$$

Subject to (1)

$$\sum_{i=1}^{m} v_i x_{ij0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0$$

$$j = 1, \ldots\ldots, n$$

$$u_r, v_i \geq 0$$

Where;

j *is the number of DMUs*

r *is the number of outputs*

i *is the number of inputs*

y_{rj} *is the* $r.$ *output value for* $j.DMU$

x_{ij} *is the* $i.$ *input value for* $j.DMU$

u_r *is the weights to be determined for output* r

v_i *is the weights to be determined for input* i

h_0 *is the relative efficiency of DMU*

Only when $h_0 = 1$, the DMU can be concluded to be efficient. Even though efficiency is a measurement unit for the classical DEA, the MCDEA model of Li and Reeves (1999) was built upon inefficiencies. d_0, which is limited to the [0, 1] range, can be regarded as a measure of "ineffectiveness" and is defined as $h_0 = 1 - d_0$. In other words, the smaller the d_0 value is, the less inefficient the DMU_0 is (i.e., more efficient). The method suggested by Li and Reeves (1999) consists of three independent objective functions including minimizing d_0, minimizing the maximum deviation and minimizing the sum of the deviations. This is stated mathematically as follows:

$$Min\ d_0\ (or \max h_0 = \sum_{r=1}^{s} u_r y_{rj0})$$

$$Min\ M$$

$$Min \sum_{j=1}^{n} d_j$$

Subject to (2)

$$\sum_{i=1}^{m} v_i x_{ij0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0$$

$$M - d_j \geq 0 \qquad j = 1, \ldots\ldots, n$$

$$u_r, v_i, d_j \geq 0$$

The solution procedure of the MCDEA model, which is used as a tool for the development of discrimination power of the classical DEA model, is an interactive approach that solves three different objective functions. The first objective function (or criterion) contains a classic DEA solution within a set of MCDEA solutions. The other two objectives, Minimax and Minsum objectives provide a more restrictive or lax efficiency solutions, respectively. This implies that a wider solution is possible with MCDEA, so as to gain more reasonable input and output weights (Ghasemi et al. 2014).

The MCDEA model is a MOLP problem, in which it is impossible to find a solution that optimizes all objective simultaneously. For this reason, the task of a MOLP solution process is not to find an optimal solution, but instead to find non-dominated solutions and to help select a most preferred one (San Cristobal 2011).

The goal of the global criterion method, which is the principal method of classification that does not require preference information, is to minimize the relative deviation of the objective functions from the feasible ideal points (Hwang and Masud 1979). All objective functions are considered to be equally important (Miettinen 1999). The global criterion method converts multi objective functions into a single-objective optimization problem. Mathematically, it can be written as (Hwang and Masud 1979):

$$\min \sum_{k=1}^{l} \left[\frac{Z_k\ (x^*) - Z_k(x)}{Z_k\ (x^*)} \right]^p \tag{3}$$

$Z_k(x)$: $Maximization - oriented\ k.objective\ function$

$Z_k(x^*)$: $PIS\ for\ k.objective\ function$

The Global Criterion Method is derived from the ratio of the difference of each objective function from its positive ideal solution to the positive ideal solution. PIS is obtained from the solution of each objective function and show the best performance of the respective objective. The best solution determined for the problem varies depending on the preferred p- value. Setting $p = 1$ as suggested by Boychuk and Ovchinnikov (1973) implies that equal importance is given to all deviations. Additionally, when $p = 1$, the global objective function becomes linear. Hwang and Masud (1979) proposed the global formulation for maximization objectives.

