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PID Controller Design for a Fractional Order System using Bode's Ideal Transfer Function

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Abstract

In this paper, an optimization method is proposed to design a fractional order system with PID controller by taking Bode's ideal transfer function as reference model. In the study, PID controllers are preferred because of their simplicity, reliability and robustness. PID controller parameters can be obtained by optimization method. In order to obtain the desired time response, it is sufficient to set two parameters in the Bode's ideal transfer function. The Bode's ideal transfer function was considered as the reference model and compared with the generated model. The error in the output signal is minimized by the integral performance criteria, and the PID controller parameters are optimized. Integral performance criteria are frequently used in evaluating the performance of control systems. In Simulink model, Matsuda's 4th order integer approximation model is used for fractional order control system. Finally, the success of the optimization method is seen in the given numerical example.

Key Words

"Controller design, Optimization, Fractional order control systems, PID Controller"

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1. INTRODUCTION

Fractional calculus is a branch of mathematics and it is possible to see many applications in the different areas. Control engineering is one of these application areas. In the control system design, the system to be controlled may be fractional order, or the controller may be fractional order (Monje et al.,2010). In this study, a fractional order system is controlled by an integer order controller. Fractional order systems can better model physical systems than integer order systems. Due to this advantage, interest in fractional order systems is increasing in today's world. There are many approaches model that are used to obtain integer order models of fractional order systems. The most well-known approach models are Matsuda, Oustaloup, Carlson, Krishna, Chareff approaches method [2-6]. In the study, Matsuda's 4th-order integer approximation model was used while modeling a fractional order system.

Controller design is one of the most fundamental issues in control engineering. In this study, an optimization method is presented about design a controller for a fractional order system. An ideal control system is required to meet expectation. In monitoring the performance of the control system, the transient behavior is often used. Percent overshoot value and time parameters of the control system in unit step response are very important for performance appraisal. In the study, Bode's ideal transfer function is accepted as the reference model. It is aimed to obtain the controller parameters by making sure that the system output to be controlled has the same characteristics as the reference model. The controller parameters are obtained by optimization method. The ITSE (integral time squared error) performance criterion is used to minimize the error in the optimization method [7].

In the study, the preferred PID (proportional integral derivative) controller structure was used because of its many advantages. PID controllers are still the most commonly used controller structure [8]. There are many tuning methods used in the literature for calculating PID controller parameters. Ziegler-Nichols, Aström-Hagglund are the most known PID tuning methods [8, 9]. In parallel with improvements in computers, the use of optimization methods is steadily increasing. Some of the optimization methods used in the literature are genetic algorithm, particle swarm optimization, ant colony algorithm. The optimization method used in the study was created in Matlab / Simulink environment.

The remainder of the paper is organized as follows. Section 2 briefly introduces Bode's ideal transfer function. Section 3 contains the PID controller design. In this section, the structure of the PID controller, the optimization approach and the application of the method are given. Finally, conclusions are drawn in Section 4.

2. BODE'S IDEAL TRANSFER FUNCTION

In 1945, H. W. Bode defined the open-loop transfer function of a feedback control system as given in Equation 1 [10]. Where ω_c is the gain crossover frequency and γ is the slope of the amplitude curve that can be expressed in fractional or integers on a logarithmic scale.

$$L(s) = \left(\frac{\omega_c}{s}\right)^{\gamma} \tag{1}$$

The block diagram of the fractional order control system with Bode's ideal transfer function is given in Figure 1. The closed loop transfer function of the block diagram in Figure 1 is given in Equation 2 [11].

