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Solution of Two-Point Reactor Kinetic Equations for Source-Driven Reflected Reactors Using the Green's Function Generation Method

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Abstract

The transient analysis of nuclear reactors is a very important issue from the safety point of view. The point reactor kinetics equations are extensively used in the analysis of the transient behavior of the nuclear reactors. The experimental results confirm that the dynamical behavior of the strongly reflected reactors cannot be adequately characterized by the conventional one-point reactor kinetics equations. In this study, a Green's function generation method is developed to solve the two-point reactor kinetics equations for a subcritical source-driven reflected reactor with one group of delayed neutron precursors during the reactor start-up. Moreover, the two-point reactor kinetics equations are solved using the prompt-jump approximation method and it is shown that the obtained results are in good agreement with the results obtained from the Green’s function generation method. The validity of the given methodologies is also demonstrated through comparison with asymptotic solutions.

Keywords: Point reactor kinetics model, Reflected reactor, Green’s function, Prompt jump approximation

Kaynak-Güdümlü Yansıtıcılı Reaktörlerin İki-Nokta Kinetik Denklemlerinin Green Fonksiyon Üretim Yöntemi ile Çözümü

Öz

Nükleer reaktörlerin zamana bağlı analizi güvenlik açısından çok önemli bir faktördür. Nokta reaktör kinetik denklemleri nükleer reaktörlerin zamana bağlı davranışlarının analizinde yaygın olarak kullanılmaktadır. Deneysel sonuçlar, yaygın olarak kullanılan tek nokta reaktör kinetik modelinin güçlü yansıtıcılı reaktörlerin dinamik davranışlarının tanımlanmasında yetersiz olduğunu göstermektedir. Bu çalışmada, reaktörün çalışmaya başlatılması sırasında bir grup gecikmiş nötron öncüleri içeren kritik altı kaynak güdümlü yansıtıcılı reaktörün iki nokta reaktör kinetik eşitliklerinin çözümüne yönelik bir Green fonksiyon üretim yöntemi geliştirilmiştir. Bunun yanı sıra, iki-nokta reaktör kinetik denklemlerini çözmek için ani sıçrama yaklaşım yöntemi kullanılmıştır. Sonuç olarak ani sıçrama yaklaşım yöntemi ve Green fonksiyon üretim yöntemi ile elde edilen sonuçların uyum içinde olduğu bulunmuştur. Önerilen yöntemlerin geçerliliği asimptotik çözümlerle karşılaştırılarak da doğrulanmıştır.

Anahtar Kelimeler: Nokta reaktör kinetik modeli, yansıtıcılı reaktör, Green fonksiyonu, Ani sıçrama Yaklaşımı

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Introduction

Source-driven systems are the subcritical reactors, in which the chain reactions are sustained by the external neutron source. In these systems, the power level strongly depends on the level of subcriticality, external source strength and position of the external sources. Increasing the source strength and bringing the multiplication factor closer to one cause the power level to increase. During the initial reactor start-up, the reactor is in a subcritical condition and the external neutron source plays a crucial role from the reactor control point of view. Since in the reactor start-up process, the average temperature of the reactor core is lower and the power added is smaller, the temperature reactivity feedback mechanism can be neglected [1-3].

The point reactor kinetics equations are a system of coupled differential equations and solve to predict the time evolutions of the reactor power and the precursor concentration. To derive the point kinetics equations, the spatial dependency of the general neutron balance equations are eliminated. Therefore, the reactor only varies in time, such that it is essentially treated as a point. The entire reactor in the conventional point reactor kinetics model is taken as a point multiplying media with an effective neutron generation time. Conventional point kinetics equations with one effective group of delayed neutron precursors and in presence of the external neutron source are expressed in the form of [4-7]:

$$\begin{cases} \dfrac{dN(t)}{dt} = \dfrac{\rho(t)-\beta}{\Lambda}\ N(t) + \lambda\, C(t) + S(t) \\ \dfrac{dC(t)}{dt} = \dfrac{\beta}{\Lambda}\ N(t) - \lambda\, C(t) \end{cases} \tag{1}$$

In this system of equations $\rho(t)$ is the system reactivity, Λ is the neutron generation time, β and λ are the delayed neutron fraction and decay constant of the precursors, respectively, $S(t)$ represents the effective external neutron source, $C(t)$ is the weighted precursor density, and $N(t)$ is the weighted neutron population which is also referred to as amplitude function and taken proportional to actual neutron density and actual reactor power.

