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ARAŞTIRMA MAKALESİ /**RESEARCH ARTICLE**

# ON THE (*k*,3)-ARCS OF *CPG*(2,25,5)

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***ABSTRACT***

In this paper, the algorithm for the classification of the $(k,3)$- arcs and some examples of the $(k,3)$-arcs in the projective plane of order 25 over the smallest Cartesian Group are given.

**Keywords:** $(k,3)$-arcs, Projective plane, Baer subplane, Cartesian group, Computer search.

## *CPG*(2,25,5) NIN (*k*,3)-YAYLARI ÜZERİNE

***ÖZ***

Bu çalışmada, en küçük Kartezyen grup üzerine kurulan 25. mertebeden projektif düzlemdeki $(k,3)$-yayların sınıflaması için bir algoritma ve bazı $(k,3)$-yayların örnekleri verilmektedir.

**Anahtar Kelimeler:** $(k,3)$-yaylar, Projektif düzlem, Baer altdüzlemi, Kartezyen grup, Bilgisayar araştırması.

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## 1. INTRODUCTION AND PRELIMINARIES

The subject of *k*-arcs is not only of interest in its purely geometrical setting but also, arcs have some applications in coding theory. They can be interpreted as linear maximum distance separable (MDS) codes, and they are related to super regular matrices to linearly independent sets of vectors in vector spaces over the finite field *K* and to optimal covering arrays. A $(k,3)$- arc is a set of *k* points such that no four of which are collinear of *PG*(2, *K*) wehere *K* is the finite field of *q* elements. A $(k,2)$-arc is called simply an arc of size *k* or a *k* arc. The largest size of an $(k,r)$-arc of *PG*(2, *q*) is indicated by $m_r(2,q)$. In (Ball 1996; Ball and Hirschfeld 2005; Coolsaet and Sticker 2010; Hirschfeld and Storme 2000), some bounds for $m_r(2,q)$ are given. In particular, $m_3(2,q) \leq 2q+1$ for $q \geq 4$ , (Thas 1975) . For a detailed description of the most important properties of these geometric structures, we refer the reader to (Hirschfeld 1998). S. Marcugini, A. Milani and F. Pambianco found the spectrum of all complete arcs in *PG*(2, 27). They also found that in *PG*(2, 29) no arc of size $k \geq 13$ exists. R.N. Daskalov and M.E.J. Contreras gave new $(k,r)$-arcs in *PG*(2, 13).

But it is well known that every projective plane has an algebraic structure obtained by coordinatization. Conversely, certain algebraic structures can be used to construct projective planes. For instance, a general method of generating Cartesian groups has been given by Panella. In (Akça 1991; Akça and Kaya 1997), the consruction of the cartesian group plane and subplanes of this plane are given. Also, in (Akça 2011), $(k,3)$-arcs in the left semifield plane of order 9 are obtained by computer. We shall be interested in the projective plane of order 5 over the smallest Cartesian Group of order 25 and write *CPG*(2,25,5).

**Definition 1.1** A projective plane π consists of a set ***P*** of points, and a set *L* of subsets of ***P***, called lines, such that every pair of points is contained in exactly one line, every two different lines intersect in exactly one point, and there exist four points, no three of which are collinear.

**Definition 1.2** A subplane of a projective plane π is a ***B*** of points and lines which is itself a projective plane, relative to the incidence relation given in π. Let π be a projective plane of order *n*. If π′ is a subplane of order *m*, then either $m^2 = n$ or $m^2 + m \leq n$. ***B*** is called Baer subplane of π if it satisfies the following conditions:

1) Every point of π is incident with a line of ***B***
2) Every line of π is incident with a point of ***B***.

It is clear that for the Baer subplane ***B*** of order *m*, $m^2 = n$.

