Süleyman Demirel Üniversitesi
İktisadi ve İdari Bilimler
Fakültesi Dergisi
Y.2011, C.16, S.2, s.1-11.

Suleyman Demirel University
The Journal of Faculty of Economics
and Administrative Sciences
Y.2011, Vol.16, No.2, pp.1-11.

A DETERMINISTIC INVENTORY MODEL WITH PRICE-DEPENDENT DEMAND CONSIDERING THE BOUNDED LEARNING

SINIRLI ÖĞRENMEYİ DİKKATE ALAN FİYAT BAĞIMLI TALEPLİ DETERMİNİSTİK BİR ENVANTER MODELİ

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ABSTRACT

In this paper, we develop an economic production quantity (EPQ) model where linear demand is affected by selling price and furthermore production learning exists. Lot sizes and price levels in each cycle have been obtained which maximize the total profit per unit time that is total revenue less total cost per unit time, which includes holding cost, labor and material cost and setup cost per unit time. Numerical analysis has been performed in order to see the effects of demand parameters and learning on total profit per unit time.

ÖZET

Bu makalede doğrusal talebin satış fiyatından etkilendiği ve üretimde öğrenmenin olduğu ekonomik üretim miktarı modeli geliştirilmektedir. Her bir partideki parti büyüklükleri ve fiyat seviyeleri birim başına toplam karı maksimize ederek elde edilmektedir. Birim başına toplam kar toplam hasılat ile toplam maliyet arasındaki farktır. Birim başına toplam maliyet ise emek ve malzeme maliyeti ile kurulum maliyetinin toplamından oluşmaktadır. Talep parametreleri ve öğrenmenin birim başına toplam kara etkisini görebilmek amacıyla nümerik analiz yapılmıştır.

Inventory, Economic Production Quantity, Price-Sensitive Demand, Learning.
Envanter, Ekonomik Üretim Miktarı, Fiyata Duyarlı Talep, Öğrenme.

INTRODUCTION

The classical Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models are used to determine the optimal lot size

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in batch production. Their major assumptions are that the setup and unit variable costs are constant and independent of order quantities. These assumptions are generally valid for items produced by machines exhibiting near-identical operational behavior (Li and Cheng, 1994). These assumptions may not be true for labor intensive production systems. In a production system, in repetitive tasks, performance of a paid worker improves over time. The phenomenon that reduction of effort and time to produce the same amount of output in a repetitive task is called learning curve (Zhou and Lau, 1998). In a labor intensive production, as the amount of production increases, setup cost and unit production time decrease due to learning. This learning effect first was revealed by Wright (1936) in the aircraft industry. Keachie and Fontana (1966) were the first researchers to combine the effect of learning on unit production time with EOQ. Since then, some researchers (Adler and Nanda, 1974; Axsater and Elmaghraby, 1981; Fisk and Ballou, 1982; Jaber and Bonney, 1996; Muth and Spremann, 1983; Smunt and Morton, 1985; Salameh vd., 1993; Sule, 1979; Wortham, A.M. Mayyasi, 1972) have made major contributions on the subject. There is only a limited amount of work in the literature considering the effects of learning on setup time (Cheng, 1991; Rachamadugu and Tan, 1997; Replogle, 1998). Li and Cheng (1994) and Cheng (1994) considered both the effects of learning on unit production time and setup time.

These above-models do not consider the shortage case. Jaber and Salameh (1995) have developed an EPQ model with shortages backordered considering the Wright unbounded learning situation. Later, Zhou and Lau (1998) have considered the EPQ model with shortages backordered considering the De Jong bounded learning case. In both models, the setup cost was assumed to be constant. None of these researchers simultaneously considered the price-dependent demand and the learning situation.

In this paper, an economic production quantity model with shortages backordered considering the effects of learning on unit production time and setup time has been developed. It has been proven that the model has a unique solution and that the total cost function per unit time has a global minimum. The effects of the parameters of learning and forgetting on the optimal solution have been investigated by using illustrative examples.

1. MODEL ASSUMPTIONS AND NOTATION

The following assumptions are used in the model:

1. Demand rate is a decreasing linear function of selling price.
2. Unit variable production time decrease as a result of learning over time.
3. Production rate is greater than demand rate.