The experimental results exhibit that the conventional one-point reactor kinetics model cannot properly predict the dynamical behavior of the strongly reflected reactors in which a small core is surrounded by a thick reflector. In such reactors, the effective neutron generation time is affected by the neutron migration time in the reflector region [8-10]. Therefore, the dynamical behavior of the reflected reactors has been analyzed by the Two-Point Reactor Kinetics Model

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(TPRKM). This model is based on the coupled reactor theory developed by Avery and was first derived by Cohn and re-derived by Van Dam and Spriggs et al. [11-13,8]. In this model, the fraction of fission neutrons leaking from the core to the reflector (f_{cr}) and fraction of reflector neutrons returning back to the core (f_{rc}), known as reflected reactor coupling parameters, describe the migration of neutrons between core and reflector. Deterministic two-point reactor kinetics equations with external neutron source in the core region and also with one group of delayed neutron precursor is expressed as [9,13]:

$$\begin{cases} \dfrac{dN_c(t)}{dt} = \dfrac{\rho(t) - \beta - f_{cr}\, f_{rc}}{\Lambda_c}\ N_c(t) + \\ \qquad\qquad \dfrac{f_{rc}}{l_r} N_r(t) + \lambda\, C\,(t) + S(t) \\ \dfrac{dN_r(t)}{dt} = \dfrac{f_{cr}}{\Lambda_c}\ N_c(t) - \dfrac{N_r(t)}{l_r} \\ \dfrac{dC\,(t)}{dt} = \dfrac{\beta}{\Lambda_c}\ N_c(t) - \lambda\, C\,(t) \end{cases} \qquad (2)$$

where $N_c(t)$ is the neutron population in the core region and is taken proportional to reactor power, $N_r(t)$ represents the neutron population in the reflector region, Λ_c is the neutron generation time in the core region, l_r is the neutron lifetime in the reflector region, $\rho(t) = \rho_\infty(t) - f_{cr}(1 - f_{rc})$ is the system reactivity, and $\rho_\infty(t)$ is the infinite core reactivity.

In the case of different perturbation scenarios in the absence and presence of temperature reactivity feedback, different deterministic numerical solution methods such as analytical inversion method, fundamental matrix method and analytical exponential technique were developed to solve the source free version of the TPRKM [9, 14-16]. Analytical solution for source free form of the TPRKM with constant kinetics parameters was presented by Holschuh et al. [17]. They showed that, by decreasing the reflector return fraction f $(= f_{cr} \times f_{rc})$ the two and one point reactor kinetics models become identical.

In this work, the dynamical behavior of subcritical source-driven reflected reactors with constant kinetics parameters during the reactor start-up (with $N_c(0) = 0$) is evaluated analytically and effects of the source strength and coupling parameters on the time evolution of the reactor power are investigated. The system response to a unit neutron pulse which is inserted to the system at time $t = t'\ s$ is called the Green's function. In the present study, without dealing with complicated numerical methods to solve the linear non-homogeneous two-point reactor kinetics equations, the corresponding Green's functions for the neutron populations and precursor concentration are generated

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analytically and used to predict the dynamical behavior of the reflected systems. Besides the corresponding expressions for the neutron population in the core and reflector regions are derived from the prompt jump approximation (PJA) method and the obtained results are compared to the results obtained from the Green's function generation method.

Material and Methods

In this section, the Green's Function Generation method and Prompt Jump Approximation method are used to solve the linear non-homogeneous form of the TPRKM.