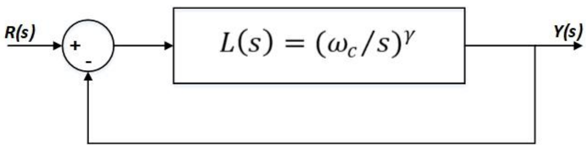

Figure 1: Fractional order transfer function with Bode's ideal transfer function

$$G(s) = \frac{L(s)}{1+L(s)} = \frac{1}{\left(\frac{s}{\omega_c}\right)^{\gamma}+1}, 1<\gamma<2 \tag{2}$$

Frequency and time domain characteristics for Bode's ideal transfer function can be obtained. The Maximum overshoot value is expressed as in Equation 3.

$$M_p \cong 0.8(\gamma-1)(\gamma-0.75), \quad 1<\gamma<2 \tag{3}$$

When the ω_c is taken as 1 rad/s, the maximum overshoot according to the γ parameter change are given in Table 1. When Table 1 is examined, it is seen that as the γ parameter increases, the maximum overshoot increases.

Table 1: Overshoot value for reference model (ω_c=1 rad/s)

γ	**Overshoot (%)**		γ	**Overshoot (%)**
1.0	0		1.5	30
1.1	2.8		1.6	40.8
1.2	7.2		1.7	53.2
1.3	13.2		1.8	67.2
1.4	20.8		1.9	82.8

The time parameters such as peak time, rise time and settling time can be calculated by the following equations, depending on the γ and ω_c parameters [12].

$$t_p \cong \frac{1.106(\gamma - 0.255)^2}{(\gamma - 0.921)\omega_c} \tag{4}$$

Table 2 and Table 3 show the change in peak time and rise time, respectively, depending on the γ and ω_c parameters.

Table 2: Peak time for reference model (ω_c=1 rad/s)

γ	**Peak time (t_p)**		γ	**Peak time (t_p)**
1.0	7.7704		1.5	2.9608
1.1	4.4118		1.6	2.9467
1.2	3.5401		1.7	2.9645
1.3	3.1868		1.8	3.0035
1.4	3.0271		1.9	3.0571

$$t_r \cong \frac{0.131(\gamma + 1.157)^2}{(\gamma - 0.724)\omega_c} \tag{5}$$

Table 3: Rise time for reference model (ω_c=1 rad/s)

γ	**Rise time (t_r)**		γ	**Rise time (t_r)**
1.0	2.2083		1.5	1.1918
1.1	1.7748		1.6	1.1367
1.2	1.5289		1.7	1.0956
1.3	1.3730		1.8	1.0645
1.4	1.2670		1.9	1.0410

When Table 2 is examined, it is noteworthy that the peak time changes slightly with respect to the γ parameter. When Table 3 is examined, it is seen that the rise time is shortened as the γ parameter grows.

Tolerance values allowed for settling time are usually values exceeding 2% and 5%. The equations for these two tolerance values are as follows. Table 4 was prepared for the 5% value of settling time.

$$t_s(\%2) \cong \frac{4}{\cos(\pi - \pi / \gamma)\omega_c} \tag{6}$$

$$t_s(\%5) \cong \frac{3}{\cos(\pi - \pi / \gamma)\omega_c} \tag{7}$$

Table 4: Settling time for reference model (ω_c=1 rad/s)

γ	**Settling time (t_s)**		γ	**Settling time (t_s)**
1.0	3		1.5	6.0000
1.1	3.12		1.6	7.8394
1.2	3.4641		1.7	10.9624
1.3	4.0080		1.8	17.2763
1.4	4.8116		1.9	36.3287

Figure 2 shows the unit step response curves of Bode's ideal transfer function according to the change of γ parameter (1<γ<2). While the ω_c parameter remains constant, the maximum overshoot value increases as the γ parameter grows. It is seen that minimum the overshoot value is γ = 1.1, while the maximum overshoot value is at γ = 1.9 [13].

