ANADOLU ÜNİVERSİTESİ

Bilim ve Teknoloji Dergisi B-Teorik Bilimler
Cilt: 2 Sayı: 2 2013
Sayfa:121-125

ARAŞTIRMA MAKALESİ / **RESEARCH ARTICLE**

Zakir KHANKISHIYEV[1]

ON THE SOLUTION OF A PROBLEM FOR A HYPERBOLIC TYPE INTEGRO-DIFFERENTIAL EQUATION

ABSTRACT

In the paper, a problem for a hyperbolic type linear integro-differential equation containing higher order derivatives in the boundary conditions is investigated, application of the straight lines method to the solution of this problem is given, and a solution algorithm of the problem, obtained after applying the straight lines method, is suggested.

Keywords: Linear integro-differential equation, Hyperbolic equation, Straight lines method

HİPERBOLİK TİPİ İNTEGRO-DİFERANSİYEL DENKLEM PROBLEMİNİN ÇÖZÜMÜ

ÖZ

Bu çalışmada sınır koşullarında yüksek dereceli türevler içeren hiperbolik tipi doğrusal integro-diferansiyel denklemi için bir problem araştırılmıştır. Problemin çözümü için doğru hat metodu uygulaması verilmiş ve doğru hat metodu uygulandıktan sonra problemin çözümü için elde edilen algoritma önerilmiştir.

Anahtar Kelimeler: Doğrusal integro-diferansiyel denklemi, Hiperbolik denklem, Doğru hat metodu

[1], Applied Math and Cybernetics Faculty at Baku State University.
E-mail: hankishiyev.zf@yandex.ru, Tel: 00994503572883

Recieved: 28 November 2012 **Accepted:** 18 June 2013

1. INTRODUCTION

Many phenomena in biological systems essentially depend on previous history of this system. As a rule, these phenomena are described by loaded differential equations (Nakhushev,1995). Consider the equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + b(x,t)u(x,t) + \int_0^t c(x,t)\frac{\partial u(x,t)}{\partial t}dt + f(x,t),\ 0 < x < l,\ 0 < t \le T\ .(1)$$

This equation is also a loaded differential equation that Volterra called a hyperbolic type integro-differential equation (Volterra, 1952).

In this paper we consider the following problem for equation (1):

Find the continuous function $u = u(x,t)$, in the closed domain $\overline{D} = \{0 \le x \le l,\ 0 \le t \le T\}$ satisfying equation (1), boundary conditions

$$\frac{\partial^2 u(0,t)}{\partial x^2} + \alpha u(0,t) = \mu_1(t), \quad \frac{\partial^2 u(l,t)}{\partial x^2} + \beta u(l,t) = \mu_2(t), \quad 0 \le t \le T\ , \tag{2}$$

and initial conditions

$$u(x,0) = \varphi_1(x), \quad \frac{\partial u(x,0)}{\partial t} = \varphi_2(x), \quad 0 \le x \le l\ . \tag{3}$$

Here $a,\ \alpha,\ \beta$ - are real numbers, $b(x,t),\ c(x,t),\ f(x,t),\ \mu_1(t),\ \mu_2(t),\ \varphi_1(x),\ \varphi_2(x)$ -are continuous functions of own arguments.

Note that such a problem with simpler boundary conditions $u(0,t) = \mu_1(t)$ and $u(l,t) = \mu_2(t)$ was solved in the paper (Khankishiyev, 2010).

2. APPLICATION OF THE STRAIGHT LINES METHOD TO THE SOLUTION OF THE INITIAL VALUE PROBLEM

To apply the straight lines method to the solution of problem (1)-(3) we divide the segment $[0, l]$ into N equal parts by the points $x_n = nh,\ n = 0,1,...,N,\ x_0 = 0,\ x_N = l$. If we consider equation (1) on the straight lines $x = x_n, n = 1,2,...,N-1,$ and replace the derivative $\frac{\partial^2 u(x_n,t)}{\partial x^2}$ by the appropriate difference expression, then after applying the integration by parts formula, we get the following equations:

$$\frac{\partial^2 u(x_n,t)}{\partial t^2} = a^2 \frac{u(x_{n+1},t) - 2u(x_n,t) + u(x_{n-1},t)}{h^2} + O(h^2) + b(x_n,t)u(x_n,t) + c(x_n,t)u(x_n,t) -$$

$$-c(x_n,0)u(x_n,0)-\int_0^t c_t'(x_n,t)u(x_n,t)\,dt+f(x_n,t),\ n=1,2,...,N-1,\ 0<t\le T. \qquad (4)$$

Let $K\ge 2$- be a fixed natural number. Divide the segment $[0,T]$ by the points $t_k=k\tau,\ k=0,1,...,K,\ t_0=0,\ t_K=T$, into K equal parts.

Let $0<t\le t_1$. Then applying to the integral in (4) the trapezoid formula, we get:

$$\frac{\partial^2 u(x_n,t)}{\partial t^2}=a^2\frac{u(x_{n+1},t)-2u(x_n,t)+u(x_{n-1},t)}{h^2}+O(h^2)+[b(x_n,t)+c(x_n,t)]u(x_n,t)-$$

$$c(x_n,0)u(x_n,0)-\frac{t}{2}\left[c_t'(x_n,t)u(x_n,t)+c_t'(x_n,0)u(x_n,0)\right]+O(\tau^3)+f(x_n,t),\ n=1,2,...,N-1,$$

$$0<t\le T. \qquad (5)$$

If in these equalities we reject the addend of order $O(h^2+\tau^3)$ and replace $u(x_n,t)$ by $y_n(t)$, then for the functions $y_n(t),\ n=1,2,...,N-1,$ we get the following system of linear differential equations:

$$\frac{d^2y_n(t)}{dt^2}=a^2\frac{y_{n+1}(t)-2y_n(t)+y_{n-1}(t)}{h^2}+d_n(t)y_n(t)+F_n(t),\ n=1,2,...,N-1,\ 0<t\le t_1, \qquad (6)$$

where

$$d_n(t)=b(x_n,t)+c(x_n,t)-\frac{t}{2}c_t'(x_n,t),\ F_n(t)=f(x_n,t)-c(x_n,0)\varphi_1(x_n)-\frac{t}{2}c_t'(x_n,0)\varphi_1(x_n),$$

$$n=1,2,...,N-1.$$

Suppose that equation (1) is fulfilled for $x=0$ and $x=l$. Then substituting $\frac{\partial^2 u(0,t)}{\partial x^2}$ and $\frac{\partial^2 u(l,t)}{\partial x^2}$ found from this equation into the appropriate boundary condition in (2), after integration by parts and replacing the obtained integrals by the trapezoids formulae and replacing $u(0,t)$ and $u(l,t)$ by $y_0(t)$ and $y_N(t)$, we get:

$$\frac{d^2y_0(t)}{dt^2}=d_0(t)y_0(t)+F_0(t),\ 0<t\le t_1,\ \frac{d^2y_N(t)}{dt^2}=d_N(t)y_N(t)+F_N(t),\ 0<t\le t_1, \qquad (7)$$

Where

$$d_0(t) = b(0,t) + c(0,t) - a^2\alpha - \frac{t}{2}c'_t(0,t), \quad d_N(t) = b(l,t) + c(l,t) - a^2\beta - \frac{t}{2}c'_t(l,t),$$

$$F_0(t) = f(0,t) + a^2\mu_1(t) - \left[c(0,0) + \frac{t}{2}c'_t(0,0)\right]\varphi_1(0),$$

$$F_N(t) = f(l,t) + a^2\mu_2(t) - \left[c(l,0) + \frac{t}{2}c'_t(l,0)\right]\varphi_1(l).$$

We have the following from the initiol conditions (3):

$$y_n(0) = \varphi_1(x_n), \ \frac{dy_n(0)}{dt} = \varphi_2(x_n), \ n = 0,1,...,N. \tag{8}$$

So, if $0 < t \le t_1$, then after applying the straight lines method we get problem (6)-(8). This is a Cauchy problem for a system of second order linear differential equations. Let the solution of this problem be found and the values $y_n(t_1), \ n = 0,1,...,N$. be defined. Then we can pass to the next step.

Let $0 < t \le t_2$. In this case, having applied the above-stated method, we get:

$$\frac{d^2 y_n(t)}{dt^2} = a^2 \frac{y_{n+1}(t) - 2y_n(t) + y_{n-1}(t)}{h^2} + d_n(t)y_n(t) + F_n(t), \ n = 1,2,...,N-1, \ 0 < t \le t_2, \tag{9}$$

$$\frac{d^2 y_0(t)}{dt^2} = d_0(t)y_0(t) + F_0(t), \ 0 < t \le t_2, \ \frac{d^2 y_N(t)}{dt^2} = d_N(t)y_N(t) + F_N(t), \ 0 < t \le t_2, \tag{10}$$

where

$$d_0(t) = b(0,t) + c(0,t) - a^2\alpha - \frac{t - t_1}{2}c'_t(0,t),$$

$$d_N(t) = b(l,t) + c(l,t) - a^2\beta - \frac{t - t_1}{2}c'_t(l,t),$$

$$F_0(t) = f(0,t) + a^2\mu_1(t) - \left[c(0,0) + \frac{t_1}{2}c'_t(0,0)\right]\varphi_1(0) - \frac{t}{2}c'_t(0,t_1)y_0(t_1),$$

$$F_n(t) = f(x_n,t) - \frac{t}{2}c'_t(x_n,t_1)y_n(t_1) - \left[c(x_n,0) + \frac{t_1}{2}c'_t(x_n,0)\right]\varphi(x_n), \ n = 1,2,...,N-1,$$

$$F_N(t) = f(l,t) + a^2\mu_2(t) - \left[c(l,0) + \frac{t_1}{2}c'_t(l,0)\right]\varphi_1(l) - \frac{t}{2}c'_t(l,t_1)y_N(t_1).$$

Let the solution of the system of differential equations (9)-(10) satisfying initial conditions (8) be found, and the values $y_n(t_2), \ n = 0,1,...,N$, be determined. Then we can pass to the next step and etc.

Thus, the solution algorithm of problem (1)-(3) by the straight lines method consists of following:

At the first the problem (6)-(8) is solved for $0 < t \le t_1$, and the value of the solution for $t = t_1$ is determined. Allowing for the found values $y_n(t_1)$, $n = 0,1,...,N$, the problem (9)-(10),(8) for $0 < t \le t_2$ is solved. After finding $y_n(t_2)$, $n = 0,1,...,N$, we can pass to the solution of the problem for $0 < t \le t_3$ and etc. At the end, allowing for the found values $y_n(t_k)$, $k = 1,2,...,K-1$, $n = 0,1,...,N$, the problem for $0 < t \le T$ is solved.

REFERENCES

Khankishiyev, Z. F. (2010). On Convergence of The Straight Lines Method by Solving A Problem for A Hyperbolic Type Integro-Differential Equation, Proceedings of The Conference “*Contemporary Problems of Mathematics, Informatics and Economics*”, Baku, 38 (Russian)

Nakhushev, A.M. (1995). Equations of Mathematical Biology Moscow “Vysshaya shkola”, 301 (Russian).

Volterra, V. (1952). Theory of Functionals, Integral and Integro-Differential Equations, Moscow, “Nauka”, 304 (Russian).