4. Each cycle length is different.

5. Production quantity, inventory level and quantity demanded are treated as continuous variables for each cycle.

6. A single item is produced in batches.

7. No shortages are allowed.

The notation used is

s = Selling price

$D(s) = D$ = Demand rate, $D = \alpha - \beta s$, $(\alpha, \beta > 0)$, is a decreasing linear function of selling price where $\alpha - \beta s > 0$ for positive demand.

T_{k-1} = Beginning point of cycle k

T_{pk} = Point that production stops in cycle k

T_k = End point of cycle k

$T_k - T_{k-1}$ = Cycle time for cycle k

q_k = Production quantity in cycle k

I_k = Maximum stock level in cycle k

Y_{jk} = Production time of j^{th} unit in cycle k

A = Fixed setup cost for each cycle

h = Inventory holding cost per unit per unit time

M_c = Material cost per unit item

L_c = Labor cost per unit time

Y = Production time of first unit in first cycle

2. MODEL FORMULATION

We assume the existence of a finite replenishment inventory system as shown in Figure 1.

We assume that learning curve is given as follows (DeJong, 1957):

$$Y_{jk} = mY + (1-m)Y_{1k} j^b \tag{1}$$

where $Y_{1k} = Y\left(1 + \sum_{n=1}^{k-1} q_n\right)^b$

b = Learning coefficient for production time, $b = \log(p)/\log(2)$

p = Learning rate for production time

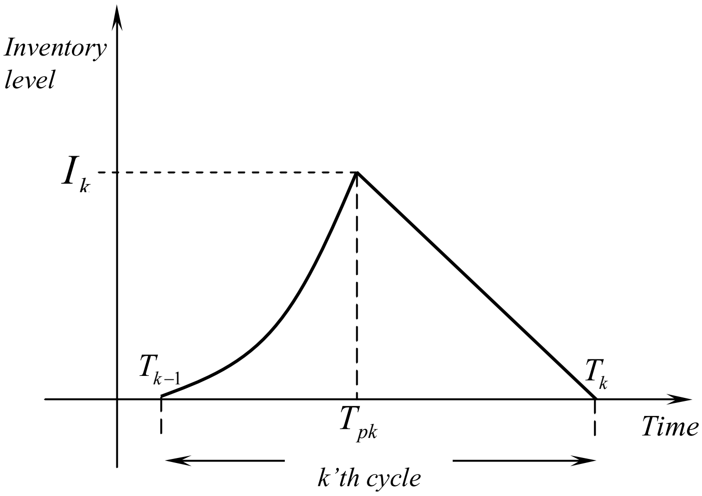

Figure 1: Inventory level with time under production learning

In many practical situations, the value of b are greater than -1 or, equivalently, $0.5 < p \le 1$ (Argote and Epple, 1990; Li and Cheng, 1994). So, we assumed that $-1 < b \le 0$ or $0.5 < p \le 1$

The cumulative time to produce X units in (T_{k-1}, t) time interval is given by

$$t - T_{k-1} = \sum_{j=1}^{X} Y_{jk} \cong \int_0^X Y_{jk}\, dj$$

$$= mYX + \frac{1-m}{(1+b)} Y_{1k} X^{1+b} \quad , T_{k-1} \le t \le T_{pk} \qquad (2)$$

Letting $X = q_k$ in equation (2), we obtain

$$T_{pk} - T_{k-1} = mY\, q_k + \frac{1-m}{(1+b)} Y_{1k}\, q_k^{1+b} \qquad (3)$$

Since the production quantity in the (T_{k-1}, T_{pk}) time interval is equivalent to demand in (T_{k-1}, T_k) time interval, we get:

$$q_k = I_k + D(T_{pk} - T_{k-1})$$

or,

$$I_k = q_k(1 - mYD) - \frac{(1-m)D}{(1+b)} Y_{1k}\, q_k^{1+b} \tag{4}$$

On the other hand, From Figure 1, the following expression can be written,

$$T_k - T_{pk} = I_k / D \tag{5}$$

Define $I(t)$ as inventory level in cycle k. Then, it can be expressed as

$$I(t) = \begin{cases} X - D(t - T_{k-1}) & , \ T_{k-1} \le t \le T_{pk} \\ I_k - D(t - T_{pk}) & , T_{pk} \le t \le T_k \end{cases}$$