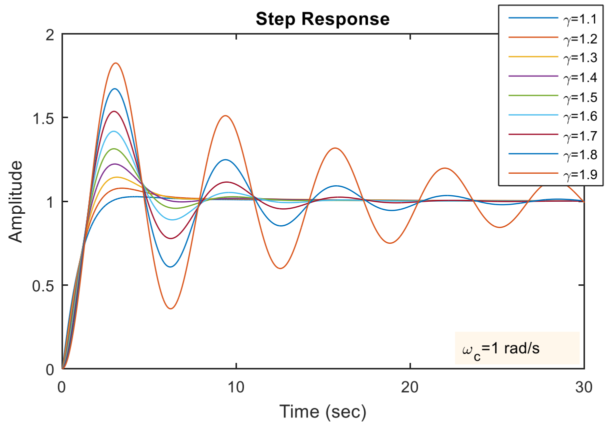

Figure 2: Step responses of reference system for different γ parameters

3. PID CONTROLLER DESIGN

3.1. PID Controller Structure

The purpose of the control system is to ensure that the signal applied to the input is followed in the shortest time and at the least error. At this point, PID controllers offer great ease of use in the best operating conditions of the system. The simplicity and durability of the structure, to be well-known controller, the low number of parameters to be calculated is the reason for the PID controller's preference [8].

The transfer function of a conventional PID controller is given in Equation 8 [14]. Here, the proportional term is Kp, the coefficient of integral term is Ki, and the coefficient of the derivative term is Kd.

$$C(s) = K_p + \frac{K_i}{s} + K_d.s \tag{8}$$

The PID controller compares the feedback signal to the input signal. According to the error between the two signals, the PID controller makes an effect to reduce the error and sends it to the output. In this way, feedback signal is performed until the error is reduced to the minimum. The block diagram of the close-loop control system with PID controller is given in Figure 3.

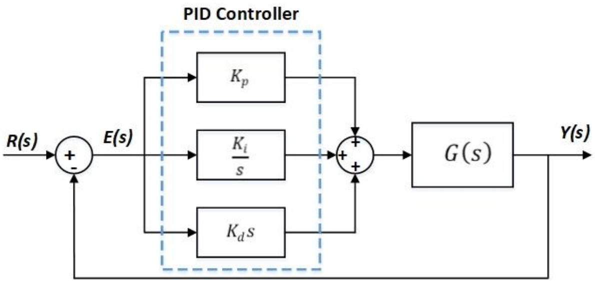

Figure 3: Block diagram of close-loop control system with PID controller

3.2. Optimization Approach

The block diagram of the model used to set the PID controller parameters is given in Figure 4.

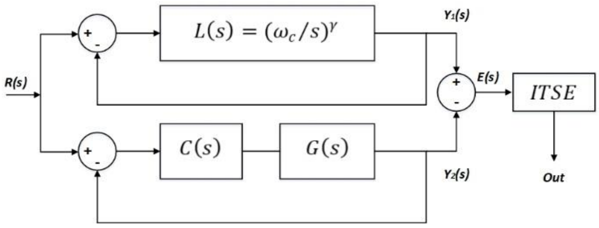

Figure 4: Block diagram of the model created

In the model, it is aimed to calculate the controller parameters by minimizing the error between output of the Bode's ideal transfer function and output of the controlled system. Integral performance criteria are often used to minimize the error. The ITSE performance criterion is chosen for this study. The ITSE criterion is given in Equation 9 [7].

$$J_{ITSE} = \int_0^\infty t.e^2(t)dt \tag{9}$$

Where, *t* denotes the time and *e(t)* denotes the error occurring in the control system. The error is defined as the difference between the input signal and the output signal in a system with unit feedback and is expressed by Equation 10 [7].

$$e(t) = r(t) - y(t) \tag{10}$$

Optimization algorithm begins by entering initial values in parameters K_p, K_i and K_d. In the reference model, If appropriate values of γ and ω_c are entered, the most suitable PID controller parameters are obtained by applying Equation 9.

3.3. Implementation of the Method

Consider the control system of Figure 4 with transfer function

$$G(s) = \frac{3}{s^{0.1}(s+0.6)} \tag{11}$$

PID controller parameters are obtained for different γ and ω_c values in Bode's ideal transfer function. When the controller parameters are obtained, the error is minimized according to the ITSE performance criterion, and the system is optimized according to the reference model. The PID controller parameters obtained for different γ and ω_c values are given in Table 5. PID controller parameters applying to the fractional order system, the unit step responses are obtained and presented graphically [13].