Green's Function Generation Method

The system response to a unit neutron pulse that is introduced to the system (core region) at time $t = t'$ gives us the corresponding Green's functions for the neutron populations and precursor concentration:

$$\frac{d}{dt}\,\bar{g}(t,t') = \bar{\bar{A}}\,\bar{g}(t,t') + \bar{f}(t,t') \tag{3}$$

where

$$\bar{g}(t,t') = \begin{pmatrix} G_c(t,t') \\ G_r(t,t') \\ G_p(t,t') \end{pmatrix},$$

$$\bar{\bar{A}} = \begin{pmatrix} \frac{\rho-\beta-f_{cr}\,f_{rc}}{\Lambda_c} & \frac{f_{rc}}{l_r} & \lambda \\ \frac{f_{cr}}{\Lambda_c} & -\frac{1}{l_r} & 0 \\ \frac{\beta}{\Lambda_c} & 0 & -\lambda \end{pmatrix}$$

and

$$\bar{f}(t,t') = \delta(t-t')\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$G_p(t,t')$ represents Green's function corresponding to the precursor density, and $G_c(t,t')$ and $G_r(t,t')$ represent the Green's functions corresponding to the neutron population in the core and reflector regions, respectively.

The 3 by 3 matrix $\bar{\bar{A}}$ can be diagonalized and written in the form of:

$$\bar{\bar{A}} = \bar{\bar{P}}\,\bar{\bar{D}}\,\bar{\bar{P}}^{-1} \tag{4}$$

where

$$\bar{\bar{P}} = \begin{pmatrix} 1 & 1 & 1 \\ \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_1\right)} & \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_2\right)} & \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_3\right)} \\ \frac{\beta}{\Lambda_c(\lambda+\omega_1)} & \frac{\beta}{\Lambda_c(\lambda+\omega_2)} & \frac{\beta}{\Lambda_c(\lambda+\omega_3)} \end{pmatrix}$$

and

$$\bar{\bar{D}} = \begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{pmatrix}$$

ω_i's are the eigenvalues of the matrix $\bar{\bar{A}}$; and $\bar{\bar{P}}$ is an invertible matrix whose each column is the corresponding eigenvector of the matrix $\overline{\overline{A}}$ corresponding to eigenvalue ω_i.

System of equations (3) is a non-homogeneous linear system of differential equations with constant coefficients, and can be decoupled by setting:

$$\bar{g}(t,t') = \overline{\overline{P}}\,\bar{u}(t,t') \quad (5)$$

where

$$\bar{u}(t,t') = \begin{pmatrix} u_c(t,t') \\ u_r(t,t') \\ u_p(t,t') \end{pmatrix}$$

The corresponding system for $\bar{u}(t,t')$ becomes as follows:

$$\frac{\mathrm{d}}{\mathrm{dt}}\,\bar{u}(t,t') = \overline{\overline{D}}\,\bar{u}(t,t') + \overline{\overline{P}}^{\,-1}\bar{f}(t,t') \quad (6)$$

It is known that there are no free neutrons at the beginning of start-up. Therefore, as initial conditions, the neutron populations in the core and reflector regions and precursor concentration are taken equal to zero. Hereby, due to initial conditions, $\bar{u}(0,t')$ becomes equal to zero and decoupled system of differential equations (6) is solved as follows:

$$u_c(t,t') = \frac{(\omega_3-\omega_2)\left(\frac{1}{l_r}+\omega_1\right)(\lambda+\omega_1)\,e^{-\omega_1(t'-t)}}{\chi(\omega_1,\omega_2,\omega_3)} \quad (7)$$

$$u_r(t,t') = \frac{(\omega_1-\omega_3)\left(\frac{1}{l_r}+\omega_2\right)(\lambda+\omega_2)\,e^{-\omega_2(t'-t)}}{\chi(\omega_1,\omega_2,\omega_3)} \quad (8)$$

$$u_p(t,t') = \frac{(\omega_2-\omega_1)\left(\frac{1}{l_r}+\omega_3\right)(\lambda+\omega_3)\,e^{-\omega_3(t'-t)}}{\chi(\omega_1,\omega_2,\omega_3)} \quad (9)$$

where

$$\chi = \omega_3^2(\omega_2-\omega_1) + {\omega_2}^2(\omega_1-\omega_3) + \omega_1^2(\omega_3-\omega_2)$$