On the other hand, the MOLP problems do not consist of only maximization objectives. While Bashiri and Tabrizi (2010) proposed minimization objective-weighted global objective function, Umarusman and Türkmen (2013) suggested equally important global objectives function for minimization objectives. In this paper, objective functions were formulated considering that they are equally important.

$$\text{Min} \sum_{s=1}^{r} \left[\frac{W_s(x) - W_s\ (x^*)}{W_s\ (x^*)} \right]^p \tag{4}$$

$W_s(x)$: $Minimization - oriented\ k - objective\ function$

$W_s(x^*)$: $PIS\ for\ k.objective\ function$

Eq. (3) and eq. (4) show the degree of the distance of each maximization and minimization objective function from the positive ideal solution, respectively. Furthermore, for multiple objective functions with different but equally important orientations (max. and min.), the Global model can be generalized as follows (Umarusman and Türkmen 2013):

$$Min\ G = \left(\sum_{k=1}^{l} \left[\frac{Z_k\ (x^*) - Z_k(x)}{Z_k\ (x^*)} \right]^p + \sum_{s=1}^{r} \left[\frac{W_s(x) - W_s\ (x^*)}{W_s\ (x^*)} \right]^p \right) \quad (5)$$

Subject to

$A_i(x) \leq b_i$

$x \geq 0$

Eq. (2) consists of three different objective functions. Therefore, the proposed algorithm was arranged according to $Max\ h_0$, $Min\ M$ and $Min\ \sum d_j$. The reason, why an arrangement was made on the algorithm in line with the $Min\ d_0$ objective, was that the denominator is equal to 0 in the eq. (4) notation used to form the global objective function. The denominator is equal to 0 as according to the $h_0 = 1 - d_0$ equation the positive ideal solution of efficient DMU objective functions equals to 0. Miettinen (1999) argued that objectives with a PIS equal to 0 cannot be involved in global objective as a part of global objective function.

The G- MCDEA objective function proposed in this paper is constituted as follows:

$$\text{Min} \left[\frac{h_{0j}^* - h_{0j}}{h_{0j}^*} + \frac{M_j - M_j^*}{M_j^*} + \frac{\sum_{j=1}^{n} d_j - d_j^*}{d_j^*} \right]^p \quad (6)$$

OR the model can be modified as follows through the simplification of the solution:

$$\text{Min} \left[\left(\frac{M_j}{M_j^*} + \frac{\sum_{j=1}^{n} d_j}{d_j^*} - \frac{h_{0j}}{h_{0j}^*} \right) - 1 \right]^p$$

Subject to (7)

$$\sum_{i=1}^{m} v_i x_{ij0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0$$

$M - d_j \geq 0 \qquad j = 1, \ldots\ldots, n$

$u_r, v_i, d_j \geq 0$

Where;

$h_{0j} = j.DMU's\ efficiency\ level$

$h_{0j}^* =\ the\ positive\ ideal\ solution\ value\ of\ h'_{0j}$

$M_j = j.the\ greatest\ deviation\ in\ the\ solution\ for\ DMU$

$M_j^* =\ the\ ideal\ solution\ value\ of\ M_j$

$$\sum_{j=1}^{n} d_j = j.total\ deviations\ in\ the\ solution\ for\ DMU$$

$$d_j^* = \ the\ ideal\ solution\ value\ of \sum_{j=1}^{n} d_j$$

The constraints of the proposed hybrid model are the same as the constraints of the model in eq (2). For achieving a minimum objective function value in eq (7), the result of $\left(\frac{M_j}{M_j^*} + \frac{\sum_{j=1}^{n} d_j}{d_j^*} - \frac{h_{0j}}{h_{0j}^*}\right)$ should be minimum. Therefore, the eq (7) model is arranged as follows:

$$\text{Min}\left[\left(\frac{M_j}{M_j^*} + \frac{\sum_{j=1}^{n} d_j}{d_j^*} - \frac{h_{0j}}{h_{0j}^*}\right)\right]^p$$

Subject to (8)

$$\sum_{i=1}^{m} v_i x_{ij0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0$$

$$M - d_j \geq 0 \qquad j = 1, \ldots\ldots, n$$

$$u_r, v_i, d_j \geq 0$$

In the light of all aforementioned explanations regarding the MCDEA and Global criterion methods, the procedure steps for the G- MCDEA model can be listed as follows:

- **Step 1:** Identification of the inputs and outputs for the MCDEA model and creation of the model,
- **Step 2:** Determination of PIS and efficiency value of each objective through eq. (2),
- **Step 3:** Arrangement of the problem in line with eq. (6) employing the PIS identified in the 2nd step; and solution of the problem$(1 \leq p < \infty)$,
- **Step 4:** Comparison of the efficiency value obtained with the G- MCDEA model and the efficiency value resulted from the MCDEA model.