Table 5: PID controller parameters for different γ

γ	K_p	K_i	K_d
	ω_c=0,5 rad/s		
1.1	0,159	0,094	0,006
1.3	0,472	0,173	0,741
1.5	0,233	0,215	0,549
1.7	0,073	0,255	0,603
	ω_c=1 rad/s		
1.1	0,334	0,2	0,0001
1.3	0,325	0,313	0,077
1.5	0,27	0,475	0,173
1.7	0,229	0,796	0,456
	ω_c=2 rad/s		
1.1	0,717	0,429	0,001
1.3	2,51	1,664	0,774
1.5	2,759	4,836	1,417
1.7	1,54	5,948	1,332
	ω_c=3 rad/s		
1.1	1,12	0,67	0,001
1.3	3,618	3,499	0,696
1.5	2,959	6,802	0,81
1.7	1,702	8,654	0,735

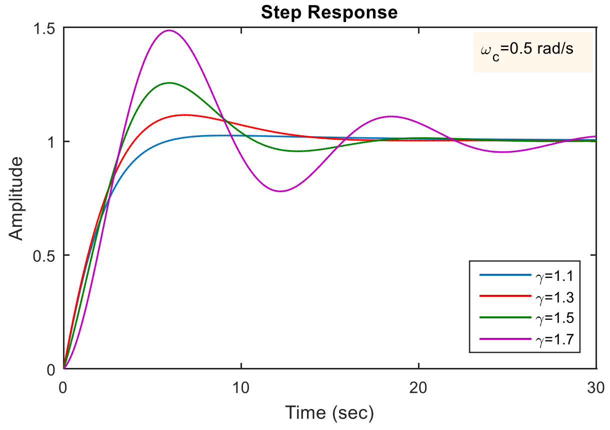

Figure 5: Unit-Step response with controller (ω_c=0.5 rad/s)

Figure 5 shows the unit step response for ω_c= 0.5 rad/s according to various γ parameter. When the figure is examined, it is clear that the maximum overshoot value of the system increases as the γ parameter grows. Moreover, as the γ parameter increases, the rise and peak time become shorter and the settling time becomes longer.

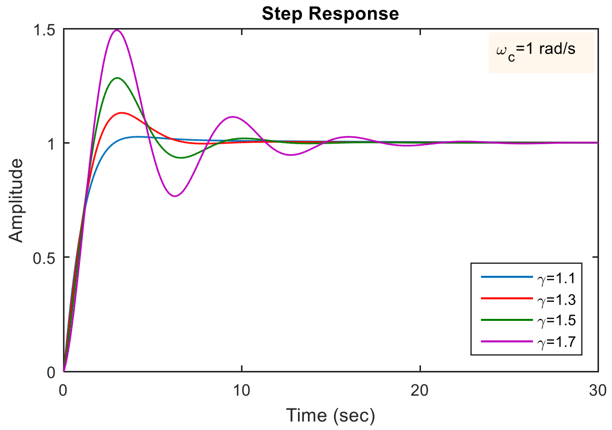

Figure 6: Unit-Step response with controller (ω_c=1 rad/s)

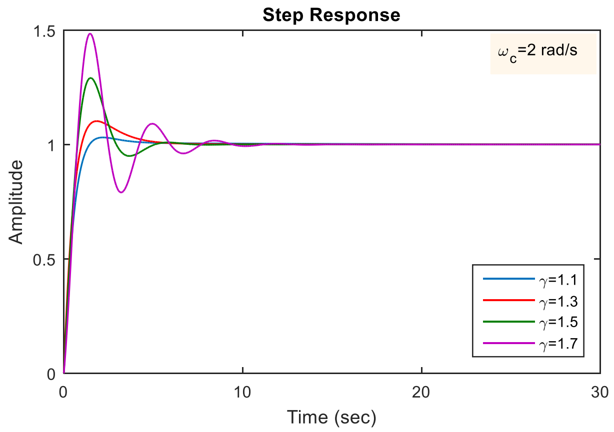

Figure 7: Unit-Step response with controller (ω_c=2 rad/s)