**Definition 1.3** A system $(S,\oplus,*)$ is a Cartesian group if and only if the following conditions are satisfied:
i) $(S,\oplus)$ is a group
ii) Each of equations $a*x=b$ and $x*a=b$ has a unique solution for all $a,b\in S-\{0\}$, where 0 denotes the additive identity.
iii) There exists an element $e\in S$ such that $e*x = x = x*e$ for all $x\in S$.
iv) $x*0 = 0 = 0*x$ for all $x\in S$.
v) Given $a,b,c,d\in S$ such that $a\neq c$, there exists a unique $x\in S$ such that

$$a*x\oplus b = c*x\oplus d$$

vi) Given $a,b,c,d\in S$ such that $a\neq c$, there exists a unique pair $(x,y)\in S^2$ such that

$$x*a\oplus y = b \text{ and } x*c\oplus y = d$$

**Example 1.1** Let $(F_5,+,.)$ be the field of integers modulo 5. Let $S=\{(a,b):a,b\in F_5\}$ and consider the addition and multiplication on *S* given by

$$(a,b)\oplus(c,d) = (a+c, b+d)$$

and

$$(a,b)*(c,d)=\begin{cases}(a.c,a.d), & if\ b=0\\ (a.c-(a^2-2).d.b^{-1},b.c-a.d), & if\ b\neq 0\end{cases}$$

The system $(S,\oplus,*)$ is a proper Cartesian group.

## 2. THE SMALLEST CARTESIAN GROUP PLANE $P_2S$

We shall be interested in the projective plane $P_2S$ over the smallest Cartesian Group of order 25. The 651 points of $P_2S$ are the elements of the set

$$\{[x,y]:x,y\in S\}\cup\{[m]:m\in S\}\cup\{[Y]\}.$$

The points of the form $[x,y]$ are called affine points and the points of the form $[m]$ and the unique point $[Y]$ are called ideal points. The 651 lines of $P_2S$ are defined to be set of points satisfying one of the three conditions:

$$\langle m,k\rangle=\{[x,y]\in S^2:y=m*x\oplus k\}\cup\{[m]\}$$

$$\langle \lambda\rangle=\{[x,y]\in S^2:x=\lambda\}\cup\{[Y]\}$$

$$\langle \infty\rangle=\{[m]\in S\}\cup\{[Y]\}$$

The 625 lines having form $y=m*x\oplus k$ and 25 lines having of the form $x=\lambda$ are called the affine lines and the unique line $\langle\infty\rangle$ is called the ideal line.

The system of points, lines and incidence relation given above defines a projective plane of order 25, which is the smallest Cartesian group plane.

In this paper, our program was written in GAP and the algorithm for the classification of the $(k,3)$- arcs in the Baer subplane which is the projective subplane of order 5 of the projective plane of order 25 over the smallest Cartesian Group, using GAP was presented and were obtained some examples of the $(k,3)$-arcs in this work has been obtained using a computer-based exhaustive search.

## 3. MAIN RESULTS

Let $G_0=\{O=[0,0],I=[1,0],X=[0,1],P=[1,1]\}$ be a quadrangle in *CPG*(2,25,5), $l_1=OX,l_2=PX,l_3=OP,l_4=IP,l_5=OI,l_6=IX$, for each *j*, $1\leq j\leq 6$, $X_j\in l_j-G_0$ and $G_j=G_{j-1}\cup\{X_j\}$.

**Theorem 3.1** If *CPG*(2,25,5) is projective plane with coordinatized by quadrangle $G_0$, then

i) The total numbers of (5,3)-arcs is 27

ii) The total numbers of (6,3)-arcs is $23\binom{27}{1}$

iii) The total numbers of (7,3)-arcs is $19\binom{27}{2}$

iv) The total numbers of (8,3)-arcs is $15\binom{27}{3}$

v) The total numbers of (9,3)-arcs is $10\binom{27}{4}$

vi) The total numbers of (10,3)-arcs is $2\binom{27}{5}$

vii) The total numbers of (11,3)-arcs is $\binom{27}{6}$.