4. Hybrid Model Analysis for CSFs

4.1. Sample

Research data was collected in 2016. In 2016 521 CSFs operate in Turkey. These CSFs are distributed in an irregular manner to cities in Turkey. For instance, while there were seven CSFs in Konya, there were 14 CSFs in a smaller city Eskişehir. In addition, the sizes of these CSFs in different cities are irregular. In other words, CSFs located in some cities are all big facilities, while TATs located in some cities are all small facilities. Therefore, in this study, Kayseri province, which has a sufficient number of facilities compared to its size and has a homogeneous structure in terms of having

TAT of any size, was chosen as a sample. Fourteen out of sixteen CFSs located in Kayseri province volunteered to participate in the study. The remaining facilities either did not continue to collect or separate packaging waste, or did not volunteer to participate in the research.

4.2. Data

The study data were generated in accordance with the information received from the facilities. Input data includes management and material costs, collection cost, separation cost, infrastructure cost, and the costs related to location, and machine and equipment costs. The total revenue is the only factor considered as output data in the study. In order to reveal current efficiency value, the data from 2015 were processed in the study.

4.3. Solution of the Model

The solution steps can be listed as follows for the proposed hybrid model:

Step 1: Identification of the inputs and outputs for the MCDEA model and creation of the model

v_1: *The cost of managemet and material* $(*\ 10^3\$)$

v_2: *The cost of collection* $(*\ 10^3\$)$

v_3: *The cost of separation* $(*\ 10^3\$)$

v_4: *The cost of infrastructure* $(*\ 10^3\$)$

v_5: *The cost related to location* $(*\ 10^3\$)$

v_6: *Machine and equipment cost* $(*\ 10^3\$)$

u: *Total revenue* $(*\ 10^3\$)$

DMU	Inputs						Output
	v_1	v_2	v_3	v_4	v_5	v_6	u
DMU_1	34.83214	47.52897	25.55774	3.40363	0.96298	6.53725	310.35432
DMU_2	215.22485	399.74810	102.23097	5.47431	2.93479	11.98496	931.06295
DMU_3	34.83214	99.63621	25.55774	9.48879	9.24395	11.98496	232.76574
DMU_4	111.50539	47.52897	68.15399	3.11712	0.25679	6.53725	77.58858
DMU_5	43.35140	47.52897	34.07699	3.56045	1.28397	6.53725	142.76298
DMU_6	17.79365	47.52897	17.03850	3.24680	0.64199	6.53725	45.46691
DMU_7	17.79365	85.90138	25.55774	5.11870	2.55620	6.53725	155.17716
DMU_8	34.83214	85.90138	85.19248	6.22708	2.56794	11.98496	465.53147
DMU_9	26.31290	169.51362	8.51925	3.46887	0.62658	6.53725	387.94289
DMU_{10}	34.83214	95.05793	59.63473	3.68592	1.54077	6.53725	130.34881
DMU_{11}	26.31290	85.90138	25.55774	3.36599	2.78438	6.53725	155.17716
DMU_{12}	456.17169	332.15982	255.57744	5.59977	0.14674	11.98496	853.47437
DMU_{13}	26.31290	122.52759	8.51925	3.37226	0.05136	6.53725	465.53147
DMU_{14}	43.35140	95.05793	76.67322	3.87411	1.92596	6.53725	232.76574

Table 1. CSFs' revenue and cost

The MCDEA model created according to eq. (2) using the input and output data in Table 1 is given in Appendix-A. The model was solved with the LINDO w32 software, which can solve linear programming-based problems.