Using the expression given in equation (5) the Green's functions are resulted as follows:

$$G_c(t,t') = u_c(t,t') + u_r(t,t') + u_p(t,t') \quad (10)$$

$$G_r(t,t') = \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_1\right)}u_c(t,t') + \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_2\right)}u_r(t,t') + \frac{f_{cr}}{\Lambda_c\left(\frac{1}{lr}+\omega_3\right)}u_p(t,t') \quad (11)$$

$$G_p(t,t') = \frac{\beta}{\Lambda_c(\lambda+\omega_1)}u_c(t,t') + \frac{\beta}{\Lambda_c(\lambda+\omega_2)}u_r(t,t') + \frac{\beta}{\Lambda_c(\lambda+\omega_3)}u_p(t,t') \quad (12)$$

Using the resulting Green's functions, the time-dependent neutron population and precursor concentrations are obtained from the following integrals:

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$$N_c(t) = \int_{t'=0}^{t} S(t')\, G_c(t,t')\, dt \qquad (13)$$

$$N_r(t) = \int_{t'=0}^{t} S(t')\, G_r(t,t')\, dt \qquad (14)$$

$$C(t) = \int_{t'=0}^{t} S(t')\, G_p(t,t')\, dt' \qquad (15)$$

Prompt Jump Approximation

In the nuclear reactors, prompt neutron average lifetime changes from $10^{-7}s$ to $10^{-5}s$ whereas the delayed neutron precursor average lifetime varies from $10^{-2}s$ to $10^{+2}s$. The effective lifetime of the neutrons is given in Equation (16) where the average lifetime of delayed neutron is roughly taken equal to precursor lifetime.

$$\bar{l}_{eff} = (1-\beta)\bar{l}_{prompt} + \beta\frac{1}{\lambda} \qquad (16)$$

The nuclear reactors are designed to operate in delayed critical and prompt subcritical conditions. In prompt supercritical ($\rho > \beta$) conditions the prompt neutrons alone make the reactor supercritical and due to small lifetime of the prompt neutrons the reactor control with mechanical equipment becomes almost impossible. In contrast, in both delayed critical ($\rho = 0$) and delayed supercritical ($\rho < \beta$) operation conditions the prompt neutrons alone are not sufficient to sustain the fission chain reactions. In these conditions, the delayed neutrons even with small fractions increase the effective lifetime of the neutrons, subsequently slow down the system behavior, and make the reactor control easier [18]. In the case of the subcritical source-driven systems the fission chain reactions are sustained by the external neutron sources.

In the case of any reactivity insertion into an initially critical system, which is operated in any condition except the prompt supercritical condition, we experience a sudden change with a sharp slope in the reactor power for a short time domain. This effect is known as prompt jump and caused due to differences between precursors and prompt neutrons' lifetimes. In a short time domain, change of the precursor concentration due to inserted reactivity is negligible, and delayed neutrons will still be emitted at a rate determined by earlier conditions. In contrast, the prompt neutron population will be fully adjusted to the new multiplication factor. Therefore, the flux will change to a level that is allowed by the new multiplication of prompt neutrons plus the generation of delayed neutrons based on the old conditions. By passing the time, the precursors start to adjust to new conditions and subsequently cause to decrease in the slope of power change and finally, after a transient time, the precursors and neutrons will have an asymptotic exponential behavior in time.

It is known that for the cases that ρ is less than β, the neutron generation time is almost equal to neutron lifetime. In the Prompt Jump Approximation method, the rapid change in power after reactivity insertion due to prompt neutrons generations is neglected; and $\Lambda_c\ (dN(t)/dt\,|_{t0})$ is taken equal to zero, where t_0 represents the reactivity insertion time [19-20] .