**Proof** Since *CPG*(2,25,5)\ $G_0$ contains exactly 27 points, the total numbers of (5,3)-arcs which contain $G_0$ is 27. For each *j*, $1 \le j \le 6$, one can easily find the total numbers (5+*j*)-arcs contain $G_0$, using GAP. Now, we will give examples of (5,3)-arcs and (5+*j*,3)-arcs, for each $1 \le j \le 6$, using GAP, respectively.

## 4. SOME EXAMPLES OF (k,3)-ARCS

Now, we will give some examples of (4+*j*,3)-arcs, for each *j*, $1 \le j \le 6$, using GAP, respectively.

**Example 4.1** (5,3)-arcs which contain $G_0$

```
gap> G0:=[[0,0],[1,0],[0,1],[1,1]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ] ]
gap> Set( G0);

[ [ 0 ], [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 3 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 4 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 1, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 1, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 3, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ 4, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 1, 0 ], [ 1, 1 ], [ Y ] ]
```

**Example 4.2** (6,3)-arcs which contain $G_1$ where it is selected $X_1 = [0,2]$.

gap> $G_1$:=[[0,0],[1,0],[0,1],[1,1],[0,2]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ], [ 0, 2 ] ]
gap> Set( $G_1$ );

[ [ 0 ], [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1 ], [ 1, 0 ], [ 1, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 3, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 4, 4 ] ]

**Example 4.3** (7,3)-arcs which contain $G_2$ where it is selected $X_2 = [2,1]$.

gap> $G_2$:=[[0,0],[1,0],[0,1],[1,1],[0,2],[2,1]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ], [ 0, 2 ], [ 2, 1 ] ]
gap> Set( $G_2$);

[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 3 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 4 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 0 ], [ 2, 1 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 3, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 3, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 3, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 3, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 4, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 4, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 4, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 4, 4 ] ]

**Example 4.4** (8,3)-arcs which contain $G_3$ where it is selected $X_3 = [2,2]$.

gap> $G_3$:=[[0,0],[1,0],[0,1],[1,1],[0,2],[2,1],[2,2]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ], [ 0, 2 ], [ 2, 1 ], [ 2, 2 ] ]
gap> Set( $G_3$);

[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 3 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 4 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 0 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 2, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 3, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 3, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 3, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 4, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 4, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 2, 1 ], [ 2, 2 ], [ 4, 3 ] ]

**Example 4.5** (9,3)-arcs which contain $G_4$ where it is selected $X_4 = [1,2]$.

gap> $G_4$:=[[0,0],[1,0],[0,1],[1,1],[0,2],[2,1],[2,2],[1,2]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ], [ 0, 2 ], [ 2, 1 ], [ 2, 2 ], [ 1, 2 ] ]
gap> Set( $G_4$);

[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 0 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 2, 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 2, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 3 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 3, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 3, 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 4 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 4, 0 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 1 ], [ 2, 2 ], [ 4, 3 ] ]

**Example 4.6** (10,3)-arcs which contain $G_5$ where it is selected $X_5 = [2,0]$.

gap> $G_5$:=[[0,0],[1,0],[0,1],[1,1],[0,2],[2,1],[2,2],[1,2],[2,0]];
[ [ 0, 0 ], [ 1, 0 ], [ 0, 1 ], [ 1, 1 ], [ 0, 2 ], [ 2, 1 ], [ 2, 2 ], [ 1, 2 ], [ 2, 0 ] ]
gap> Set( $G_5$);

[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2 ], [ 2, 0 ], [ 2, 1 ], [ 2, 2 ] ]
[ [ 0, 0 ], [ 0, 1 ], [ 0, 2 ], [ 1, 0 ], [ 1, 1 ], [ 1, 2 ], [ 2, 0 ], [ 2, 1 ], [ 2, 2 ], [ 3 ] ]

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