Step 2: *Determination of PIS and efficiency value of each objective through eq (2),* As a result of the solution for the h_0 objective, which maximizes the efficiency value of each DMU, the $DMU_1, DMU_2, DMU_8, DMU_{12}$ and DMU_{13} were found to be efficient. When the efficiency value in the classical DEA model was compared with the h_0 MCDEA model, the $1-d_0$ equivalent result was obtained. That is to say that interpretation can be also made according to the $d_0$classical DEA model, which minimizes the deviation from the efficiency value. Table 3 shows the solutions for the M objective that aims to minimize the maximum deviation. According to the solution of this objective, only the DMU_1 was found to be efficient. Table 4 presents the solutions for the $\sum d_j$ objective, which targets at minimizing the total deviation. The solution of this objective indicated only the DMU_{13} to be efficient.

DMU	Inputs						Output	Efficiency
	v_1	v_2	v_3	v_4	v_5	v_6	u	
DMU_1	0.000912	0.006949	0	0	0	0.097588	0.00322	1
DMU_2	0	0.000476	0	0	0	0.067565	0.00107	1
DMU_3	0.009961	0.006092	0.001803	0	0	0	0.00219	0.511932
DMU_4	0	0.017552	0	0	0.645514	0	0.00469	0.363963
DMU_5	0	0.007497	0	0	0	0.098461	0.00322	0.460000
DMU_6	0.020022	0.012245	0.003625	0	0	0	0.00442	0.201001
DMU_7	0.056200	0	0	0	0	0	0.00317	0.492926
DMU_8	0.007410	0.005305	0	0.045955	0	0	0.00214	1
DMU_9	0.000998	0	0	0	0	0.148952	0.00214	0.833333
DMU_{10}	0	0.005063	0	0	0	0.079354	0.00244	0.318939
DMU_{11}	0.009200	0.006587	0	0.057059	0	0	0.00266	0.413875
DMU_{12}	0	0.000722	0	0.135183	0.021181	0	0.00117	1
DMU_{13}	0	0.004685	0.001060	0	0	0.063770	0.00214	1
DMU_{14}	0	0.005063	0	0	0	0.079354	0.00244	0.569534

Table 2. $Max\ h_0$ MCDEA results

DMU	Inputs						Output	Efficiency
	v_1	v_2	v_3	v_4	v_5	v_6	u	
DMU_1	0	0.007028	0.001590	0	0	0.095655	0.00322	1
DMU_2	0	0.001718	0.000363	0	0	0.023036	0.00077	0.724366
DMU_3	0	0.003724	0.000843	0	0	0.050683	0.00170	0.397331
DMU_4	0.002493	0	0	0	0	0.110454	0.00169	0.131279
DMU_5	0	0.006934	0.001569	0	0	0.094376	0.00317	0.453843
DMU_6	0	0.007125	0.001612	0	0	0.096968	0.00326	0.148494
DMU_7	0	0.005588	0.001182	0	0	0.074924	0.00253	0.392753
DMU_8	0	0.003727	0.000843	0	0	0.050728	0.00170	0.795593
DMU_9	0	0.003916	0	0	0	0.051428	0.00168	0.652907
DMU_{10}	0	0.005063	0.001146	0	0	0.068904	0.00232	0.302539
DMU_{11}	0	0.005588	0.001182	0	0	0.074924	0.00253	0.392753
DMU_{12}	0	0.001808	0.000409	0	0	0.024607	0.00082	0.707530
DMU_{13}	0	0.004799	0	0	0	0.063024	0.00206	0.959925
DMU_{14}	0	0.004966	0.001124	0	0	0.067585	0.00227	0.530007

Table 3. Minimax (M) MCDEA results

DMU	Inputs						Output	Efficiency
	v_1	v_2	v_3	v_4	v_5	v_6	u	
DMU_1	0	0.001025	0	0	0	0.145518	0.00231	0.717849
DMU_2	0	0.001702	0	0	0	0.026675	0.00082	0.766264
DMU_3	0	0	0	0	0.108179	0	0.00001	0.002793
DMU_4	0.000920	0	0	0	0	0.137279	0.00198	0.153625
DMU_5	0	0.001025	0	0	0	0.145518	0.00231	0.330210
DMU_6	0	0.001025	0	0	0	0.145518	0.00231	0.105164
DMU_7	0	0.001410	0	0.171702	0	0	0.00161	0.250611
DMU_8	0	0.000559	0	0	0	0.079428	0.00126	0.587966
DMU_9	0	0.003677	0	0	0	0.057631	0.00177	0.689374
DMU_{10}	0	0.005063	0	0	0	0.079354	0.00244	0.318963
DMU_{11}	0	0.005309	0	0	0	0.083212	0.00256	0.398184