In case of two-point reactor kinetics model by taking both $\Lambda_c\ (dN_c(t)/dt\,|_{t=0})$ and $\Lambda_c\ (dN_r(t)/dt\,|_{t=0})$ equal to zero, and after some mathematical operations the neutron population in the core and reflector regions are obtained as follows:

$$N_c(t) = \frac{\Lambda_c}{\rho(t)-\beta}\left[-S(t) + S(t_0-)e^{-\int_{t_0}^{t}\frac{\lambda\rho(t)}{\rho(t)-\beta}dt}\right] + \frac{\rho(t_0-)-\beta}{\rho(t)-\beta}e^{-\int_{t_0}^{t}\frac{\lambda\rho(t)}{\rho(t)-\beta}dt}N_c(t_0) + \frac{\lambda\,\Lambda_c}{\rho(t)-\beta}e^{-\int_{t_0}^{t}\frac{\lambda\rho(t)}{\rho(t)-\beta}dt}\times \left[\int_{t_0}^{t}\frac{\beta}{\rho(t)-\beta}\,S(t)\left[e^{-\int_{t_0}^{t}\frac{\lambda\rho(t)}{\rho(t)-\beta}dt}\right]dt\right] \quad (17)$$

$$N_r(t) = \frac{f_{cr}\,l_r}{\Lambda_c}\,N_c(t) \quad (18)$$

Precursor concentration is also obtained in the form of:

$$C(t) = -\frac{\rho(t)-\beta}{\lambda\,\Lambda_c}\,N_c(t) - \frac{S(t)}{\lambda} \quad (19)$$

Results and Discussions

As a study case, the zero-power research reactor PROTEUS consists of a relatively small core (about 1 cubic meter) surrounded by a thick graphite reflector is taken into consideration [9,13]. The kinetics parameters in subcritical condition are presented in table (1). Using the expression which relates the system reactivity with the coupling parameters, the system reactivity becomes equal to -0.003.

Table 1. *The kinetics parameters for the subcritical reflected reactor.*

$\boldsymbol{\rho_\infty}$	0.397
$\boldsymbol{\Lambda_c(ms)}$	0.4
$\boldsymbol{l_r\ (ms)}$	4.0
$\boldsymbol{f_{rc}}$	0.5
$\boldsymbol{f_{cr}}$	0.8
$\boldsymbol{\lambda\ (s^{-1})}$	0.07696
$\boldsymbol{\beta}$	0.00723

As the first test problem, it is assumed that at $t = 0\ s$ an external neutron source of the strength of $200\,n/s$ is inserted into the system. The time-dependent neutron population in the core and reflector regions as well as the time-dependent precursor concentration are calculated from the Green's function generation method and presented in equations (20) through (22).

$$N_c(t) = 26.667 - 0.12694\, e^{-1270.5\,t} - 7.5284\, e^{-5.086\,t} - 19.011\, e^{-0.02233\,t} \quad (20)$$

$$N_r(t) = 213.33 + 0.24877\, e^{-1270.5\,t} - 61.478\, e^{-5.086\,t} - 152.1\, e^{-0.02233\,t} \quad (21)$$

$$C(t) = 6262.99 + 0.001806\, e^{-1270.5\,t} + 27.166\, e^{-5.086\,t} - 6290.16\, e^{-0.02233\,t} \quad (22)$$

Using the prompt jump approximation, the results are as follows:

$$N_c(t) = 26.667 - 18.847\, e^{-0.02257\,t} \quad (23)$$

$$N_r(t) = 213.33 - 150.77\, e^{-0.02257\,t} \quad (24)$$

$$C(t) = 6262.99 - 6262.99\, e^{-0.02257\,t} \quad (25)$$

As it is seen, in compare with Green's function generation method, in the prompt jump approximation method the two exponential terms with more negative exponents are omitted, and only the dominant exponential term remains. Due to the problem subcriticality, the attenuation rate of the remained term is less than the other two terms. In the case of positive reactivity insertion, the exponent of the dominant exponential term is positive in which by increasing the reactivity its value goes up.

The time evolutions of the neutron population in the core and reflector as well as the precursor concentration time evolution are plotted in Fig. (1). It is observed that the neutron population in the core and reflector regions are calculated as 7.82 and 62.56, respectively, with the PJA method at $t = 0\ s$.