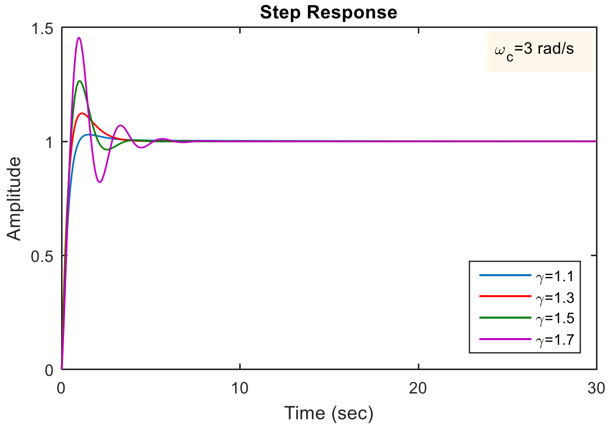

Figure 8: Unit-Step response with controller (ω_c=3 rad/s)

Similarly, Figures 6, 7 and 8 show the unit step response curves of the controlled system for ω_c= 1 rad/s, ω_c= 2 rad/s and ω_c= 3 rad/s, respectively. When the figures are examined, it is noticed that the maximum overshoot values do not change with ω_c parameter but vary with γ parameter. The maximum overshoot values and time parameters for the figures are given in Table 6 in detail.

Table 6: Time response parameters and overshoot value

	γ=1.1	γ=1.3	γ=1.5	γ=1.7
ω_c=0,5 rad/s				
Rise time	3,59	2,93	2,58	2,38
Settling time	5,30	13,39	15,78	28,28
Peak time	9,09	6,83	5,94	5,95
Overshoot (%)	1,95	11,20	25,51	45,51
ω_c=1 rad/s				
Rise time	1,78	1,46	1,29	1,18
Settling time	5,33	5,95	8,15	16,71
Peak time	4,18	3,27	3,00	2,98
Overshoot (%)	2,49	13,06	28,38	49,20
ω_c=2 rad/s				
Rise time	0,90	0,71	0,59	0,57
Settling time	3,11	4,41	4,55	7,28
Peak time	2,19	1,84	1,50	1,47
Overshoot (%)	2,98	10,20	29,09	48,44
ω_c=3 rad/s				
Rise time	0,59	0,43	0,39	0,38
Settling time	2,21	2,82	3,04	4,74
Peak time	1,52	1,14	1,00	0,97
Overshoot (%)	2,92	12,32	26,49	45,49

The PID controller parameters obtained for constant γ and variable ω_c values are given in Table 7. The graphs obtained by applying the PID controller parameters to the fractional order system are given in Figures 9, 10 and 11.

Table 7: PID controller parameters for different ω_c

ω_c	K_p	K_i	K_d
	γ=1.2		
0.5	0,457	0,133	0,624
1	0,359	0,254	0,062
2	2,273	0,952	0,585
3	2,654	1,323	0,425
	γ=1.5		
0.5	0,239	0,217	0,561
1	0,27	0,475	0,173
2	0,578	1,286	0,075
3	1,384	3,241	0,199
	γ=1.8		
0.5	0,136	0,459	1,431
1	0,211	1,172	0,821
2	1,011	6,505	1,361
3	1,189	10,183	0,82

Figure 9 shows the unit step response curves of the system controlled according to variable ω_c values. When the figure is examined, it is seen that the maximum overshoot values do not changed by changing the ω_c parameter. However, the change in time parameters is clearly visible.

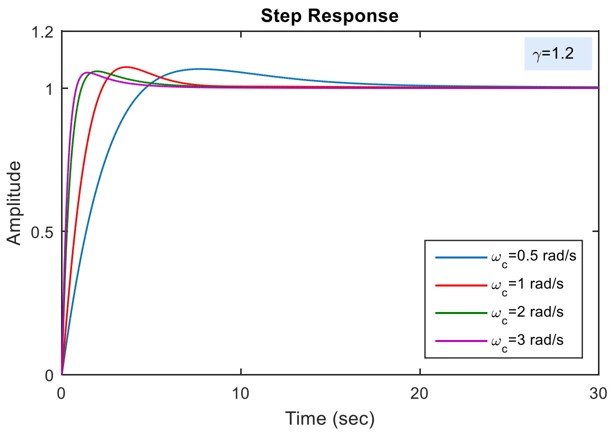

Figure 9: Unit-Step response with controller (γ=1.2)

Similarly, Figure 10 and 11 are obtained according to γ =1.5 and γ =1.8, respectively. It is noticed that the maximum overshoot values do not change while the γ parameter remains constant. Time parameters are seen to be shortened as ω_c values increase.