DMU12	0.001875	0.000436	0	0	0	0	0.00022	0.188617
DMU13	0	0.004445	0	0	0	0.069666	0.00214	1
DMU14	0	0.005063	0	0	0	0.079354	0.00244	0.569577

Table 4. Minsum (Σdj) MCDEA results

The efficiency results obtained from the solutions of the classical DEA h_0 or d_0 were optimistic as compared to the efficiency values calculated through the minimax and minsum objectives. Table 5 demonstrates the PIS values to be used for the arrangement of the G- MCDEA objective function according to Eq. (9) in the 3rd step of the model.

	PIS		
	$Max\ h_0$	$Min\ M$	$Min \sum d_j$
DMU1	1	1.137312	7.686164
DMU2	1	0.275396	2.248449
DMU3	0.511932	0.602609	2.922826
DMU4	0.363963	1.016785	7.509866
DMU5	0.460000	1.122109	7.686164
DMU6	0.201001	1.152932	7.686164
DMU7	0.492926	0.895721	5.895686
DMU8	1	0.603150	4.195349
DMU9	0.833333	0.614792	4.857690
DMU10	0.318939	0.819253	6.688752
DMU11	0.413875	0.895721	7.013888
DMU12	1	0.292570	1.861958
DMU13	1	0.753416	5.872125
DMU14	0.569534	0.803568	6.688752

Table 5. PIS for MCDEA model

Step 3: 2. Arrangement of the problem in line with (6) employing the PIS identified in the 2nd step; and solution of the problem: Global objective function was arranged considering $p = 1$.

The aim of resolving each objective in the MCDEA model in the G-MCDEA model was to reach three different efficiency values for three different objectives. The purpose of the Global Criteria Method is to convert the objective function into a single-objective optimization problem, in other words, to find the compromise result. When the global criterion objective function is constructed, the objective function results of the MCDEA model objectives are used. For instance, the global objective function for the DMU_1 is as follows:

$$Min \frac{1-h_1}{1} + \frac{m-1.137312}{1.137312} + \frac{\sum d_j - 7.686164}{7.686164} \tag{9}$$

The model is as follows after simplification:

$$Min\ (0.8792661996M + 0.1301039114d_1 + 0.1301039114d_2 + 0.1301039114d_3 + 0.1301039114d_4 + 0.1301039114d_5 + 0.1301039114d_6 + 0.1301039114d_7 + 0.1301039114d_8 + 0.1301039114d_9 + 0.1301039114d_{10} + 0.1301039114d_{11} + 0.1301039114d_{12} + 0.1301039114d_{13} + 0.1301039114d_{14} - 310.35432u_1) - 1$$

The G- MCDEA model converted into a single objective was constructed separately for each DMU. The constraints of the G- MCDEA model are the same with the constraints of the MCDEA model. Table 6 indicates the input-output, solution and efficiency values reached after the solution. Deviation variables reached the global solution result, performed for DMU_1 , are found as $M = 1.166667,\ d_1 = 0,\ d_2 = 0.91745,\ d_3 =$

1.166667, $d_4 = 0.75$, $d_5 = 0.54$, $d_6 = 0.8535$, $d_7 = 0.755821$, $d_8 = 0.3251$, $d_9 = 0.563246$, $d_{10} = 0.896866$, $d_{11} = 0.755821$, $d_{12} = 0.716853$, $d_{13} = 0$ and $d_{14} = 0.566866$.