Therefore, the inventory holding cost for cycle k is expressed as

$$HC_k = h\int_{T_{k-1}}^{T_k} I(t)dt$$

$$= h\left[\frac{q_k^2(1 - mYD)}{2D} - \frac{(1-m)Y_{1k}\, q_k^{2+b}}{(1+b)(2+b)}\right]$$

The production cost for cycle k is expressed as

$$PC_k = A + M_c\, q_k + L_c\left(T_{pk} - T_{k-1}\right)$$

$$= A + M_c\, q_k + L_c\, q_k\left[mY + \frac{(1-m)}{(1+b)} Y_{1k}\, q_k^b\right]$$

Therefore, total cost for cycle k can be given as

$$TC_k = PC_k + HC_k$$

On the other hand, total revenue, total profit and total profit per unit time for cycle k can respectively be given as

$$TR_k = s\,q_k$$

$$TP_k = TR_k - TC_k\,. \tag{6}$$

$$TPU_k = \frac{TP_k}{T_k - T_{k-1}} = \left(\frac{D}{q_k}\right) TP_k$$

$$= sD - \left\{ \frac{DA}{q_k} + DM_c + DL_c \left[mY + \frac{1-m}{(1+b)} Y_{1k}\, q_k^b \right] \right.$$

$$\left. + h\left[\frac{q_k}{2}(1 - mYD) - \frac{(1-m)DY_{1k}\, q_k^{1+b}}{(1+b)(2+b)} \right] \right\} \tag{7}$$

where, since the production quantity in any cycle k is equivalent to that cycle's demand,

$$q_k = D(\mathrm{T_k} - T_{k-1})$$

or

$$T_k - T_{k-1} = q_k / D \tag{8}$$

Our aim is to find the optimal values of s and q_k in order to maximize the total profit per unit time for cycle k, TPU_k. By partially differentiating equation (7) with respect to s and q_k, the following first order optimality conditions are obtained:

$$s=\left\{\frac{\alpha}{\beta}+\frac{A}{q_k}+M_c+L_c\left(mY+\frac{1-m}{1+b}Y_{1k}\,q_k^b\right)\right.$$
$$\left.-\frac{hmY\,q_k}{2}-\frac{h(1-m)}{(1+b)(2+b)}Y_{1k}\,q_k^{1+b}\right\}/2 \tag{9}$$

$$D\left\{\frac{A}{q_k^2}-\frac{bL_c(1-m)Y_{1k}\,q_k^{b-1}}{1+b}+\frac{hmY}{2}+\frac{h(1-m)Y_{1k}\,q_k^b}{2+b}\right\}-\frac{h}{2}=0 \tag{10}$$

Next, substituting (9) into (10), the two-dimensional problem determining the optimal values of s and q_k can be reduced to a one-dimensional problem determining the optimal value of q_k as follows:

$$D\left\{\frac{A}{q_k^2}-\frac{bL_c(1-m)Y_{1k}\,q_k^{b-1}}{1+b}+\frac{hmY}{2}+\frac{h(1-m)Y_{1k}\,q_k^b}{2+b}\right\}-\frac{h}{2}=0 \tag{11}$$

Thus, we solve expression (11) by using any one-dimensional search method.

$$\frac{d^2TPU_k}{dq_k^2}=\frac{dD}{dq_k}\left\{\frac{A}{q_k^2}-\frac{bL_c(1-m)Y_{1k}\,q_k^{b-1}}{1+b}+\frac{hmY}{2}+\frac{h(1-m)Y_{1k}\,q_k^b}{2+b}\right\}$$
$$+D\left\{-\frac{2A}{q_k^3}-\frac{b(b-1)(1-m)L_cY_{1k}\,q_k^{b-2}}{1+b}+\frac{hb(1-m)Y_{1k}\,q_k^{b-1}}{2+b}\right\} \tag{12}$$

where

$$\frac{dD}{dq_k}=\frac{\beta}{2}\left\{\frac{A}{q_k^2}-\frac{b(1-m)L_cY_{1k}\,q_k^{b-1}}{1+b}+\frac{hmY}{2}+\frac{h(1-m)Y_{1k}\,q_k^b}{2+b}\right\}$$

3. NUMERICAL EXAMPLE

In this section, the proposed model will be illustrated while assuming the following: the time to produce the first unit in the first cycle, Y, is 0.0625 days, setup cost per cycle, A, is $200\$$, labor cost, L_c , is

80 \$/day, material cost, M_c, is 100 \$/unit item, inventory holding cost, h, is 0.2 \$/unit/day, learning coefficient, b, is -0.1 and so the learning rate, p, is 0.933, incompressibility factor, m, is 0.25 and demand function parameters, α and β are 30 and $0,1$ respectively.