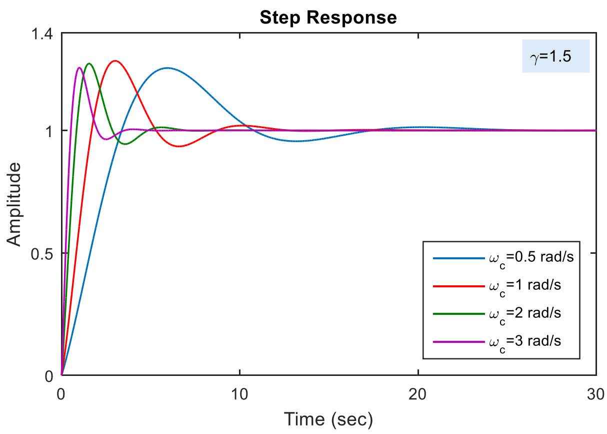

Figure 10: Unit-Step response with controller (γ=1.5)

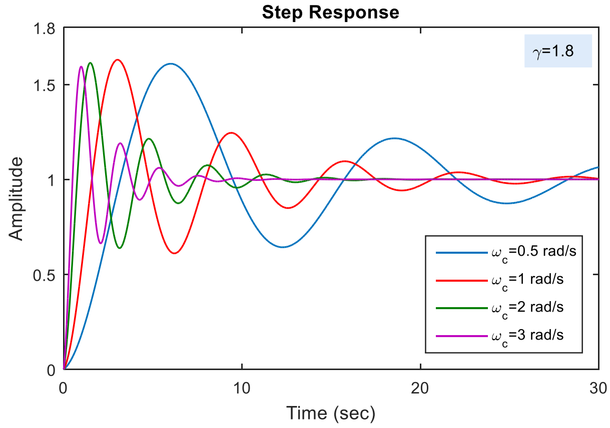

Figure 11: Unit-Step response with controller (γ=1.8)

The maximum overshoot values and time parameters for the figures are detailed in Table 8.

Table 8: Time response parameters and overshoot value

	ω_c=0, 5	ω_c=1	ω_c=2	ω_c=3
		γ=1.2		
Rise time	3,25	1,60	0,73	0,53
Settling time	14,55	6,44	4,67	3,44
Peak time	7,74	3,59	1,96	1,45
Overshoot (%)	6,29	7,21	5,88	5,44
		γ=1.5		
Rise time	2,57	1,29	0,64	0,40
Settling time	15,75	8,15	4,42	2,98
Peak time	5,93	3,00	1,54	1,00
Overshoot (%)	25,44	28,38	27,35	25,67
		γ=1.8		
Rise time	2,37	1,14	0,56	0,37
Settling time	29,28	26,04	11,71	6,79
Peak time	5,99	3,02	1,49	0,98
Overshoot (%)	51,13	62,47	61,32	59,37

4. RESULTS

In this paper, an optimization method is presented related to determine PID controller parameters using Bode's ideal transfer function. The Bode's ideal transfer function is considered as the reference model. In the reference model, the desired unit step response can be obtained by setting two parameters (γ ve ω_c). The unit step response of the controlled system was obtained as having the same characteristic as the Bode's ideal transfer function. The controller parameters are obtained by minimizing the difference between the reference model and the system. A number of simulation studies have been carried out in the study and the results are presented in graphical and tabular form.

It has been shown that the PID controller parameters can be obtained according to the desired unit step response output for a fractional order system. The method has been shown to be successful from the results obtained.

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