	Inputs						Output	G-MCDEA	
	v_1	v_2	v_3	v_4	v_5	v_6	u	Solution	Efficiency
DMU_1	0	0.006667	0	0	0	0.104499	0.003222	0.171790	1
DMU_2	0.000245	0.001652	0	0	0	0.023949	0.000785	0.299055	0.73088
DMU_3	0	0.003478	0	0	0	0.054521	0.001681	0.818040	0.39127
DMU_4	0.000933	0.006296	0	0	0	0.091285	0.002992	0.569044	0.23214
DMU_5	0	0.006667	0	0	0	0.104499	0.003222	0.185688	0.46000
DMU_6	0	0.006667	0	0	0	0.104499	0.003222	0.429040	0.14649
DMU_7	0	0.005309	0	0	0	0.083212	0.002566	0.419108	0.39818
DMU_8	0	0.003653	0	0	0	0.057256	0.001765	0.388309	0.82166
DMU_9	0	0.003677	0	0	0	0.057631	0.001777	0.219308	0.68937
DMU_{10}	0	0.005063	0	0	0	0.079354	0.002447	0.081402	0.31896
DMU_{11}	0	0.005309	0	0	0	0.083212	0.002566	0.075168	0.39818
DMU_{12}	0.000258	0.001744	0	0	0	0.025281	0.000829	0.597181	0.70753
DMU_{13}	0	0.004445	0	0	0	0.069666	0.002148	0.032335	1
DMU_{14}	0	0.004966	0.001124	0	0	0.067585	0.002277	0.098756	0.53000

Table 6. G- MCDEA solution and efficiency values

Since the efficiency values are output-oriented, the result of the G-MCDEA model for each DMU is multiplied by the monetary expression of the variable values to obtain the efficiency result. The efficiency results are indicated in Table 6.

The variable values of the solutions reached for the G- MCDEA model objectives were placed in each DMU objective function to reach the non-dominated results. The non-dominated solutions were applied to calculate the normalized distance of the each DMU_1 and DMU_2 objective function from the PIS and the results are shown in Table 7.

DMU	Normalized degree of distance from the PIS	MCDEA Efficiency	G- MCDEA Efficiency
DMU_1	$Max\ h_1 = \frac{1-1}{1} = 0$	1	1
	$Min\ M = \frac{1.16667 - 1.137312}{1.13712} = 0.025813$	1	
	$Min \sum d = \frac{8.80819 - 7.686164}{7.686164} = 0.145980$	0.717850	
DMU_2	$Max\ h_2 = \frac{1-0.73084}{1} = 0.269116$	1	0.730884
	$Min\ M = \frac{0.277438 - 0.275396}{0.275396} = 0.007415$	0.724367	
	$Min \sum d = \frac{2.298872 - 2.248449}{2.248449} = 0.022426$	0.766265	
DMU_3	$Max\ h_3 = 0.235681$	0.511932	0.391279
	$Min\ M = 0.010101$	0.397331	
	$Min \sum d = 0.572305$	0.002793	
DMU_4	$Max\ h_4 = 0.362174$	0.363963	0.232145
	$Min\ M = 0.040022$	0.131280	
	$Min \sum d = 0.166779$	0.153625	
DMU_5	$Max\ h_5 = 0$	0.460000	0.460000
	$Min\ M = 0.039709$	0.453844	
	$Min \sum d = 0.145979$	0.330211	
DMU_6	$Max\ h_6 = 0.271177$	0.201001	0.146494
	$Min\ M = 0.011915$	0.148495	
	$Min \sum d = 0.145979$	0.105165	
DMU_7	$Max\ h_7 = 0.192201$	0.492926	0.398185