By using equation (11), the optimal production quantities, q_k, for $k = 1,2,\ldots$, can be obtained by employing the Newton-Raphson method. Then, the corresponding optimal values of s, I_k, TPU_k, TP_k, production time, $(T_{pk} - T_{k-1})$, and cycle time, $(T_k - T_{k-1})$, can be attained by using equations (9), (4), (7), (6), (3) and (8). Optimal solution results for cycles 1 through 6 are presented in Table 1.

Table 1 suggests that the effect of learning on optimal solution results is rather obvious at the initial cycles and fades during the later cycles. For example, when relative variation rates (in percentage) in optimal production quantities expressed as $V_k = 100\left(1 - \frac{q_{k+1}}{q_k}\right)$, relative variation rate (13.2, 1.27, 0.50, 0.45, 0.34) tapers.

Table 1: Effect of learning on optimal solution

k	q_k	I_k	s	D	$(T_{pk}$-$T_{k-1})$ Production Time	$(T_k$-$T_{k-1})$ Cycle Time (day)	TPU_k	TP_k	$\frac{d^2TPU_k}{dq_k^2}$
1	209.2	114.4	201.82	9.82	9.65	21.31	942.92	20093.0	-0.00039
2	181.6	121.3	201.59	9.84	6.13	18.45	950.37	17537.2	-0.00062
3	179.3	121.8	201.56	9.84	5.87	18.25	951.03	17351.9	-0.00065
4	178.4	122.1	201.55	9.85	5.72	18.12	951.41	17240.6	-0.00066
5	177.6	122.1	201.54	9.85	5.61	18.01	951.68	17137.7	-0.00067
6	177.0	122.2	201.53	9.85	5.53	17.94	951.88	17072.5	-0.00068

TPU=949.68, TP=106432.8, TS=112.072

CONCLUSION

In this paper, an economic production quantity model with shortages backordered considering the learning effects has been developed. By solving the model, optimal lot sizes have been obtained. It has been proven that the total cost function per unit time of the model is convex. The effects of the parameters of production and setup learning on optimal solution have been investigated by using illustrative examples. The solution results indicate that the effects of learning on production and setup are rather obvious at the initial cycles and fade in later cycles. It can also be noted that as the learning retention rate increases, the effect of learning increases. Moreover, as learning improves, i.e. as learning rate decreases, the average unit cost and production lot size decrease and the cost difference between classical EPQ and learning models gets larger.

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APPENDIX

Derivation of Inventory Holding Cost:

$$HC_k = h\int_{T_{k-1}}^{T_k} I(t)dt$$

$$= h\left\{\int_{T_{k-1}}^{T_{pk}} \left[X - D\left(t - T_{k-1}\right)\right]dt + \int_{T_{pk}}^{T_k} \left[I_k - D\left(t - T_{pk}\right)\right]dt\right\}$$

$$= h\left\{\int_0^{q_k} X\left[mY + (1-m)Y_{1k}X^b\right]dX\right.$$
$$\left. - \int_{T_{k-1}}^{T_{pk}} D\left(t - T_{k-1}\right)dt + \int_{T_{pk}}^{T_k} \left[I_k - D\left(t - T_{pk}\right)\right]dt\right\}$$

$$= h\left\{\frac{mYq_k^2}{2} + \frac{(1-m)Y_{1k}q_k^{2+b}}{2+b} - \frac{D}{2}\left(T_{pk} - T_{k-1}\right)^2\right.$$
$$\left. + I_k\left(T_k - T_{pk}\right) - \frac{D}{2}\left(T_k - T_{pk}\right)^2\right\}$$

$$= h\left\{\frac{mYq_k^2}{2} + \frac{(1-m)Y_{1k}q_k^{2+b}}{2+b} - \frac{D}{2}\left(\frac{q_k - I_k}{D}\right)^2 + \frac{I_k^2}{2D}\right\}$$

$$= h\left\{\frac{q_k^2(1-mYD)}{2D} - \frac{(1-m)Y_{1k}\, q_k^{2+b}}{(1+b)(2+b)}\right\}$$