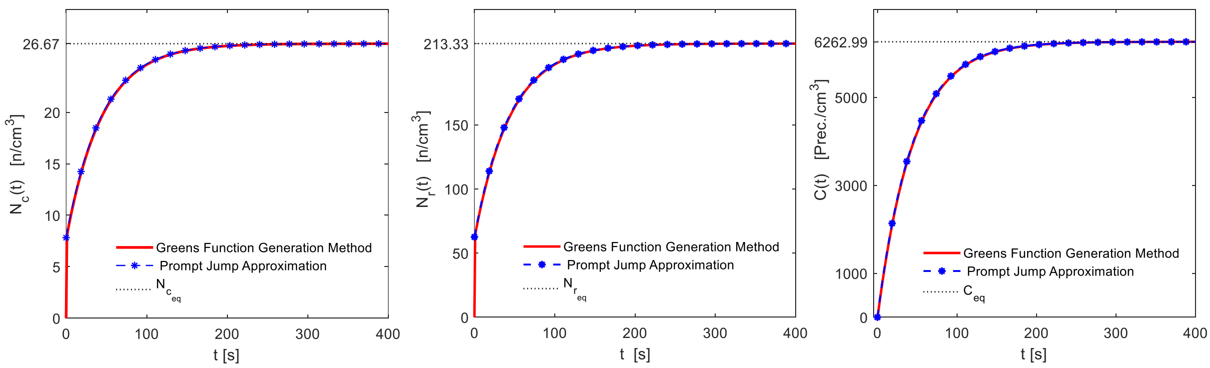

Figure 1. *Neutron population and precursor concentration for the first test case*

It is known that for a constant neutron source, the system reaches the constant equilibrium condition after a long time. Under the equilibrium condition, from the system of equations (2), $N_{c_{eq}}$ becomes equal to $-(\Lambda_c\, S)/\rho$. Subsequently $N_{r_{eq}}$ and C_{eq} become equal to $(l_r f_{cr}/\Lambda_c)N_{c_{eq}}$ and $(\beta/\lambda\Lambda_c)N_{c_{eq}}$, respectively. As it is seen in Fig. (1) the results obtained from the Green's function generation method and PJA method asymptotically reach to the equilibrium conditions. This, in turn, indicates the correctness of our proposed solution methods.

The differences between the neutron populations obtained from the Green's function generation method and PJA method, in the core and reflector regions, are plotted in Fig. (2). It is seen that these differences asymptotically tend to zero. Therefore, if we are interested in long time behavior of the nuclear reactors the PJA method can be an efficient method.

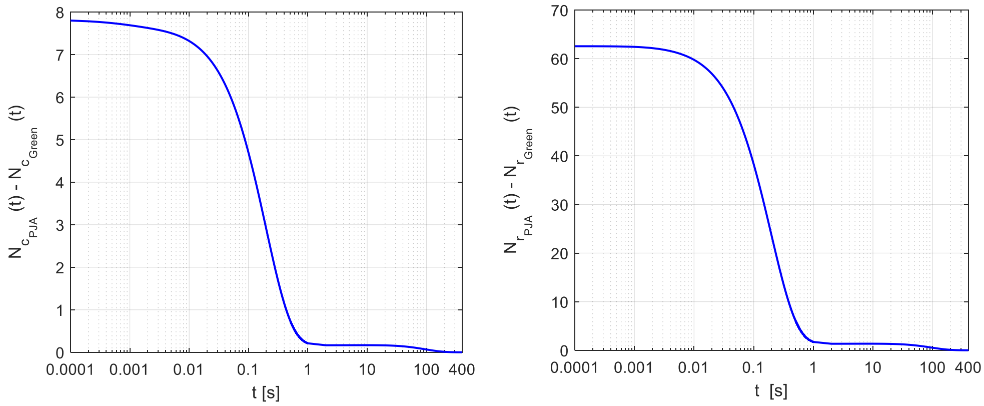

Figure 2. *The differences between neutron populations obtained from the Green's function generation method and PJA method in the core and reflector regions*

As the second test problem, it is assumed that at $t = 0\ s$ a time-dependent external source, $S(t) = 200\ e^{0.005\ t}\ n/s$, is inserted into the system. As seen in Fig. (3) the Green function generation method and PJA method are in good agreement. It is seen that despite the system subcriticality the power and reflector region neutron population increase with time. This increase is due to the increase in external neutrons with time.