DMU	Normalized degree of distance from the PIS	MCDEA Efficiency	G- MCDEA Efficiency
	$Min\ M = 0.037161$	0.392753	
	$Min \sum d = 0.189664$	0.250611	
DMU₈	$Max\ h_8 = 0.178337$	1	0.821663
	$Min\ M = 0.059827$	0.795593	
	$Min \sum d = 0.150355$	0.587966	
DMU_9	$Max\ h_9 = 0.172749$	0.833333	0.689375
	$Min\ M = 0.046553$	0.652908	
	$Min \sum d = 0$	0.689375	
DMU_{10}	$Max\ h_{10} = 0$	0.318939	0.318964
	$Min\ M = 0.081402$	0.302540	
	$Min \sum d = 0$	0.318964	
DMU_{11}	$Max\ h_{11} = 0.037909$	0.413875	0.398185
	$Min\ M = 0.037161$	0.392753	
	$Min \sum d = 0$	0.398185	
DMU_{12}	$Max\ h_{12} = 0.29247$	1	0.707530
	$Min\ M = 0.001008$	0.707530	
	$Min \sum d = 0.303308$	0.188618	
DMU_{13}	$Max\ h_{13} = 0$	1	1
	$Min\ M = 0.032335$	0.959925	
	$Min \sum d = 0$	1	
DMU_{14}	$Max\ h_{14} = 0.069400$	0.569534	0.530008
	$Min\ M = 0$	0.530008	
	$Min \sum d = 0.029188$	0.569578	

Table 7. Normalized grades of distance from the PIS

The normalized degree of distance from the PIS is between 0 and 1. As this value approximates 0, the G-MCDEA efficiency result becomes closer to the efficiency value found as a result of the PIS. If the result is 0, then the solution occurs over the ideal solution of the objective function. If none of the results are equal to 0, the solution occurs at the point, the closest to 0. As a result of the calculations performed for DMU_1, the normalized degree of the $Max\ h_1$ objective was found to be 0. It implies that the efficiency of the G- MCDEA was over the efficiency value obtained in the solution of the $Max\ h_1$ objective. As none of the normalized degrees determined for DMU_2 was equal to 0, the solution occurred at the point which was the closest to zero as well as to the $Min\ M$ objective.

Step 4: Comparison of the efficiency value obtained with the G- MCDEA model and the efficiency value resulted from the MCDEA model.

When the G-MCDEA model efficiencies were examined considering the normalized distances from the PIS, it was concluded that the solution of the objective function occurred at the closest distance to 0. This is because the global model has the property of selecting "the best" among the multi objective functions. Considering the normalized distance from the PIS, the global efficiency value of DMU_1 and DMU_5, $Max\ h_0$ objectives; $DMU_2, DMU_3, DMU_4, DMU_6, DMU_7, DMU_8, DMU_{12}$ and DMU_{14} $Min\ M$ objectives and DMU_9 and DMU_{11} occurred over or close to the $Min \sum d_j$ objective. The DMU_{10} and DMU_{13} global efficiency value, nevertheless, occurred over or near both $Max\ h_0$ and $Min \sum d_j$ objective efficiency values.

5. Conclusion

Waste recovering has become one of the most studied topics in recent years due to both preventing environmental pollution and reducing the use of virgin raw materials. In this study has investigated the efficiency of CSF which the first stage of waste recovering in Turkey. In addition, in this study, a model which is thought to be an alternative to MCDEA in terms of not requiring subjective interpretation and conventional DEA in terms of the power of discrimination is proposed. In this regard, the study has produced original results.

As a result of the study, two of the fourteen CSFs included in the solution came to the fore as efficient CSFs. These CSFs, called DMU1 and DMU13, earned more income with less cost than other CSFs. Therefore, inefficient CSFs must either control their costs or find ways to increase their income in order to be efficient.

As it can be seen in the MCDEA model solution, minimax and minsum models gave less efficient DMU results. However, the DMUs found to be efficient in the minimax and minsum models were certainly efficient in DEA (in d_0 for our model). Li and Reeves (1999) argued that, in evaluation of DMUs, the minimax and minsum criteria do not give as feasible results as in the classical DEA methodology. For this reason, the efficiencies defined within the scope of minimax and minsum criteria yield stricter results than those in the classical DEA: Achieving DMU efficiency on minimax or minsum criteria is more difficult than in the conventional DEA. If a DMU is efficient according to the minimax or minsum models, it must be certainly efficient. However, minimax or minsum may not be efficient if the classical DEA is efficient. On this basis, wit can be concluded that minimax or minsum criteria usually give less efficient DMU. By incorporating these new criteria into the classic DEA model, the discriminative power of the model can be enhanced.