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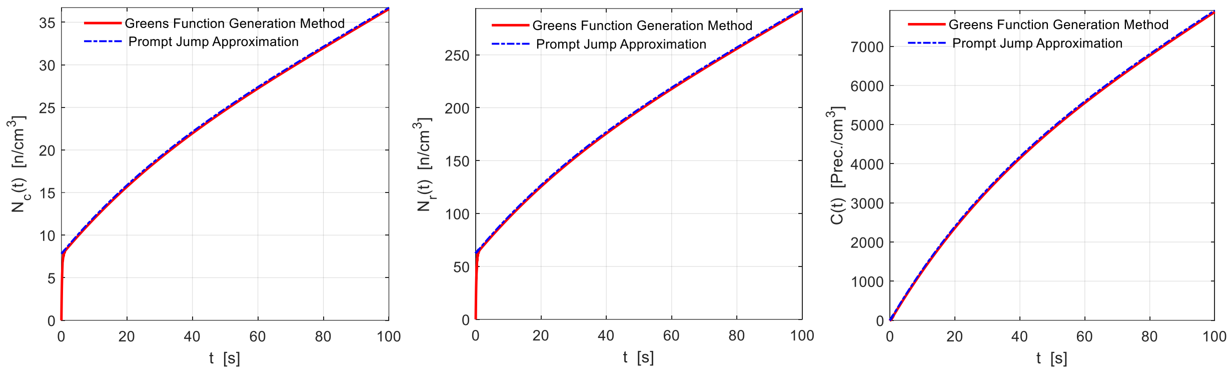

Figure 3. *Neutron population and precursor concentration for the second test case*

The Green's function generation method is also used to obtain the neutron population in the core and reflector regions for different f_{rc} values where the $f_{cr}(= 0.8)$ is kept constant. As it is seen in Fig. (4), any increase in f_{rc} causes an increase in the reactivity and then an increase in neutron population.

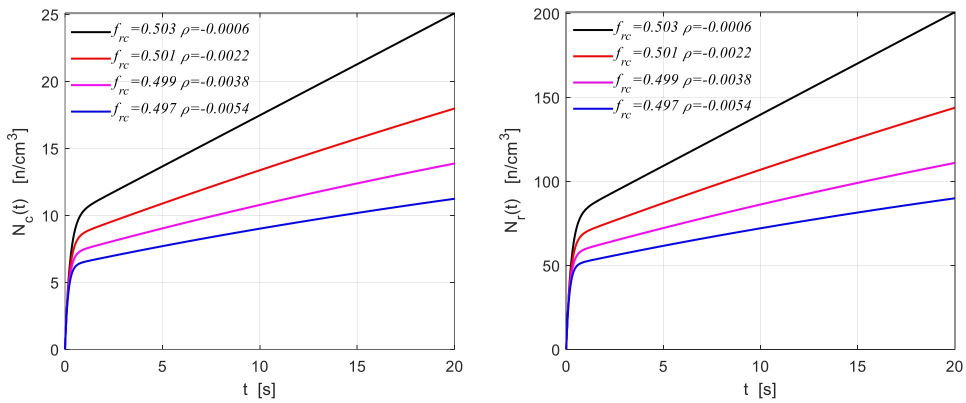

Figure 4. *Neutron population with varying* f_{rc}

Conclusion

The dynamical behavior of the subcritical source-driven reflected reactor during the reactor start-up is investigated by solving two-point reactor kinetics equations. In deterministic numerical solution methods, the time domain is divided into small time-intervals in which the non-homogeneous system of differential equations is solved in each time-interval and used to estimate the dynamical behavior of the source-driven reflected reactors. In this work, without dealing with such cumbersome solution methods, the corresponding Green's function for the neutron population and precursor concentration are derived and used to predict the dynamical behavior of

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the reflected systems. In the present study, the neutron populations in the core and reflector regions and precursor concentration are also calculated using the Prompt Jump Approximation method in which the rapid change in power after reactivity insertion due to prompt neutrons generations is neglected. Although the PJA method does not predict the dynamical behavior in the prompt jump region it is seen that obtained results for the dynamical behavior of the exponential region from both PJA and Green's Function Generation methods are in good agreement with each other; the difference between the results of these methods asymptotically tends to zero. In addition, in the case of a neutron source with constant intensity, the dynamical behavior of the system asymptotically reaches the constant equilibrium condition where both PJA and Green's function generation methods exactly predict the asymptotic values. These results are strong evidence for the validity of the proposed solution methods.

Although the Green's function generation method is useful to solve the linear non-homogeneous system of differential equations, in the presence of the temperature reactivity feedback mechanism, the TPRKM converts to a non-linear and non-homogeneous system of differential equations, and Green's function generation method is unable to solve such problems.

References

[1] Polo-Labarrios MA, Espinosa-Paredes G, 2012. Application of the fractional neutron point kinetic equation: Start-up of a nuclear reactor. Annals of Nuclear Energy, 46: 37-42.

[2] Polo-Labarrios MA, Espinosa-Paredes G, 2012. Numerical analysis of startup PWR with fractional neutron point kinetic equation. Progress in Nuclear Energy, 60: 38-46.

[3] Sathiyasheela T, 2010. Inhomogeneous point kinetics equations and the source contribution. Nuclear engineering and design, 240(12): 4083-4090.

[4] Henry AF, 1975. Nuclear Reactor Analysis. MIT Press, Cambridge, MA.

[5] Duderstadt JJ, 1976. Nuclear reactor analysis. Wiley.

[6] Goluoglu S, Dodds HL, 2001. A time-dependent, three-dimensional neutron transport methodology. Nuclear Science and Engineering, 139(3): 248-261.

[7] Ball TA, 2017. The Inverse Kinetics Method and Its Application to the Annular Core Research Reactor. MSc. Thesis, The University of New Mexico.

[8] Spriggs GD, Busch RD, Williams JG, 1997. Two-region kinetic model for reflected reactors. Annals of Nuclear Energy, 24(3): 205-250.

ISSN: 2536-4383

[9] Aboanber AE, Nahla AA, 2018. Mathematical treatment for two-point reactor kinetics model of reflected systems. Progress in Nuclear Energy, 105: 287-293.

[10] Wang X, Huff KD, Aufiero M, Peterson PF, Fratoni M, 2016, June. Coupled reactor kinetics and heat transfer model for fluoride salt-cooled high-temperature reactor transient analysis. In International Conference on Nuclear Engineering (Vol. 50053, p. V005T15A055). American Society of Mechanical Engineers.

[11] Avery R, 1958. Theory of coupled reactors (No. A/CONF. 15/P/1858). Argonne National Lab., Lemont, Ill.

[12] Cohn CE, 1962. Reflected-reactor kinetics. Nuclear Science and Engineering, 13(1): 12-17.

[13] Van Dam H, 1996. Inhour equation and kinetic distortion in a two-point reactor kinetic model. Annals of Nuclear Energy, 23(14): 1127-1142.

[14] Aboanber AE, El Mhlawy AM, 2009. Solution of two-point kinetics equations for reflected reactors using Analytical Inversion Method (AIM). Progress in Nuclear Energy, 51(1): 155-162.

[15] Aboanber AE, El Mhlawy AM, 2008. A new version of the reflected core inhour equation and its solution. Nuclear Engineering and Design, 238(7): 1670-1680.

[16] Aboanber AE, 2009. Exact solution for the non-linear two-point kinetic model of reflected reactors. Progress in Nuclear Energy, 51(6-7): 719-726.

[17] Holschuh TV, Marcum WR, Palmer TS, 2017. One-group analytical solution to two-region reactor kinetic model. Annals of Nuclear Energy, 99: 199-205.

[18] Pázsit I, 1996. On the kinetic response of a reactor with delayed neutrons. Annals of Nuclear Energy (Oxford), 23(4-5): 407-412.

[19] Almenas K, Lee R, 2012. Nuclear engineering: an introduction. Springer Science & Business Media.

[20] Oka Y, Suzuki K, 2013. Nuclear Reactor Kinetics and Plant Control (Vol. 10, pp. 978-4). Tokyo: Springer.