Although the discrimination power of the MCDEA model was improved, it is almost impossible to mention an optimum solution because of the three different efficiency values found in the solution. The G-MCDEA model, which was introduced in this phase, will lead the researcher to find an optimum solution by demeaning the MCDEA problem into a single objective.

The proposed G-MCDEA model successfully solved the problem of investigating the efficiencies of CSFs. However, it should not be forgotten that efficiency investigations using different input or output combinations will produce different efficiency results. In future studies, the proposed G-MCDEA model can be used to investigate the efficiencies of different DMUs. In addition, steps could be taken to develop the G-MCDEA model in future studies. First, emphasis will be placed on the possibility of the global solution to be an alternative to the "super efficiency model".

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Appendix

Appendix A. The MCDEA model (for DMU_1)

Max $310.35432u$
Min M
Min $d_1+d_2+d_3+d_4+d_5+d_6+d_7+d_8+d_9+d_{10}+d_{11}+d_{12}+d_{13}+d_{14}$

Subject to

$34.83214 v_1+47.52897v_2+25.55774v_3+3.40363v_4+0.96298v_5+6.53725v_6= 1$
$310.35432u-34.83214v_1-47.52897v_2-25.55774v_3-3.40363v_4-0.96298v_5-6.53725v_6+d_1= 0$
$931.06295u-215.22485v_1-399.74810v_2-102.23097v_3-5.47431v_4-2.93479v_5-11.98496v_6+d_2= 0$
$232.76574u-34.83214v_1-99.63621v_2-25.55774v_3-9.48879v_4-9.24395v_5-11.98496v_6+d_3= 0$
$77.58858u-111.50539v_1-47.52897v_2-68.15399v_3-3.11712v_4-0.25679v_5-6.53725v_6+d_4= 0$
$142.76298u-43.35140v_1-47.52897v_2-34.07699v_3-3.56045v_4-1.28397v_5-6.53725v_6+d_5= 0$
$45.46691u-17.79365v_1-47.52897v_2-17.03850v_3-3.24680v_4-0.64199v_5-6.53725v_6+d_6= 0$
$155.17716u-17.79365v_1-85.90138v_2-25.55774v_3-5.11870v_4-2.55620v_5-6.53725v_6+d_7= 0$
$465.53147u-34.83214v_1-85.90138v_2-85.19248v_3-6.22708v_4-2.56794v_5-11.98496v_6+d_8= 0$
$387.94289u-26.31290v_1-169.51362v_2-8.51925v_3-3.46887v_4-0.62658v_5-6.53725v_6+d_9= 0$
$130.34881u-34.83214v_1-95.05793v_2-59.63473v_3-3.68592v_4-1.54077v_5-6.53725v_6+d_{10}= 0$
$155.17716u-26.31290v_1-85.90138v_2-25.55774v_3-3.36599v_4-2.78438v_5-6.53725v_6+d_{11}= 0$
$853.47437u-456.17169v_1-332.15982v_2-255.57744v_3-5.59977v_4-0.14674v_5-11.98496v_6+d_{12}= 0$
$465.53147u-26.31290v_1-122.52759v_2-8.51925v_3-3.37226v_4-0.05136v_5-6.53725v_6+d_{13}= 0$
$232.76574u-43.35140v_1-95.05793v_2-76.67322v_3-3.87411v_4-1.92596v_5-6.53725v_6+d_{14}= 0$
$M-d_1 \geq 0$
$M-d_2 \geq 0$
$M-d_3 \geq 0$
$M-d_4 \geq 0$
$M-d_5 \geq 0$
$M-d_6 \geq 0$
$M-d_7 \geq 0$
$M-d_8 \geq 0$
$M-d_9 \geq 0$
$M-d_{10} \geq 0$
$M-d_{11} \geq 0$
$M-d_{12} \geq 0$
$M-d_{13} \geq 0$
$M-d_{14} \geq 0$
all variables ≥ 0